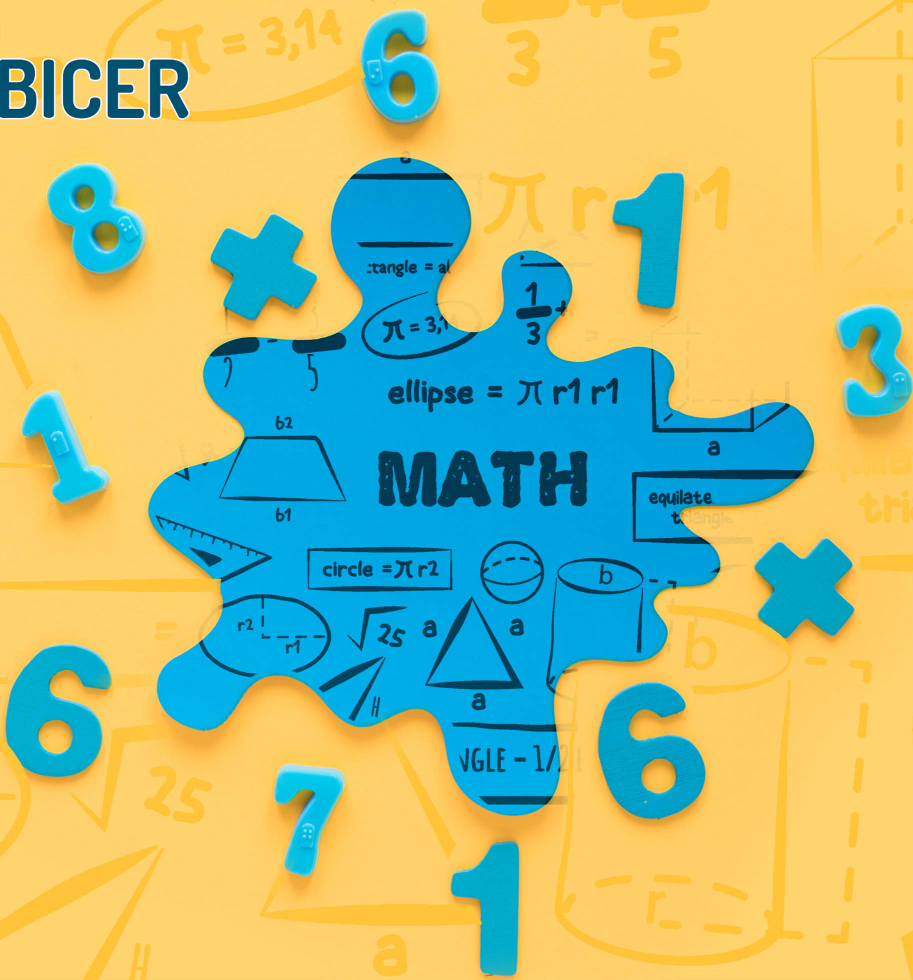
CREATIVITY-DIRECTED MATHEMATICS TASKS FOR 5TH GRADE COMMON CORE CLASSROOMS

EDITOR
DR. ALI BICER



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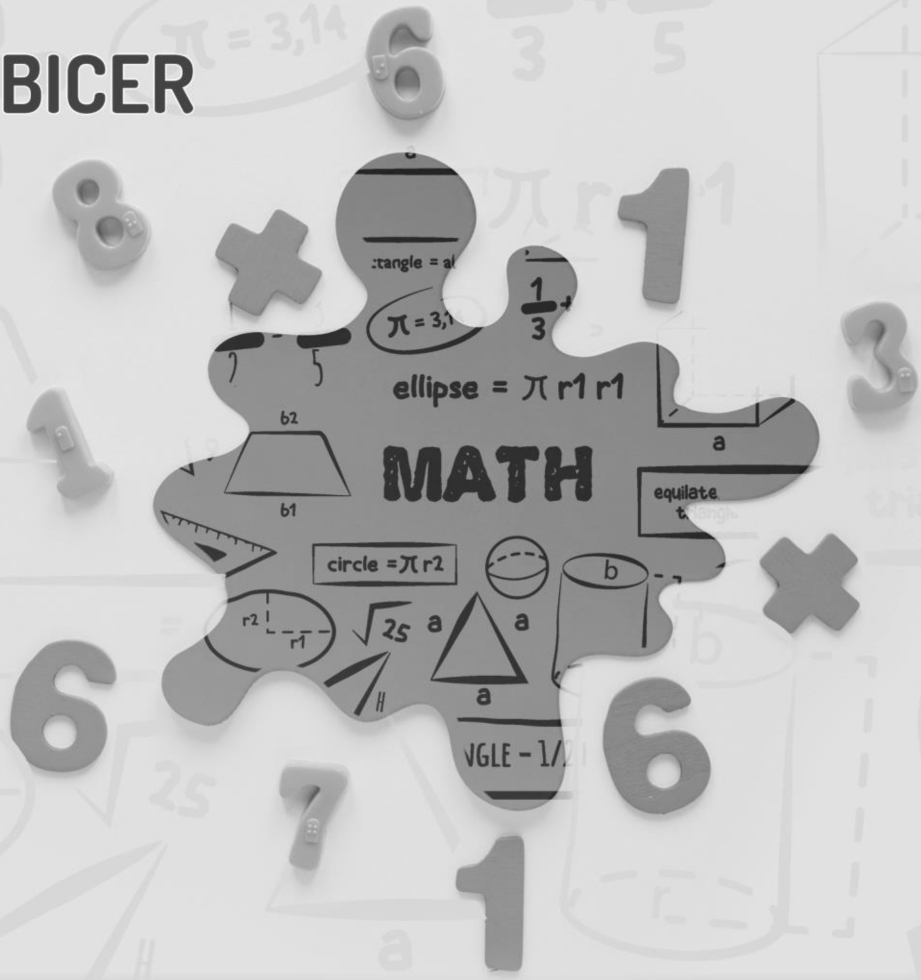

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Creativity-Directed Mathematical Tasks for 5th Grade Common Core Classrooms

Editor
Dr. Ali Bicer, University of Wyoming, United States

Cover Designed by
Mevlut Unal, Necmettin Erbakan University, Turkiye

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istesoffice@gmail.com

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Editor

Dr. Ali Bicer is an assistant professor in the School of Teacher Education at the University of Wyoming (UW), where he teaches undergraduate and graduate courses in mathematics education. His research interests have centered on mathematical creativity, creativity-directed tasks, STEM education, and writing in mathematics. Dr. Bicer received his Ph.D. in Curriculum and Instruction with a mathematics specialization from Texas A&M University in 2016. After receiving his Ph.D., he worked as a postdoctoral research assistant in Aggie STEM at Texas A&M University until 2018. Prior to starting his master's and doctoral program at Texas A&M University, he taught mathematics for three years in secondary and middle schools. Dr. Bicer currently received the Early Promising Faculty Award at the University of Wyoming. He published his research results in top-tier journals: Educational Studies in Mathematics (ESM), Thinking Skills and Creativity, Journal of Creative Behavior, ZDM-Mathematics Education, and Mathematics Education Research Journal. He is an international acquisition editor of the Journal of Urban Mathematics Education.

Authors

Traci Jackson is a National Board Certified Teacher in Mathematics. She is dual credentialed in mathematics and science and taught grades K-12 for over 16 years in various combinations of mathematics and science. Traci developed a hands-on science curriculum for K-5 students, designed a science lab, and facilitated hands-on science lessons for elementary students for 8 years. In Traci's current position as a Secondary Math Teacher on Special Assignment for the Poway Unified School District, she facilitates professional learning for secondary math teachers as well as continues to teach an Integrated 2 high school math course. Traci was the 2016 Recipient of the Rosenthal Award for Teaching Innovation and Inspiration in Mathematics for her lesson Creating Color Combos focusing on proportional reasoning. She is passionate about helping people see themselves as curious mathematicians. Currently Traci is a doctoral student studying mathematics education at the University of Wyoming with a research focus on curiosity in mathematics classrooms.

Michael Gundlach is a mathematics teacher at South High School in Cheyenne, Wyoming, where he teaches concurrent enrollment mathematics classes along with regular high school classes. Past teaching experience includes teaching at both the high school and community college level. While working at the community college level, Michael specialized in preparing prospective elementary teachers to teach mathematics. He has a bachelor's degree in mathematics education from Brigham Young University and a master's degree in mathematics from Michigan State University. Currently, Michael is pursuing a doctorate degree in curriculum and instruction with a concentration in mathematics education from the University of Wyoming. His research interests are in the intersection of authentic educational experiences, mathematical knowledge for teaching, and improving calculus instruction.

Jennifer Kellner is a Professor of Mathematics at Central Wyoming College (CWC), where she teaches a variety of community college level courses in mathematics and statistics. Jennifer is currently pursuing her Doctorate of Education in Mathematics Education from the University of Wyoming (UW). Jennifer received her Master of Education Degree in Secondary Curriculum and Instruction in Mathematics from Chadron State College in 2008. She has taught mathematics at Central Wyoming College for over 15 years where she started with the Adult Education and High School Equivalency Program. Jennifer received the Excellence in Teaching Award in 2017 for the National Society of Leadership and Success. In 2022, she completed the Academic Management Institute leadership training.

Melena Osborne is a Mathematics Instructor at Central Wyoming College (CWC), where she teaches mathematics and mathematics education courses. Melena received her BS (2012) and MS (2016) in Mathematics from Tarleton State University (TSU). Additionally, she received her MA in Teaching Mathematics from Western Governors University in 2017. Currently, Melena is working on her Ed.D. in Mathematics Education at the University of Wyoming. She has a passion for changing students' mindset about mathematics, particularly students placed into developmental mathematics courses, and her current research interests include finding pathways to prevent developmental mathematics courses from being barrier courses for students.

Michelle Tudor is an Assistant Principal at Northern Elementary in Lexington, KY. She is currently a doctoral student in Mathematics Education at the University of Wyoming. In addition to her administrative role, Michelle also coordinates interventions services for students, works with teachers to plan math instruction and teaches 1st grade math groups. Before moving to administration, Michelle taught 5th grade for 10 years. Her research interests include building foundational math skills for students from disadvantaged backgrounds.

Fay Quiroz is a 5th grade teacher pursuing her doctoral degree in mathematics education at the University of Wyoming. Her research interests center on her mathematics intervention process and how it is interrelated to mathematics creativity. Fay received her master's degree in Curriculum and Instruction with a reading endorsement, and her undergraduate degree in elementary education from the University of Wyoming. She has taught for 15 years in various positions as a teacher, and private tutor, in pre-school, second, fourth and fifth grades. In her free time she enjoys cooking, reading and camping with her family.

Helen Aleksani is a first year PhD student in Mathematics Education at the University of Wyoming. She received a bachelor's degree in mathematics from California State University of Stanislaus, master's degree in educational technology from California State University of Stanislaus, and master's degree in mathematics education from Northern Arizona University. She is currently a math instructor, department chair, and an instructional coach at Waterford High School in California. She is also a lecturer at California State University of Stanislaus. She was awarded the 2015 Teacher of the Year award of Stanislaus County. She is interested in teacher educational programs and curriculum preparation in mathematics..

Geoff Krall is a PhD student and teacher of mathematics education at University of Wyoming. Geoff is an experienced educator, blogger, author, speaker, professional developer, student, and mathematician. He taught math in Texas for eight years before moving to Colorado to obtain a master's degree. After that, he joined the New Tech Network (NTN) of schools as a Math Coach, delivering coaching, training, and professional development to teachers across the country. While there he pioneered Problem-Based Learning as the primary mode of instruction at NTN schools, training hundreds of teachers and dozens of colleagues along the way. He lives in Fort Collins with his wife, two kids, two cats, and four chickens. His first book, *Necessary Conditions: Teaching Secondary Math with Academic Safety, Quality Tasks, and Effective Facilitation* was published in 2018 and is currently available from Stenhouse Publishers.

Chuck Butler is an instructional coach and secondary mathematics department chair at Memphis Teacher Residency (MTR), a non-profit organization that recruits, trains, and supports teachers to work in under-resourced schools in Memphis, TN. He also serves as the STEM program coordinator, where he supports the Marjorie Lee Browne STEM Education Fellowship and the STEM Discovery Camp. One of the goals of The Browne STEM Fellowship is to recruit college students of color to teach in Memphis, initially through teaching at a math-focused, middle school STEM camp. Prior to working at MTR, Chuck also taught 9-12 mathematics for ten years.

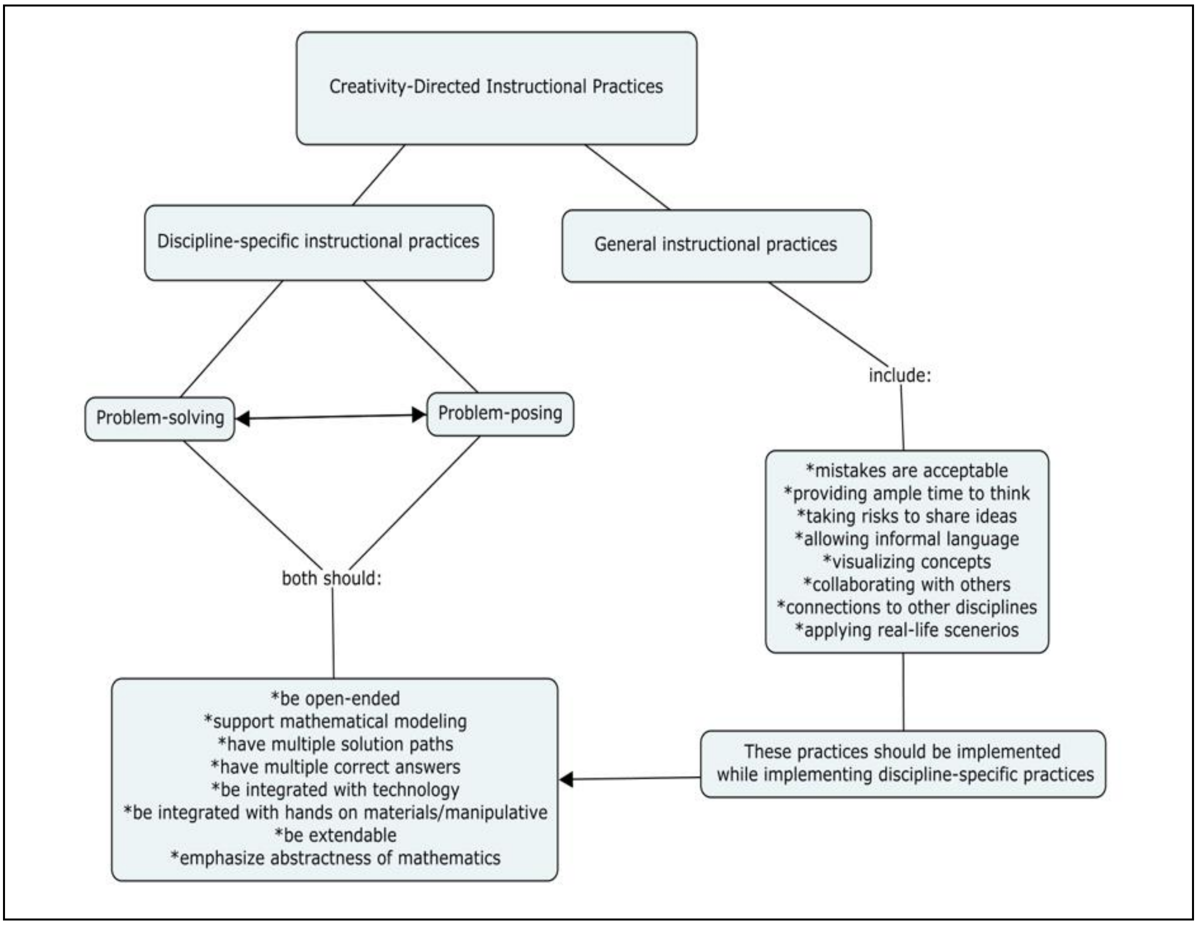
Aylin S. Carey is an education and mathematics adjunct lecturer at Laramie County Community College (LCCC) at Albany County Campus (ACC). She has a Master of Arts in Education with a concentration in Curriculum and Instruction, a certification in Online Instruction, a Post-Baccalaureate Certification in Secondary Mathematics Education and a Bachelor of Science in Mathematics from University of Wyoming (UW). She served three years as a chair in the Accountability Committee for Laramie Montessori School (LMS) and one year as a vice chair in the LMS Board of Trustees. She is currently pursuing a doctoral degree in Mathematics Education at UW. At the conclusion of intensive training in Mathematics Education and receiving her doctoral degree, she will have a strong foundation for her continued pursuit of becoming a professor and work in academia. Her research areas include diversity and equity in mathematics education, students' affect toward mathematics, mathematical creativity, and online mathematics learning.

Amy Kassel is currently the Secondary Mathematics Curriculum Coordinator and mathematics instructional coach for Laramie County School District 1 in Cheyenne, Wyoming. She is a National Board Certified teacher and taught high school mathematics for 26 years. She has completed the Math Teacher Leadership program at the University of Northern Colorado, earned the Arch Coal Teacher Achievement Award, and is the recipient of the 2017 Presidential Award for Excellence in Mathematics and Science Teaching. She earned her B.S.Ed. in Mathematics from Southeast Missouri State University, M.S. in Mathematics from Western Illinois State University, and is currently a Ph.D. student in Mathematics Education at the University of Wyoming.

Table of Contents

INTRODUCTION

This book offers creativity-directed tasks that aim to develop students' creative thinking skills while they construct new mathematical ideas, knowledge, and concepts. The creativity-directed tasks were adopted from Bicer's (2020) framework (see Figure below).



This book is for any teacher and parent who want to engage their fifth-grade students in creative thinking in mathematics by implementing a meaningful combination of some discipline-specific and general instructional practices within a mathematical task/lesson. The goal of this book is to combine creativity in mathematics and CCSS-M content and practice standards as we illustrated in each lesson plan. It is important to note that CCSS-M practice standards "do not dictate curriculum or teaching methods" (NGACPB & CCSSO, 2014c, p. 5). Rather, they provide the context for teachers so that they can apply whatever curricular approaches they follow. Beghetto and Kaufman (2014) argued that whether creativity is supported or suppressed in classrooms depends to a greater extent on how students experience the messages that are sent by the tasks and a particular classroom environment than on what curriculum their teachers follow. Several common instructional practices and the way teachers inadvertently implement mathematical tasks constrain student creativity in mathematics classrooms. This book offers several 5th-grade mathematics lessons that enable teachers to encourage their students to truly think creatively while they cover 5th-grade CCSS-M content and practice standards. Each lesson in this book emphasizes different practices (e.g., visualization, generalization, problem-posing) to foster 5^{th} graders' creative thinking in mathematics.

References

Beghetto, R. A., Kaufman, J. C., & Baer, J. (2014). *Teaching for Creativity in the Common Core Classroom*. New York, NY: Teachers College Press.

Bicer, A. (2021). A systematic literature review: Discipline-specific and general instructional practices fostering the mathematical creativity of students. *International Journal of Education in Mathematics, Science, and Technology (IJEMST), 9*(2), 252-281. https://doi.org/10.46328/ijemst.1254

Common Core State Standards Initiative (CCSSM). (2010). *Common Core State Standards for Mathematics.* National Governor's Association (NGA) & Council of Chief State School Officers Retrieved on April 10, 2020 from http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf.

Citation

Bicer, A. (2023). Introduction. In A. Bicer (Ed.), *Creativity-Directed Mathematical Tasks for 5^{th} Grade Common Core Classrooms* (pp. 1-2). ISTES Organization.

SECTION 1 - WRITE AND INTERPRET NUMERICAL EXPRESSIONS

Task 1 - Headbands

Michelle Tudor, Melena Osborne, Michael Gundlach

Mathematical Content Standards

CCSS.MATH.CONTENT.5.OA.A.1

Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.

CCSS.MATH.CONTENT.5.OA.A.2

Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. For example, express the calculation “add 8 and 7, then multiply by 2” as $2 \times (8 + 7)$. Recognize that $3 \times (18932 + 921)$ is three times as large as 18932 + 921, without having to calculate the indicated sum or product.

Mathematical Practice Standards

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Model with mathematics.

Materials

Note cards for the game (see Appendix A), white boards, dry erase markers, post-it notes, and chart paper.

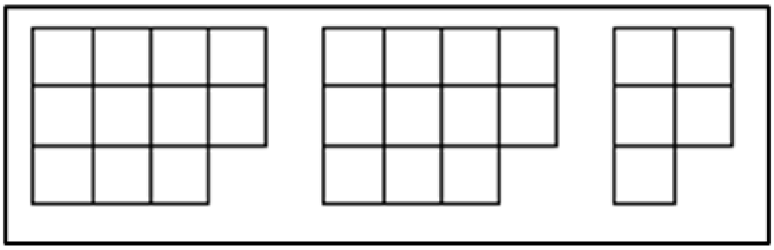
Lesson Objective

Students will gain a conceptual understanding of the order of operations by connecting images to numerical expressions. This will be done by having students create an image for a given expression and create an expression for a given image. Students will then be asked to create an image and expression with a small group to demonstrate understanding. This is intended to be a 2 day lesson.

Engagement

(2 minutes) Students will play the game “Headbands”. Half of the students will be given a note card with an image on it and half will be given a note card with a numerical expression on it. The object of the game is to find the person who has the match for your card. Students place the card on their forehead (or on a headband if the teacher has them) and walk around the classroom trying to find the student with the corresponding image or expression. This can be difficult for some students, so listen for frustration after a minute or two and call the students back together.

Example note cards:



$[(3 \times 3) + 2] \times 2 + 5$

See Appendix A for more image and expression cards

Explain

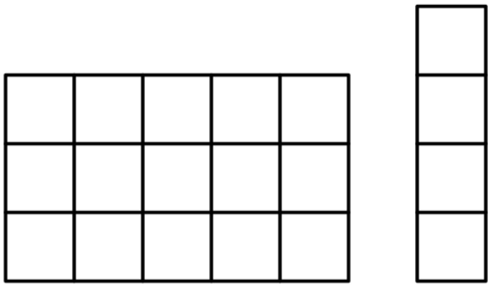
(15 minutes) Bring students back together, ask them what was difficult about that task. Continue to ask questions that lead them to understand, they need to find the matching image or expression on their own, before looking for their partner. The teacher will have examples of expressions and images on the board. (See examples below) Teacher will model how to draw an image for an expression or write an expression for an image. Be sure to model more than one image for the same expression (and more than one expression for the same image). Students will complete one of the examples on a white board to check for understanding and

the class will discuss the images and the expressions to clarify any misconceptions. To check if students understand the concept, they will be sent to their desks to start the game again. If they are still struggling to understand, model again and have students color the image to allow them to see the parts of the equation. Check for understanding again.

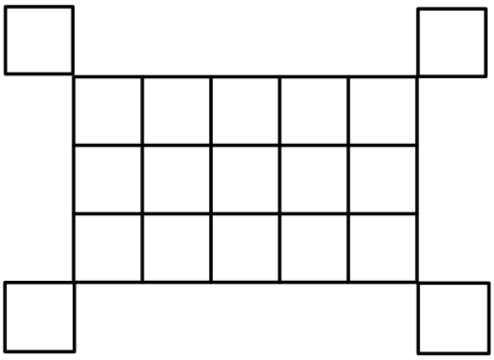
- Example 1 Possible Student Answers (Explain)
  For the given expression, create an image that represents the expression.

$$[3 \times 5] + 4$$

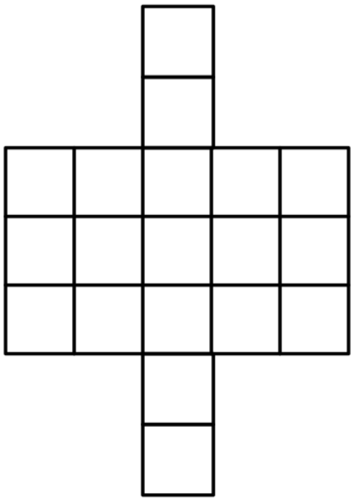
Student 1



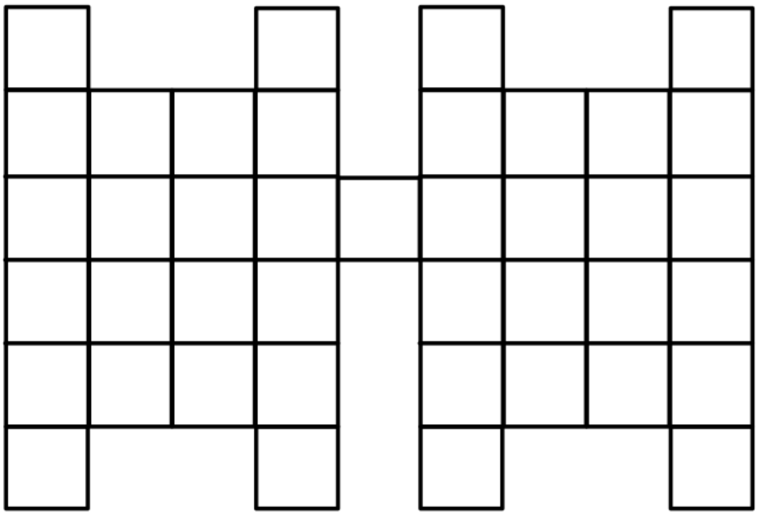
Student 2



Student 3



- Example 2 Possible Student Answers (Explain)
  For the given image, create an expression that represents the image.



Student 1

$$[(4 \times 4) + 4] \times 2 + 1$$

Student 2

$$(4 \times 5) \times 2 + 1$$

Student 3

$$[(6 \times 4) - 4] \times 2 + 1$$

Explore

(10 minutes**)** Students will determine the image or expression that matches what is on their note card and then begin looking for their partner. The teacher will circulate during this time to 1). Make sure students have the correct image or expression to look for and 2). Listen to student conversation to assess understanding and address misconceptions. After students find their partner, ask them to confirm that the expression and the image are a match.

Extend

(60 minutes/ divided between 2 class periods) Students will be introduced to a real-life problem they are going to solve. Each week at our school, students from each class are selected to be a PrinciPal. The PrinciPals get a certificate and have their picture taken with Ms. Jones, the principal. Ms. Jones would like to display all the pictures in the hallway by grade level. Each grade level has different numbers of PrinciPals, so she needs ideas for how to make a nice display. Since Ms. Jones loves math, she wants the display to have a mathematical expression to describe it.

Task: Students will be placed into groups of 3 students and assigned a grade level. The groups will be given squares of paper to represent the pictures they need to arrange. Students will work together to create a design for the display. Once the students have created a design, each group will share their designs with the whole class. Students will then be given the following question: How do you know which group has the most PrinciPals *without* counting each individual picture?

Each group will have a few minutes to discuss this question. The hope is that students will realize they can create numerical expressions to quickly find which group has the most PrinciPals. Once the students have created numerical expressions for their peers' designs, each group will then need to create a unique numerical expression for their own design to display with the design on a poster. The poster will be displayed around the room and the students will do a gallery walk to see what other groups have done. To make it more real-life, you could invite the principal to the gallery walk.

Evaluate

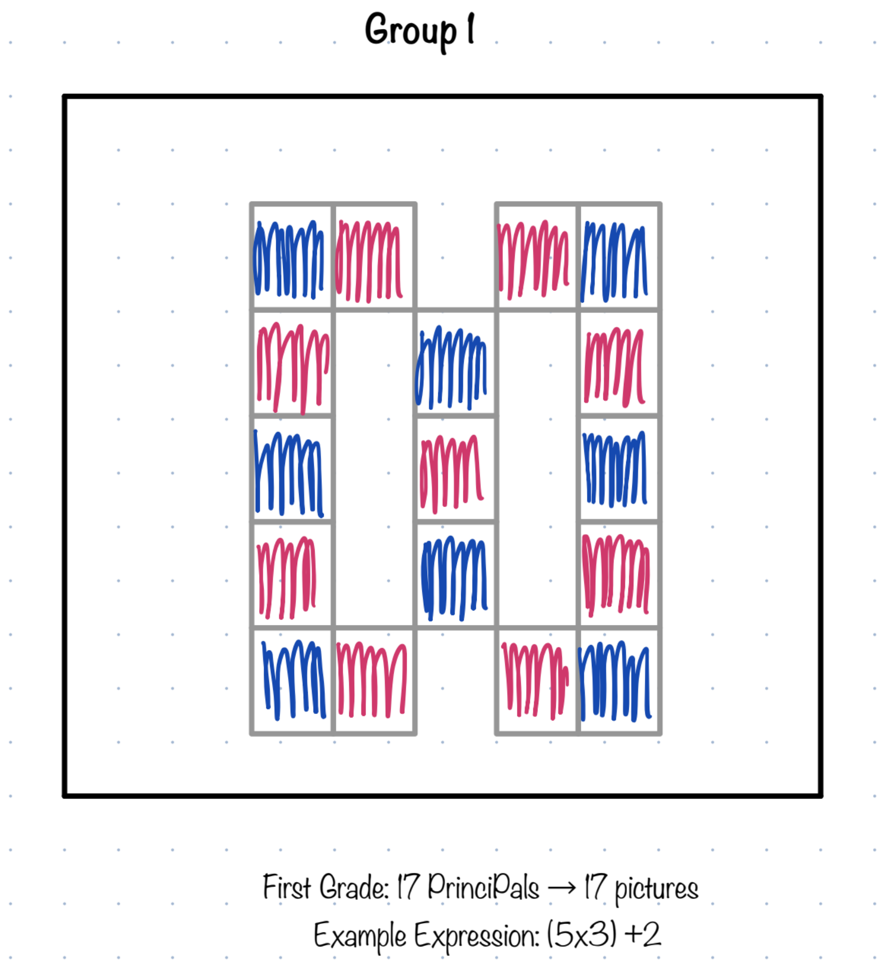
During the headband game, the teacher should check students' matches for accuracy. Also, listen to student discussion for understanding and/or confusion. If a student is having trouble with the headband game, have them try to come up with their own expression/image that corresponds to whichever they have. Once they have done this, they may be able to identify their match. If he/she is still struggling, show the student some different examples of matches.

It is important for the teacher to look for multiple designs and solutions during the Extend part of the lesson. Since the students are working in groups, encourage each student to identify unique expressions for the design their group created. If students are having a difficult time creating an expression that corresponds to their design, the teacher can suggest creating a new design that may be easier to create an expression for. The teacher could also allow different groups to collaborate to get ideas from one another.

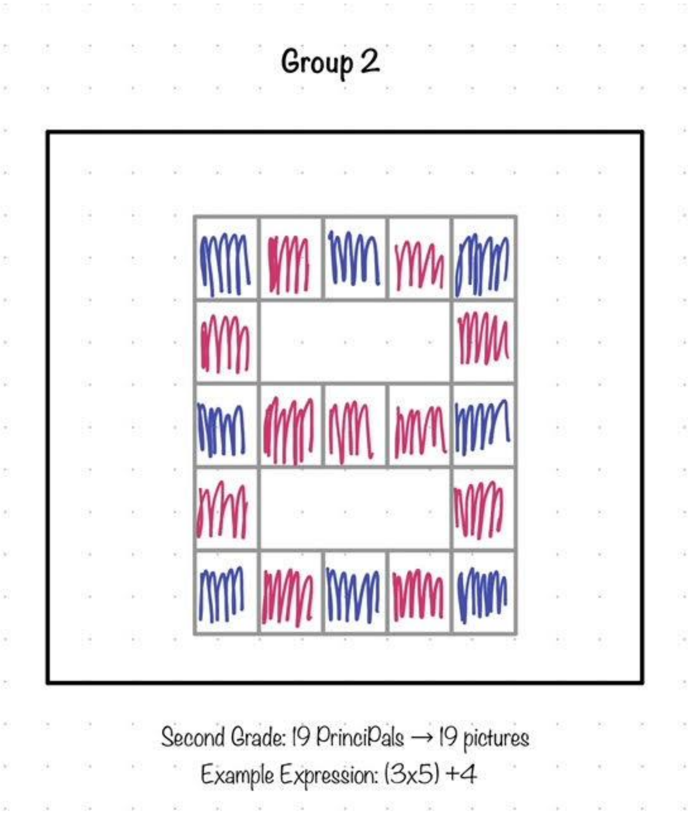
Additionally, formative evaluation will be ongoing in the explain, explore, and extend parts of the lesson. The teacher will be listening to student conversation and observing student work to determine the level of understanding and correct any misconceptions at that time. Summative evaluation will be in the form of the students' final posters for the project.

Student Sample Poster

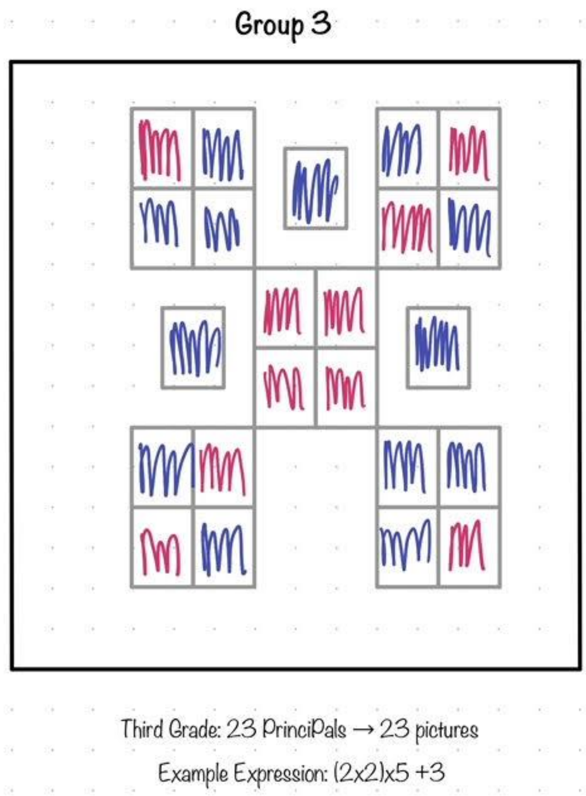
First Grade: 17 PrinciPals



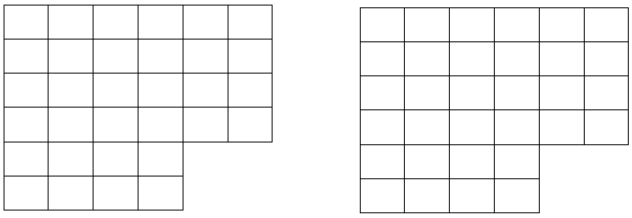
Second Grade: 19 PrinciPals



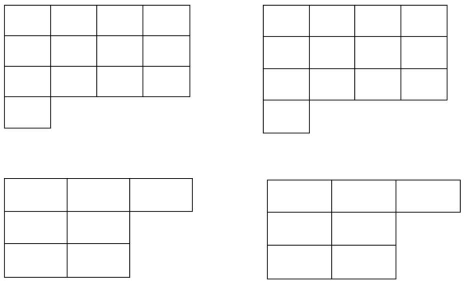
Third Grade: 23 PrinciPals



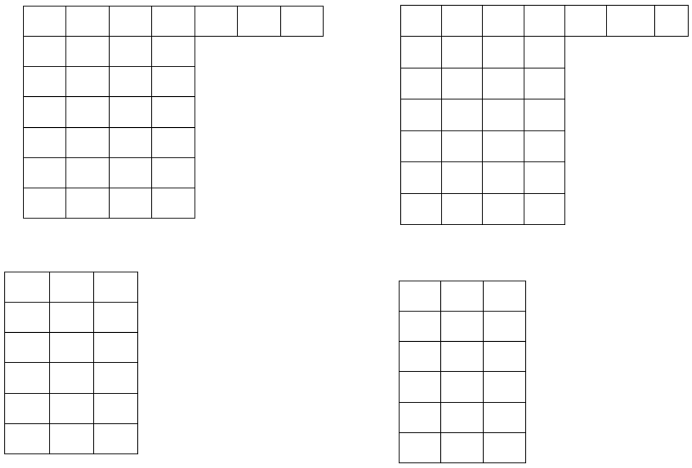
Appendix A



[(6x4) +(4x2)]x2



[(3x4) +1]x2 + [(3x2) +1]x2



[(7x4) +3]x2 + (3x6) x2

Citation

Tudor, M., Osborne, M., & Gundlach, M. (2023). Headbands. In A. Bicer (Ed.), *Creativity-Directed Mathematical Tasks for 5th Grade Common Core Classrooms* (pp. 3-12). ISTES Organization.

Task 2 - Mysterious Dominoes

Traci Jackson, Aylin S. Carey, Fay Quiroz

Mathematical Content Standards

CCSS.MATH.CONTENT.5.OA.A.1

Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.

CCSS.MATH.CONTENT.5.OA. A.2

Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. For example, express the calculation "add 8 and 7, then multiply by 2" as 2 × (8 + 7). Recognize that 3 × (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product.

Mathematical Practice Standards

1) Reason abstractly and quantitatively
2) Look for and make use of structure

Materials

Dominoes or printout of Dominoes

Vocabulary

Parentheses, brackets, expressions

Lesson Objective

Students will learn how to use parentheses, brackets, or braces in numerical expressions by: a) creating multiple expressions for the total number of dots on a given set of dominoes and b) drawing a set of dominoes that would match given expressions. The task's aim is to develop student's creative thinking by arranging the dominoes in multiple ways to group and count the total number of dots. Student creativity will also be fostered as they draw a set of dominoes that could represent given expressions. This task will be completed in about 2 hours.

Engagement

(30 minutes) Ask students whether they ever played dominoes before and what they know about dominoes. After this brief conversation, introduce dominoes using figure 1 (see Appendix A for full size.) Ask students to examine the image. Then, ask students which one doesn't belong and why (there are reasons for each one to not belong.) For example, a student may say the green doesn't belong because it is the only one with equal values on both sides, or that the blue domino is the only one without a prime number in a section. Have students do a think-pair-share about this question. Ask for a few groups to share their explanations with the whole class.

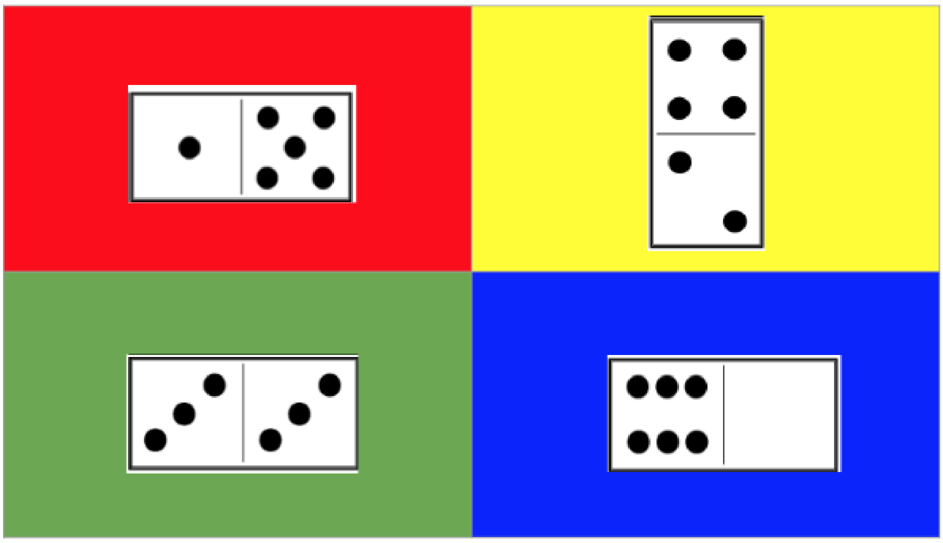


Figure 1. Which one Doesn't Belong?

Give each pair of students a set of dominoes. These can be actual dominoes or cut out dominoes (see Appendix B.) Virtual Dominoes can be found at NRICH math https://nrich.maths.org/6361. Colored dominoes (see figure 3) help with grouping in the explore portion of the lesson. Ask students to determine if they have a full set of dominoes by organizing them. They may group them in several different ways (figure 2a, 2b). This gives a common understanding of domino patterns, and engages students in identifying structure before grouping dominoes.

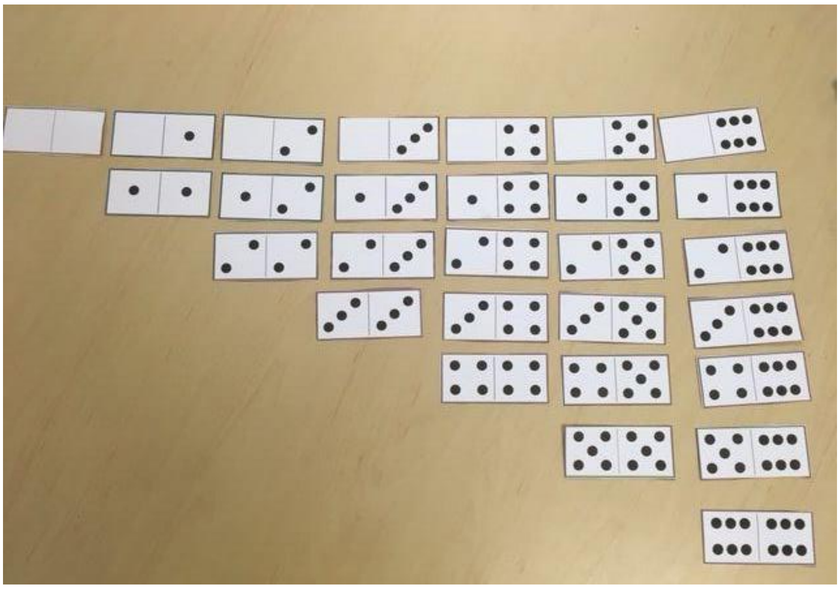


Figure 2a. Grouped by Doubles then All Like Left Sides

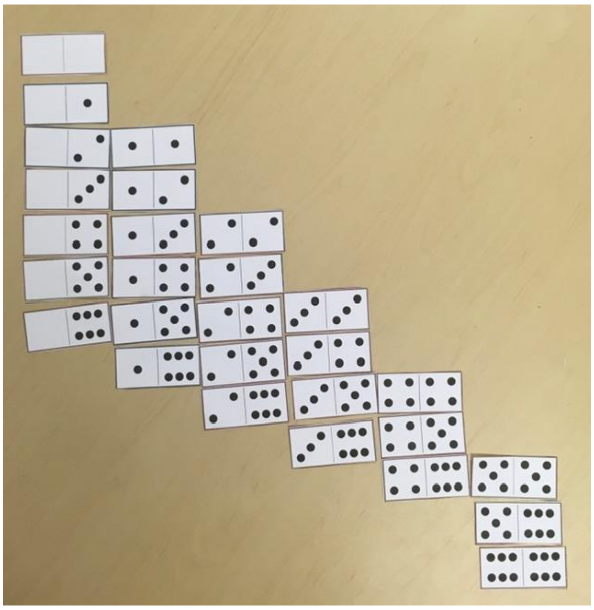


Figure 2b. Grouped by Total Dots

Math Talk Activity: Have students select all dominoes that have 5 or 6 dots (except the double six) and set the rest aside (see figure 3). Ask students, "How can you arrange the dominoes to count all the dots?" The teacher can circulate between the groups to listen and ask guiding questions. For example,

- "What do you notice about the numbers that makes it easier to count?"
- "Are there any connections between the dominoes?"

- "Do you notice any patterns?"

Bring the whole class together and ask groups or individual students to share out how they counted. The teacher should record student thinking on the board in several ways to count and student generated expressions. When students mention grouping, the teacher can introduce parentheses as a formal way of grouping. Based on the given image in figure 3, the teacher will write an expression that corresponds to the image and then asks the students to write their own expressions. If students create 6 groups of 5 and 6 dots and 2 of the numbers 1-5, their thinking may be represented by expression 2 or 3. If students group the 6 dominoes sections and 5 dominos sections and then combine the remaining dots into 2 more groups of 5 (1+4, 2+3) their thinking may be better represented by expression 1.

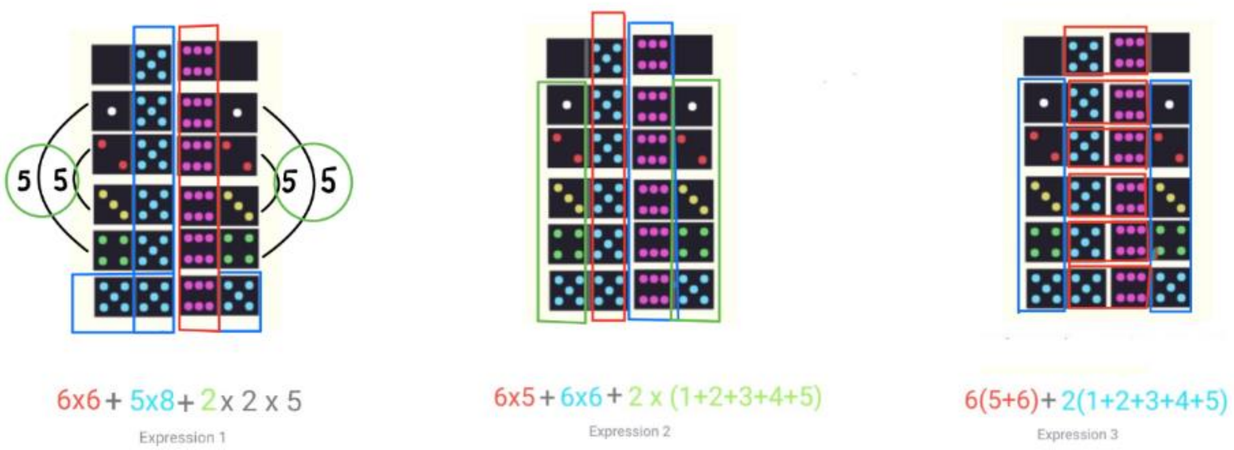


Figure 3. Examples of Possible Grouping and Expressions

Explore

(30 minutes) *Discussion:* Write the expression $2 \times 6 + 2 \times (2 + 3)$ on the board.

Ask students to create a set of dominoes (using all dominoes now) that shows this relationship (there are many ways to show this see example domino groups in figure *4*.) As students work, ask where they see the two sixes and the two groups of two plus three.) Ask pairs what would happen if they combined their dominoes for the expression $2 \times 6 + 2 \times (2 + 3)$ with another pair? Ask how the expression would change

$2[2 \times 6 + 2 \times (2 + 3)]$

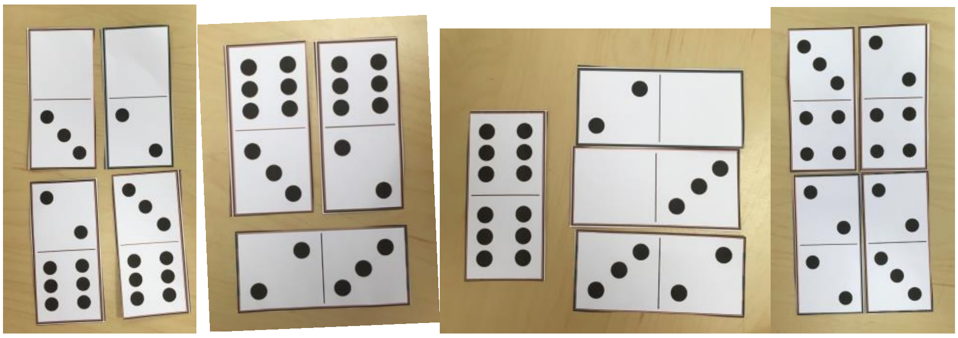


Figure 4. Different Domino Groups for the Same Expression

After the discussion, put students in groups of 4. Have each student create their own group of dominoes and write an expression using grouping symbols in the "Expression #1" box (See Appendix C.) After all students write their own expression, students pass their papers to the right for the next student to draw a domino representation to match the expression. After all students draw their representation, students pass their papers to the right. The next student generates a new expression based on the domino representation. After all students write their new expression, have students pass their papers to the right. The final student will draw a new domino representation to match the expression #2. After the activity, have students evaluate each expression and domino representation to test for equivalence.

Domino Representations and Expressions

Expression #1	Domino Representation #1	Expression #2	Domino Representation #2

Explain

(20 minutes) Bring the whole class together for students to share their domino expressions and representations under a document camera. The teacher can choose a couple groups of students to explain how they wrote their expressions and drew their representations. Encourage groups to share the conversations they had with their groups. For example, if their partner did not understand, how would they explain their thinking in a different way? Or, how did students reach a new understanding about how the parentheses were useful? The teacher may choose to begin the explanation with the most accessible to least accessible representations or the most common expressions or domino representations to model the use of parentheses. Emphasize how the representations are connected (numbers used connected with the groups of dots.)

Extend

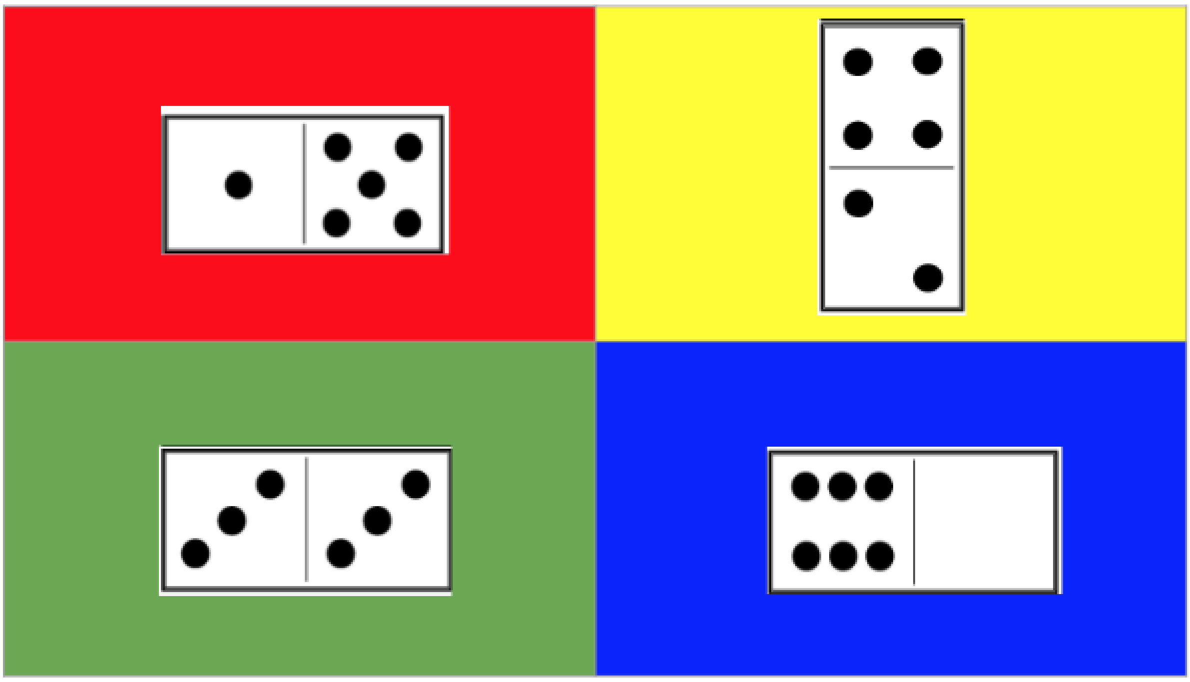
(20 minutes) Groups of students now group dominoes with a different arrangement and write expressions to exchange with other groups so that they can represent the grouping and expressions in multiple ways. Challenge the groups to make different arrangements and write expressions that require using parentheses and as many operations as possible. Provide each group with opportunities to test their arrangement and expression out before sharing with other groups. To help students apply and extend previous understandings of arithmetic to algebraic expressions, extend this to a 6th grade level expression and equations Common Core standards by asking students to identify if the given two expressions are equivalent (CCSS.MATH.CONTENT.6.EE.A.4). For example, ask students “Is this statement true: $2 \times (2 + 3 + 6) = 2 \times (4 + 3 + 4)$? (using two domino groups as presented in figure 4). In the end, bring the class together and ask them to share interesting and/or creative arrangements they received from other groups to find expressions. Or, ask students to share the expressions they created that lead others to find interesting or creative domino arrangements.

Evaluate

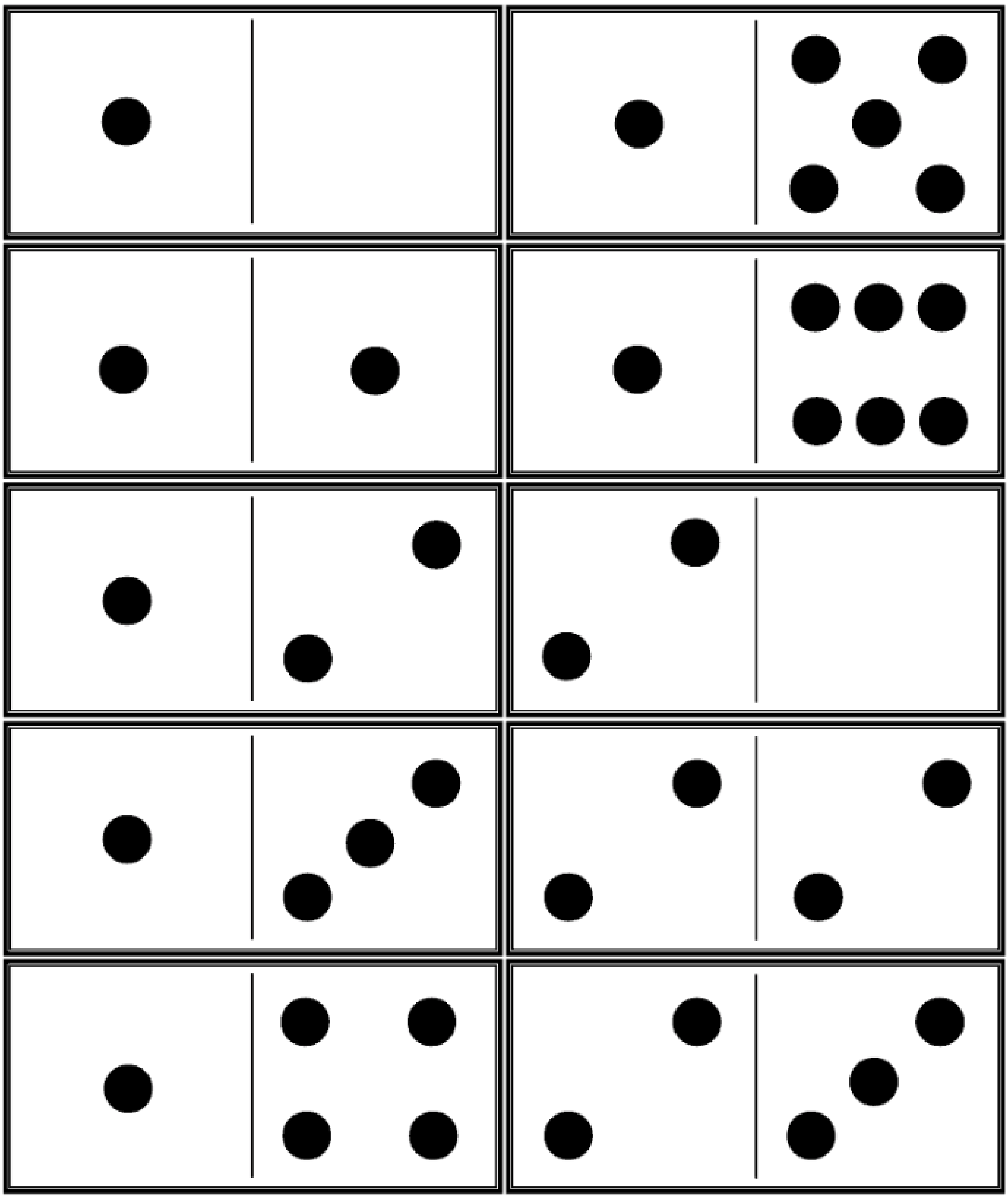
Formative assessment will take place as by closely observing each group while students describe their ideas to their group members. Formative assessment will also take place during the whole-class math talk activity as the groups share their ideas, solutions, and questions. It is important to look for whether students correctly use the symbols (e.g., parentheses, operation symbols). Students may forget to close the parentheses or overuse them as this lesson might be the first one they are introduced to parentheses. Scaffold students' understanding of using parentheses in mathematical expressions by providing them feedback when they forget to use necessary parentheses and when they use parentheses redundantly. To emphasize creativity in this task, Look for whether students are flexible and fluent in making multiple arrangements for given expressions or multiple expressions for given arrangements. Ask guiding questions students cannot make more than one or two arrangements or expressions. For summative assessment purposes, take photos of domino arrangements from each group or provide posters to students and have them draw their arrangements on to the paper to be collected at the end. This addresses possible difficulties or misconceptions that a student may have that were not caught during the class.

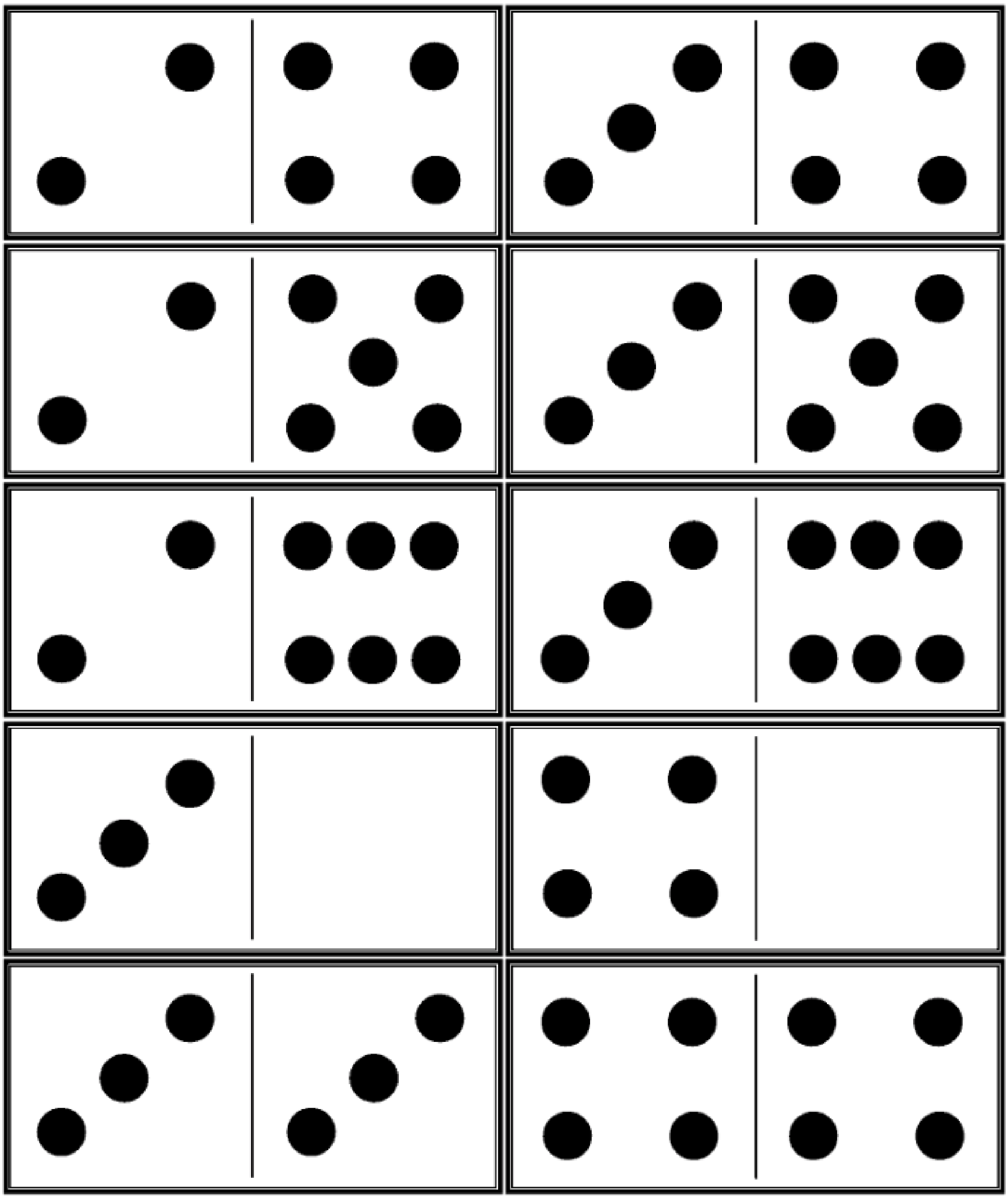
Appendix A.

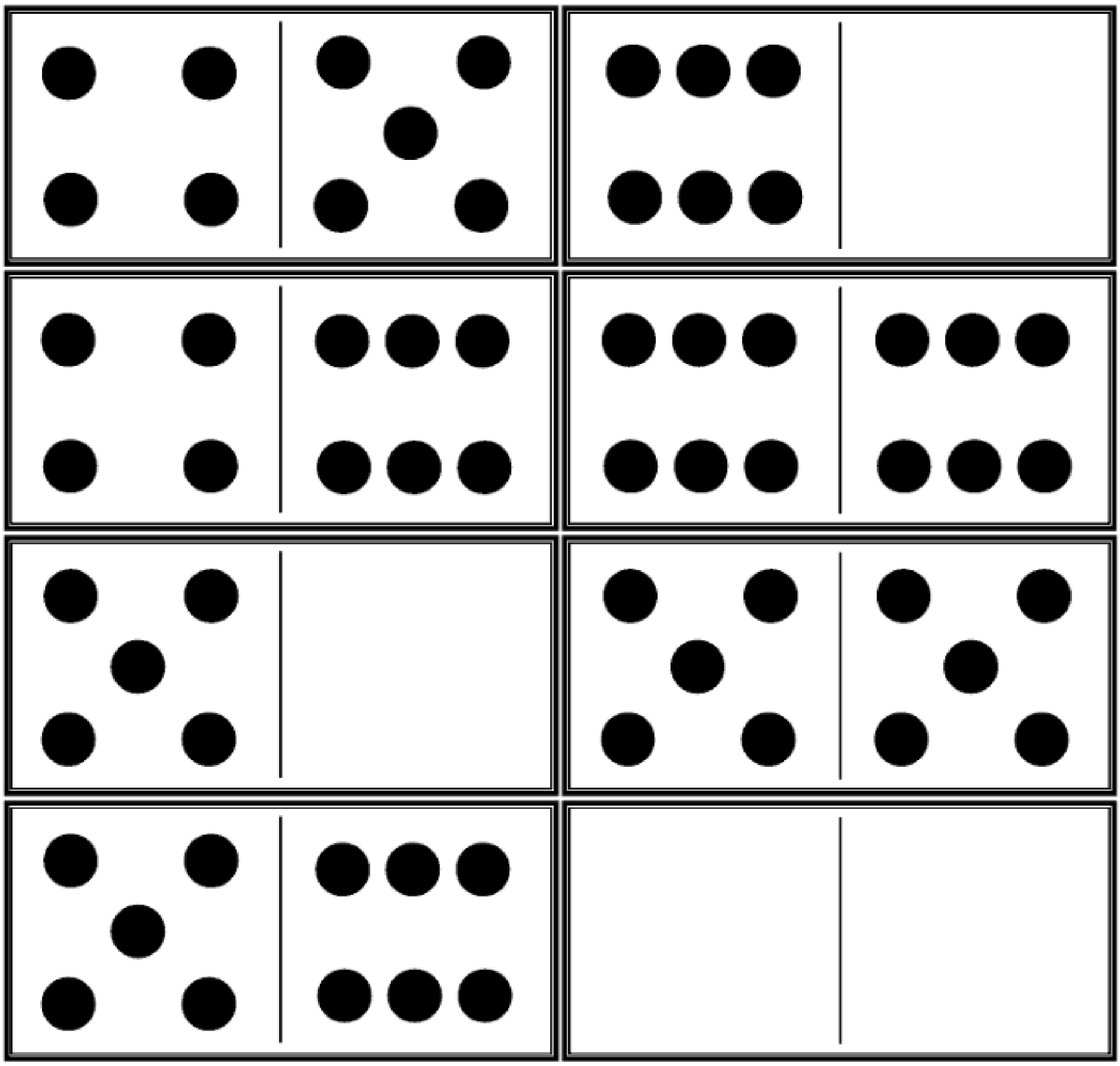
Which one doesn't belong? Why?



Appendix B







Appendix C

Domino Representations and Expressions

Expression #1	Domino Representation #1	Expression #2	Domino Representation #2
Name ____________	Name ____________	Name ____________	Name ____________

Domino Representations and Expressions

Expression #1	Domino Representation #1	Expression #2	Domino Representation #2
Name ____________	Name ____________	Name ____________	Name ____________

Citation

Jackson, T., S. Carey, A. S., & Quiroz, F. (2023). Mysterious Dominoes. In A. Bicer (Ed.), *Creativity-Directed Mathematical Tasks for 5th Grade Common Core Classrooms* (pp. 13-26). ISTES Organization.

Task 3 - Date Math

Chuck Butler, Jennifer Kellner, Amy Kassel

Mathematical Content Standards

CCSS.MATH.CONTENT.5.OA.A.1

Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.

Supporting Content Standard

CCSS.MATH.CONTENT.6.EE.A.1

Write and evaluate numerical expressions involving whole-number exponents.

Mathematical Practice Standards

1) Reason abstractly and quantitatively,
2) Look for and make use of structure, and
3) Construct viable arguments and critique the reasoning of others.

Lesson Objective

Students will write and evaluate numerical expressions using parentheses, brackets, or braces and order of operations. Students will do this by creating numerical expressions that evaluate to numbers 1-20 using a fixed set of digits, mathematical operations, and grouping symbols. Students will compare equivalent expressions. The task's aim is to develop students' mathematical creative thinking by challenging them to create as many expressions as possible that yield the given value(s).

Engagement

(20 minutes) Capture students' interest by finding something interesting that happened on the date of this lesson. Visit Famous Daily (Famous Daily, n.d.) to find examples. This activity can also be completed by choosing a birthdate or using the current year or any other grouping of 4 digits that students may find interesting.

Frame the problem to students as "I have lost some numbers (1-20) and I need your help finding them. The only tools we have are the operations (+, -, ×, ÷) and grouping symbols and the numbers from your groups' favorite date (the four digits in the month and the day) in history."

For example, if the date of the lesson is September 15, you might discuss the first spaceship to fly around the moon and return to Earth. On September 15, 1968, Zond 5 was launched and made a 3-day journey around the moon and back to the Earth (NASA, 2018). Put students in groups of three. Have each group briefly research Zond 5. Students may find pictures of the night launch of Zond 5 and the historical photos of the space shuttle (NASA, 2018). Ask each group to share one interesting fact about Zond 5. As a whole class discussion, show students a picture of a recent space shuttle and ask students to describe how space crafts have changed since 1968. Following a short discussion, state that their task will be to use each of digits from the date (0, 9, 1, 5) exactly one time with the mathematical operations of add, subtract, multiplication, division, and grouping symbols to create numerical expressions for as many different whole numbers as possible.

Ask students to use the tools of operations and grouping symbols to see what numbers you can create using each of the digits 0, 1, 5, 9 only once. Allow students a minute or two of individual think time.

Then, if necessary, the teacher may verbally lead the students through one example such as:

T: What number do we want to create?

S: 4

T: How can we use the digits 0, 9, 1, 5 each exactly one time to get 4?

S: We could subtract them.

T writes: $9 - 1 - 5 - 0$ on the board.

S: That gives us 3.

T: Could we change anything to get a different value?

S: What if we add 0 and 9 and then subtract 5? That would give us 4.

S: Then we could multiply by 1.

T: How would you write that?

S: 0 plus 9 minus 5 times 1.

T writes: $0 + 9 - 5 \times 1$.

S: Hey, that's 4.

T: Does that represent what you said? See if you can use parentheses and still get four.

S: If we write it like this $(0 + 9 - 5) \times 1$, then we still get four.

T: writes: $(0 + 9 - 5) \times 1$

S: That works! We get 4×1 which is 4.

T: Awesome. Do you think you could find a way to express a different whole number?

T: Think of a different whole number. Individually, work for a few minutes on trying to find a way to generate your whole number.

While students are working individually, the teacher should monitor students observing their thinking. Teachers should be prepared to ask students to explain their thinking. Teachers should also be prepared to give students hints and help students recognize that it may not be possible to generate every whole number. After 4-5 minutes of individual work time, bring the students back together to give their next task.

T: Was everyone able to generate the whole number they selected?

S: Some will say yes, and some will say no.

T: That's okay. Sometimes working together can generate more ideas. It also may not be possible to generate every whole number. So, it is your group's job to use the digits $0, 1, 5, 9$ to create as many whole numbers from 1 – 20 as you can find. Please use your whiteboard space to work together. Record your expressions on the paper provided (matches the visual that will be posted on the classroom display).

Explore

(30 minutes) Allow students to work together to create expressions for as many of the whole numbers for 1 – 20 as they can find (Appendix A). During this time, the teacher should monitor the groups, observing their progress and listening for creative approaches or different expressions to prepare for the consolidation of the lesson.

As the groups begin to explore, the teacher should be prepared to:

- ask questions that foster thinking, for example, "Can you explain why you chose the operation of multiplication here?" or "Could you tell me why you chose to put parentheses around this part of the expression?" or "How would your expression be different if you moved the parenthesis?".
- support groups in generating discourse about how the order of operations and grouping symbols may change the value of an expression.
- observe if students are using a variety of operations and grouping symbols. Be prepared to support groups that may be using only one operation or not using an operation.
- observe if students use a new operation like exponentiation; prepare to share that expression.
- allow time for a group to self-correct errors. Students may find their errors as solutions are shared.
- mobilize knowledge by encouraging students to visit other groups (Liljedahl, 2021).
- identify expressions that will create rich mathematical expressions.
- ask students to write their expressions on the classroom display, paying particular attention that all groups/students have work represented on the display.
- ask students to write equivalent expressions in a different color in the same box. After students have represented a number three times, call the number "shut down" and tell students to focus on the remaining values.
- allow students who may find an error in an expression to adjust the expression in a different color (do not allow them to erase other students' work).
- if it becomes apparent that students are unproductively struggling, it is acceptable for the teacher to give a "new rule", such as allowing repetitive use of numbers, exponentiation, factorial, or any other creative rule the teacher wishes to allow.
    - The teacher will need to explain exponents as a topic they will cover in 6th grade. Exponents are a short hand way to write repeated multiplication. On the board, write 3 x 3 x 3 x 3 = ______. Tell students that in mathematics a quicker way to write this is 3^4. Allow students to explore other exponents such as 2 x 2 x 2 x 2 x 2^5 or 4^2.
    - Next, show the students 0 as an exponent. We know now that $3^1 = 3$ and $3^2 = 9$ and $3^3 = 27$. To get to the next exponent value, we are multiplying by

3. Conversely, we divide by 3 to get the previous exponent value. So, if I have $3^1 = 3$, then 3^0 would be 3 divided by 3 which is 1. Therefore, any number to the 0 power would equal 1.

The goal of this time is to engage in number talks with and between students as they develop their fluency in using grouping symbols and evaluating expressions with grouping symbols. When the students have found expressions for as many of the whole numbers as possible or new solutions stall, the exploration stops.

Explain

(20 minutes) Ask students to gather around the classroom display. In turn, ask each group to share an expression that they find interesting (not their own). Ask the group to explain what they think the group who wrote the expression was thinking (Liljedahl, 2021). For example, if the group chose $(9 + 1) \div (5 + 0)$, they may explain that the thinking was $9 + 1 = 10$ and then divide by 5 to get 2.

After a couple of groups take a turn explaining an expression, the teacher should move to equivalent expressions focusing on grouping symbols. For example, if another group wrote $(9 + 1) \div 5 + 0$ for 2, the teacher may ask a different group to explain how that expression is different than the expression $(9 + 1) \div (5 + 0)$ and if it is an equivalent expression. During this time, teachers should be prepared to discuss when and if grouping symbols are necessary. For example, the teacher may ask "Are parentheses necessary in the expression $(5 + 1) + (9 \times 0)$?".

During consolidation of the lesson, the teacher should be prepared to address solutions from the least complex, for example $0 + 1 + 5 + 9$, to the most complex, such as $2^0(9 + 5) = 14$, emphasizing the use of grouping symbols throughout (Liljedahl, 2021).

Extend

(15 min) Lonely Ones

Some numbers from 1 to 20 might not be found. Called the Lonely Ones, the teacher instructs students to use the tools of operations, grouping symbols, and as many 1's as necessary to find the missing numbers. Once students have found all the missing numbers compare two equivalent expressions and have students discuss which expression they think is the best (The University of Adelaide, n.d.).

$$(1+1)1+1+1+1-(1+1+1+1)=1$$

$$1(1+1)1+1+1+(1+1+1+1)=8$$

$$(1+1+1+1)^{(1+1)}=16$$

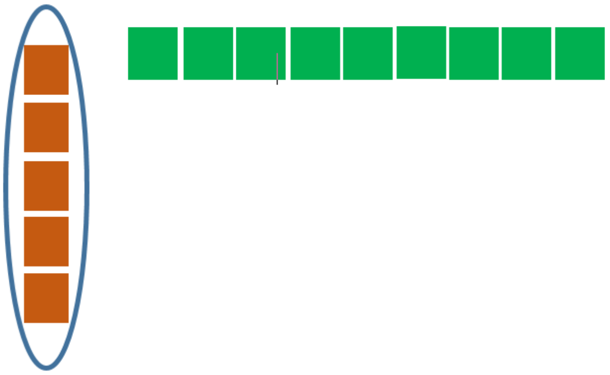
$$(1+1+1+1)^{(1+1)}+1+1=18$$

Visualize

For the number 2 or 10, draw a picture that represents the expression. Use different colors for each term in the expression. For example,

$(5\times 0)+9+1=10$

$5^0+9\times 1=10$



Evaluate

Formative assessment for creativity will occur as students select their famous day, specifically listening for students that are collaborating with others, sharing ideas, and making connections to other disciplines. Formative assessment for the standard will occur with the whole class during the opening sequence through questioning, focusing on the students' use of symbols to find specific values. When a value has 2-3 different expressions, the teacher may pause the class and ask students to pick the most creative one and why. Students will discuss in their groups and then share their responses with the whole group.

The teacher may then show an example of an expression that is incorrect and ask students to find the error and explain why it is an error. For example, students may write $5 \times 1 + 9 \times 0$ and $5 \times (1 + 9) \times 0$ and $(5 \times 1 + 9) \times 0$ without realizing how each expression is calculated differently. Teachers should be prepared to support students when working with grouping symbols.

Again, students will discuss in groups and then share out. Students will then continue finding the other values (1-20). As students are creating a variety of expressions that simplify to each value, the teacher may challenge students to use different operations if they continually use the same operations. For example, students may have used multiplication and addition repeatedly in a pattern such as $5 \times 1 + 9 + 0 = 14$ or $9 \times 1 + 5 + 0 = 14$. The teacher may ask the students what would happen if they used division and/or subtraction. Some students may ask if there are other operations that they may use. For example, some students may have been exposed to exponents or factorial. If so, the teacher should be prepared to explain how these operations are performed on numbers. For example, students may use $1^9 \times 5^0$ to generate 1. The teacher should be prepared to discuss that 1^9 is repeated multiplication and that 5^0 is equal to one because $\frac{5^a}{5^a} = 5^{a-a} = 5^0$ and $\frac{5^a}{5^a} = 1$. The teacher also could use a series of equations to help students understand, such as $5^3 = 125, 5^2 = 25, 5^1 = 5$, and extending this division pattern to show that $5^0 = 1$. Teachers may also recommend the use of factorial as an operation if students are struggling creating some numbers. Teachers should be prepared to give students an example of factorial $(5! = 5 \times 4 \times 3 \times 2 \times 1)$ and the fact that $0! = 1$.

With about ten minutes left, the teacher will ask students to make a visual of one of the numbers that has several expressions. The teacher will monitor the class for understanding, and if needed, show a visual of a different expression for that value. Examples will be color-coded so students can see the expression more clearly in the visual.

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Appendix A

= 5	= 10	= 15	= 20
= 4	= 9	= 14	= 19
= 3	= 8	= 13	= 18
= 2	= 7	= 12	= 17
= 1	= 6	= 11	= 16

Citation

Butler, C., Kellner, J., & Kassel, A. (2023). Date Math. In A. Bicer (Ed.), *Creativity-Directed Mathematical Tasks for 5th Grade Common Core Classrooms* (pp. 27-36). ISTES Organization.

Task 4 - I Do Chores for Money!

Helen Aleksani, Geoff Krall

Mathematical Content Standards

CCSS.MATH.CONTENT.5.OA.A.1

Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.

CCSS.MATH.CONTENT.5.OA. A.2

Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. For example, express the calculation "add 8 and 7, then multiply by 2" as $2 \times (8 + 7)$. Recognize that $3 \times (18932 + 921)$ is three times as large as $18932 + 921$, without having to calculate the indicated sum or product.

Mathematical Practice Standards

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
4. Model with mathematics.
6. Attend to precision.

Lesson Objective

Students will learn how to use parentheses, brackets, or braces in numerical expressions by creating an expression that demonstrates a situation or context. Also, students will practice writing expressions with a given value. This task is aimed to challenge students' creativity by encouraging them to develop their expressions and use the table to organize their work to help them discover a pattern. Depending on students' background skills and the amount of reteaching a teacher has to provide, this task can be completed in one or two class sessions.

Engagement

(30 minutes) Have students do the following sequence of operations:

- Write down any number.

- Add to it the number that comes after it.
- Add 9
- Divide by 2
- Subtract the number you began with.

Now you can "magically" read their minds. Everyone ended up with 5!

The task is to get students thinking and have them discover how the trick works. If students need a hint or are struggling with the concept, have them use boxes instead of actual numbers. The box represents a number, but even they do not need to know what the number is. Start with a square and follow the steps so students can see the process. Students then can pick their numbers to start with.

Ask students if they have done chores for money. If they haven't they can pair share with an elbow partner to get some ideas for earning money for chores. After that, have students share their experience with earning money doing chores, and if they thought the earning was fair. After a class discussion, introduce Sarah to the class. Sarah is a student who earns money for chores so she can buy her new bicycle. She offered to do extra chores around the house so she can save the needed money faster. Her mother offered to pay her $8 for cleaning a door and $4 to clean a window. If Sarah earned $40 from cleaning, how many doors and windows could she have cleaned? Draw a picture of the number of doors and windows to help you write an expression showing how much Sarah will make from her cleaning chores. If students have a hard time drawing a picture, provide them with a set of paper images of doors and windows (see Figure 1).

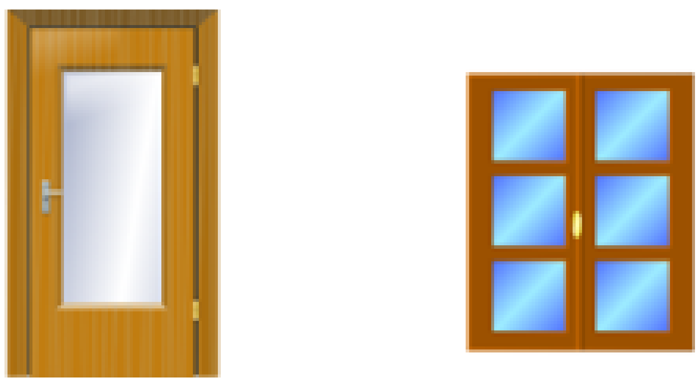


Figure 1. An Image of a Door and a Window (see Appendix A)

Explain

(10 minutes) While helping students either draw or use the images of the doors and windows to represent the number of doors and windows they would clean to meet their goal, ask them to pay attention to the relationship between the numbers 8 and 4. Encourage students to not just keep counting the number of doors and windows. Instead, have them develop a strategy that would lead them to find the number of doors and windows faster. Then, provide students with 4 to 6 minutes of individual thinking time to prepare their strategy. After that, put students in groups of three to share their strategies while the teacher circles the room listening to group discussions. Teachers then pick some of the strategies presented by the groups in a way that would lead the students to learn the objective of the day and have groups present their strategies. Encourage students to use the images and different colored pens or pencils to explain and color code their process. Figure 2 represents one example of possible student work.

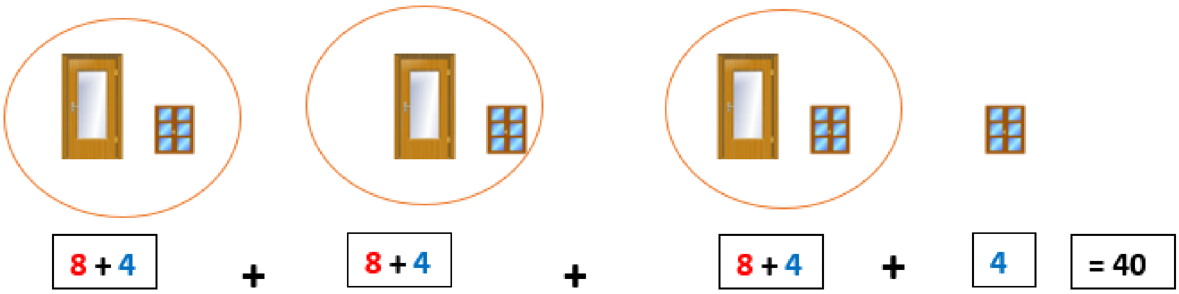


Figure 2. An Example of Student Work demonstrates the Addition of Money for Chores

Students in class record the different representations presented by their classmates in their notes. Students will be then given the recording sheet (see Table 1) so they can record all the possible outcomes and try to come up with a pattern.

In this task, students are not expected to find all possible solutions. The main goal is to lead students to help find one solution. Through classroom discussion and group work, students will be able to recognize that earning $40 and painting more than 5 doors won't be possible because 8x5=40. Also, students are encouraged to use their background knowledge on the concept of doubling to see the connection that the payment for one door is equal to the payment for two windows.

In this task, students may choose the wrong operation due to their lack of knowledge of each of the four operations. Reteaching and reviewing the context for each operation before starting the task may be helpful.

Explore

(20 minutes) Have students craft problems of their own based on this work (problem-posing). Tell students, "We found all these different combinations based on $40. I'd like you to come up with a *different* total value and then challenge one another to find the possible combinations of chores. Start by finding a total other than $40 by using different combinations of chores. And, make sure it has more than one possible combination."
For example, one student may try three doors and five windows washed:
8 dollars/door x 3 doors + 5 dollars/window x 4 windows = $24 + $20 = $44

The student then challenges another student: "If I earned $44, what's one possible combination of doors and windows washed?" Students can work in pairs and try to solve one another's exploration problems. They can then try to find additional possible combinations of chores.

Table 1. A Recording Sheet for Students to Use to Find the Possible Outcomes of the Task (Appendix B)

Number of doors	Number of windows	Work space	Amount of money earned
0	5		
2	4		
4	3		
6	2		
8	1		
10	0		

Extend

(20 minutes) Incorporate additional chores for different amounts of money. Instead of two chores for the task, now students will have three, all with varying amounts of money associated with the chore. You may wish to ask students what kinds of chores they have to do at home (or use the responses from the *Engagement* section). Look specifically for chores that you can repeat multiple times. Some examples of additional chores might include vacuuming rooms, doing a load of laundry, or feeding a pet. Assign a monetary value to the chore: vacuuming ($6 per room).

Ask students: “Sarah washes five windows, four doors, and vacuums three rooms. Write an expression showing how much money she made.” Repeat this prompt with different numbers. Extend further by asking students to generate problems, such as in the *Explore* section.”

Evaluate

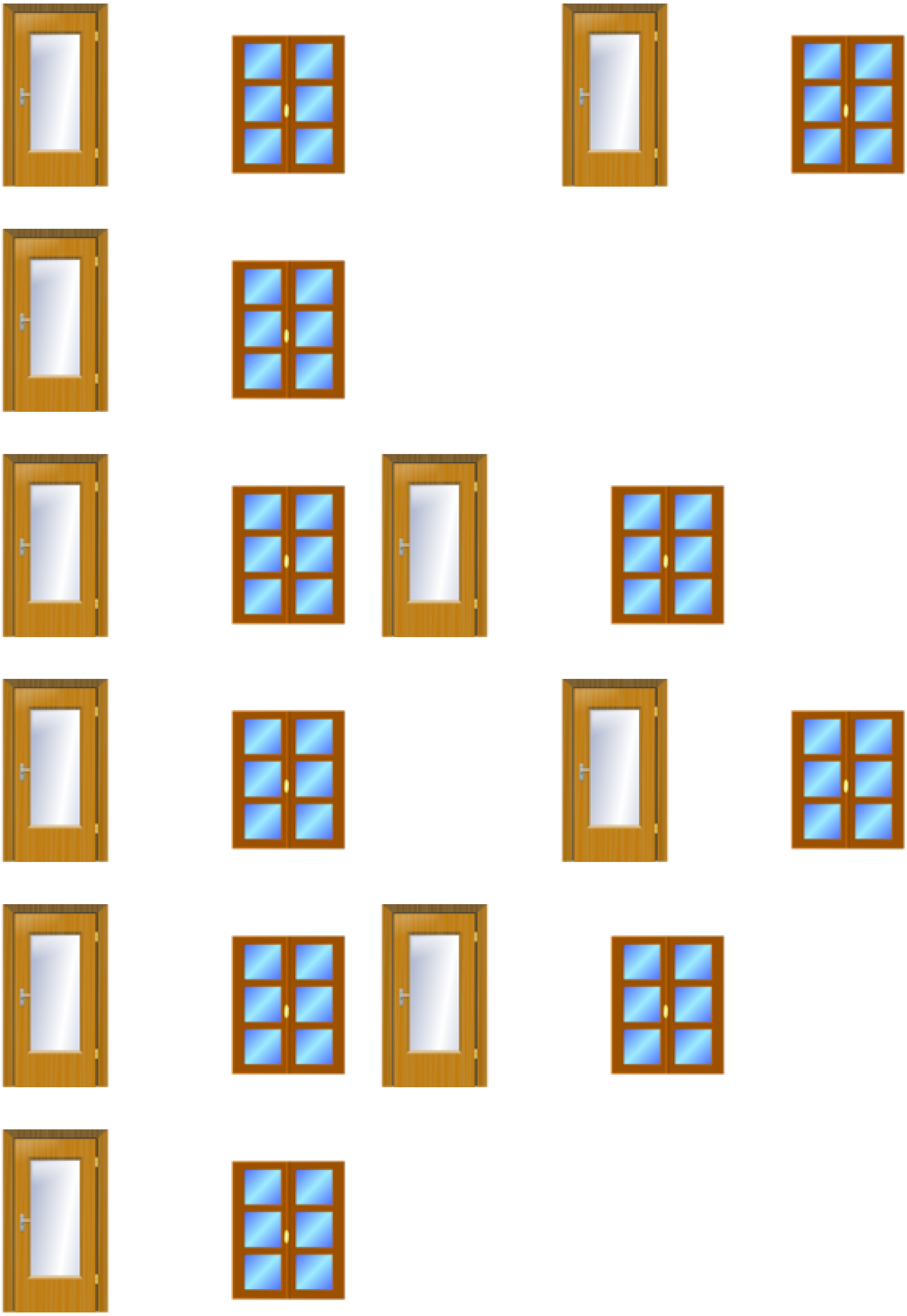
Using a combination of formative and summative assessments, check for students’ growth in understanding. For the formative assessment, move around the rooms and listens to student conversations, and monitor their work. If a student or group of students are struggling with a particular aspect of the activity, pair or group them with students who have demonstrated understanding. Manipulatives such as counters may also help guide students in their thinking. Look for students or groups of students that are not progressing with the recording sheet or failing to complete aspects of the recording sheet before intervening. Do allow *some* time to let students’ problem-solve or share their ideas. If students still continue to struggle with completing the table or finding the pattern that leads to the equation, start posing some direct questions to help them think. For example, ask the struggling student,

- What is the cost of cleaning one window?
- What is the cost of cleaning one door?
- Now record the price in your table.
- How about if you clean two windows, what is the cost?
- How about if you clean two doors, what is the cost?

Pose similar questions to direct students' thinking so they can see the pattern.

Teachers can assess summatively by evaluating the students' work on their recording sheet. Examine for creativity, student thinking, and visual representation. Students should have the recording sheet fully filled out with multiple combinations for each combination of chores, visual representations of each combination, and mathematical expressions

Appendix A. An Image of a Door and a Window for Student Use



Appendix B. A Recording Sheet for Students to Use to Find the Possible Outcomes of the Task

Number of doors	Number of windows	Work space	Amount of money earned
0	5		
2	4		
4	3		
6	2		
8	1		
10	0		

Citation

Aleksani, H. & Krall, G. (2023). I Do Chores for Money! In A. Bicer (Ed.), *Creativity-Directed Mathematical Tasks for 5th Grade Common Core Classrooms* (pp. 37-46). ISTES Organization.

SECTION 2 - ANALYZE PATTERNS AND RELATIONSHIPS

Task 5 - According to Coded Quilts

Michael Gundlach, Michelle Tudor, Melana Osborne

Mathematical Content Standards

CCSS.MATH.CONTENT.5.OA.B.3

Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns and graph the ordered pairs on a coordinate plane. For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.

Supporting Content Standards

CCSS.MATH.CONTENT.5.G.A.1

Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates.

Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate).

CCSS.MATH.CONTENT.5.G.A.2

Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.

Mathematical Practice Standards

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.

3. Look for and make use of structure.

Lesson Objective

Students will determine rules for numerical patterns shown in visual growing patterns. This means using the context of the pattern to describe how the pattern is growing. Creativity will be emphasized as students develop their own growing patterns and challenge their peers to describe the created patterns.

Engagement

(8 minutes) Tell the students that there are legends that some slaves were helped along on the underground railroad by secret messages encoded in certain patterns of quilts. Read the picture book, *The Secret to Freedom* by Marcia Vaughan. Show some of the different quilt patterns from the book (Appendix B) and discuss how the quilts would be hung out a window or on a washing line to send messages to runaway slaves. Although no one knows for sure if such coded quilts existed, we can look at some quilting patterns with pretend messages and figure out patterns behind how the quilts are made.

Explore

Give students the handout in Appendix A and talk about how quilts often get very large. The three patterns shown in the handout are the first three stages of creating certain coded patterns. Have students try to figure out the pattern and then draw the sixth and eighth stages. Encourage students to determine the number of boxes in each pattern and to justify their pattern in the figure. This justification should reference the pattern seen in the figure. In other words, instead of just describing numerical patterns, students should be able to describe precisely how the pattern is growing.

Explain

Once students have found patterns in the Appendix A handout, focus on the Bread in the Oven quilt. Tell students they can organize and examine the relationship of the pattern(s) they found by using a table that compares two quantities. Provide students with the pattern identification table (Appendix C) and allow them time to work on the table while viewing their visual patterns. Once students are finished filling their tables, ask them if they notice a pattern or relationship between the two columns. These relationships may include "adding twelves" for "Bread in the Over" or "adding the stage number and four more" for "Whirlpool." Give students time to discuss what they see happening in their tables.

Once the students have had time to create and discuss their tables, tell them they can represent these quantities in an additional way - with a coordinate plane. This allows for them to visually see the relationship between the quantities using a graph.

Since some students may not be familiar with the coordinate plane, explain to them what it is. The coordinate plane is created of a horizontal and vertical axis (perpendicular lines). Each axis represents a number line, with the center (intersection point of axes) representing the origin and the point (0, 0). The x-axis is the horizontal axis and the y-axis is the vertical axis. Explain to the students that the coordinate plane is a visual way to represent data. Show the students how to plot points using (3, 2), (-3, 4), (-4, -7), (2, -7), (0, 2), and (-4, 0). Explain to them that ordered pairs are written as (x, y) and the x value moves first, horizontally, followed by the y value, vertically.

Once you have explained the coordinate plane, have students plot the data from their table they created on the coordinate plane handout (Appendix D). Encourage students to discuss the relationship/patterns they see between the table and graph. An example of what the table and graph would look like for the Bread in the Oven quilt is in Appendix E.

As a check for understanding, ask students to create a table and graph (Appendix C & D) for the Whirlpool quilt. Circulate the classroom to answer questions and clarify misconceptions. Discuss student findings as a whole class.

Extend

Once students have had time to develop ideas, tables, and graphs about the Bread in the Oven and Whirlpool quilts, tell the students they are going to create their own quilt pattern with a complete table and graph that describes the pattern. Once students have a pattern created, have them create the corresponding table and graph (Appendix C and D). Now, prompt students to find the 20th or 50th stage in their pattern. The students might realize that this would be very time consuming (and difficult) to draw out each stage this many times. Encourage students to discuss and share ideas with one another about different methods. The idea here is that students will be able to write a description that corresponds to the pattern they create so that they do not have to draw out each stage in the pattern. Since variable expressions are not introduced until 6th grade, this description could be something like the following: "For "Bread in the Oven," the figure starts with 16 squares. In each stage, 12 squares are added. We count by 12's from 16 to get the total number of squares." If students are struggling here, have them revisit the relationships they identified between quantities in their graphs/tables. Students may work in pairs or individually. If time allows, have students also create a secret message with their quilt (or this could possibly be a take home assignment to turn in the following day).

Evaluation

Students should be summatively evaluated based on their work on the tables from the handout in Appendix C as well as by their own secret message quilts. Formative assessment will occur as the teacher monitors discussions and construction of patterns on the Appendix A handout.

Specifically, during the "Explain" stage, listen for student discussions about the relationship/patterns identified when creating the tables/graphs. Students may have different ways of explaining the relationship, which is fine as long as what they are saying corresponds with the pattern. If students are having trouble identifying the relationship between quantities, give them some hints of what to look for; have them look at the graph (sometimes it is easier for students to look at a graph rather than the table) and ask them what they notice about the graph and then ask them if they see a pattern in the numbers.

During the "Extend" stage, some students may struggle with creating a quilt pattern. If they do, prompt them to revisit the examples done in class to give them some ideas. Once students have unique quilts created, circulate around the room and check that students' quilt creations correspond to their tables and graphs. Some students may also struggle with creating an expression corresponding to their patterns. If they do, go back to the Bread in the Oven example done in class and discuss the expression that would correspond to this example. Now, have students create an expression for the Whirlpool example as practice. Once they have done this, have them go back to their quilt creation.

References

Vaughan, Marcia K. (2001) *The Secret to Freedom*. Lee & Low Books Incorporated.

Appendix A. Engagement Handout

The three growing patterns below show the first three stages of creating special coded patterns put into quilts.

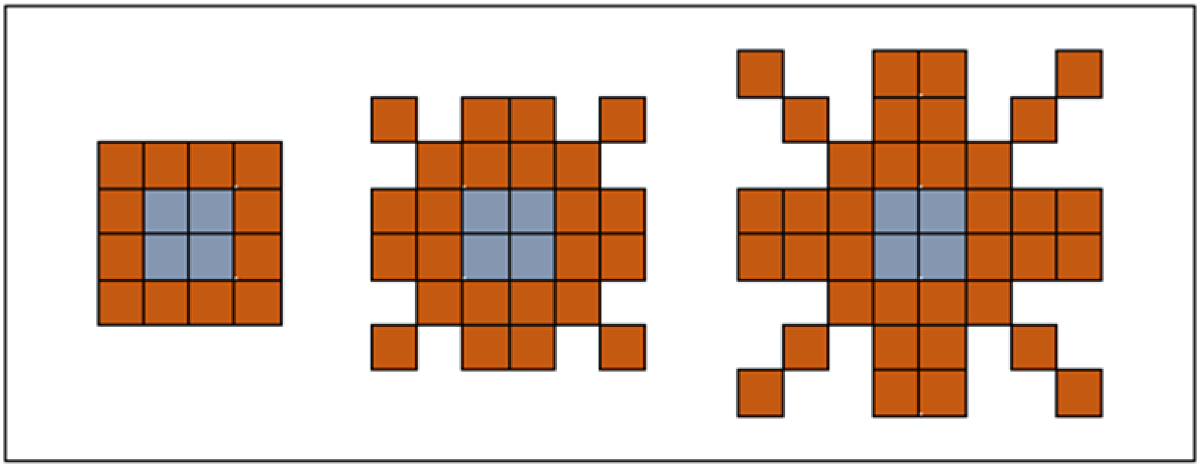
Although these patterns are made up, some legends say that runaway slaves on the underground railroad would follow coded patterns like these to help them find safety.

Each coded pattern grows in a unique way.

Determine the pattern used to create each code and see if you can draw the fifth stage and tenth stage.

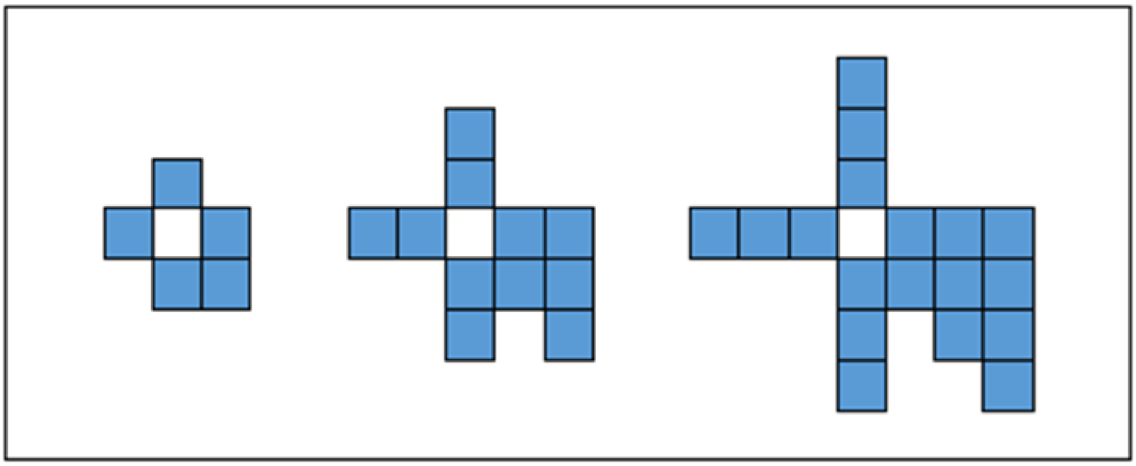
Use colored pencils to help explain how each pattern grows.

Pattern 1: “Bread in the Oven”: a pattern to show that runaway slaves could find a warm meal and a safe place to sleep for the night in the house.



Draw stages 6 and 8 below.

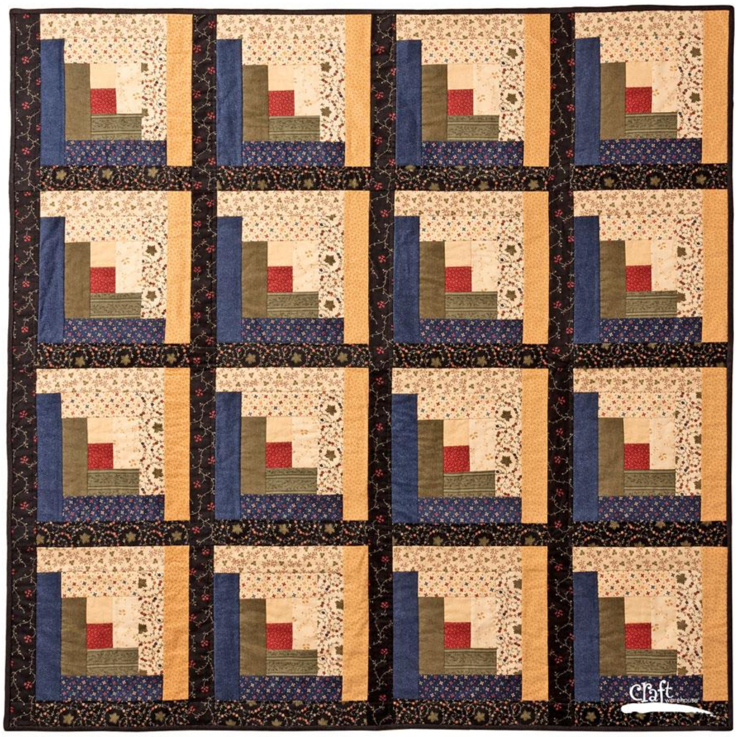
Pattern 2: “Whirlpool”: A pattern to show runaway slaves there was danger in the town and that they should move on to the next town.



Draw stages 6 and 8 below:

Appendix B. Quilted Messages

All images in this appendix are taken from Vaughan (2001).



Log Cabin Quilt - safe places to stay

Flying Geese - follow the migrating geese north

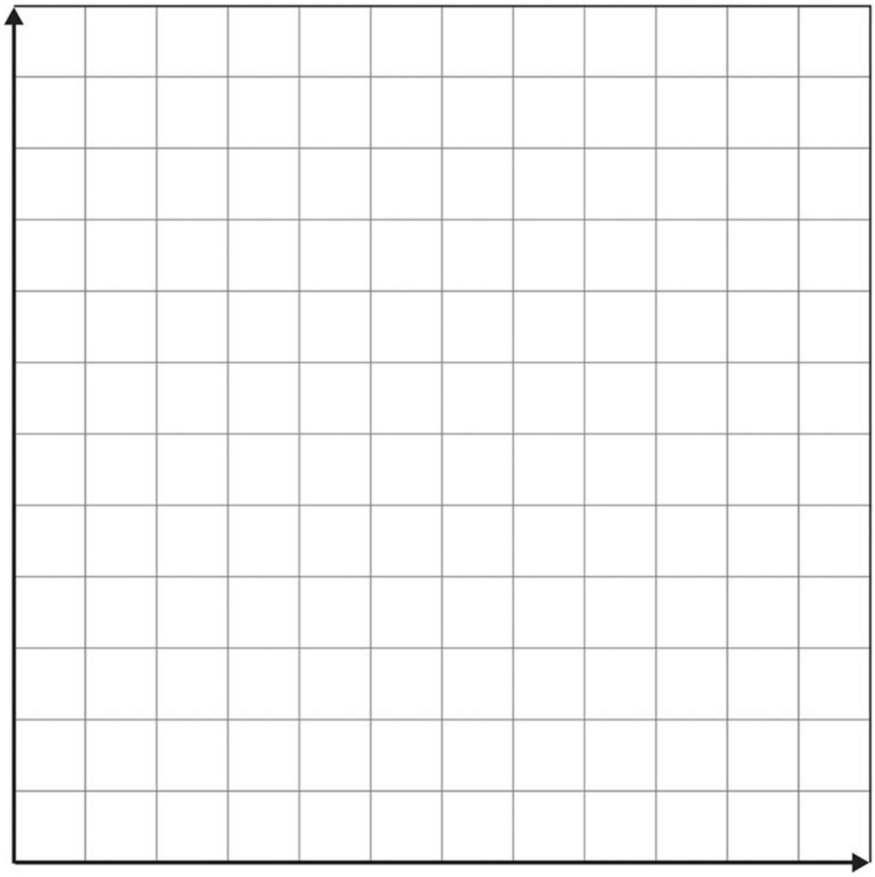
Monkey Wrench Quilt - gather tools and supplies

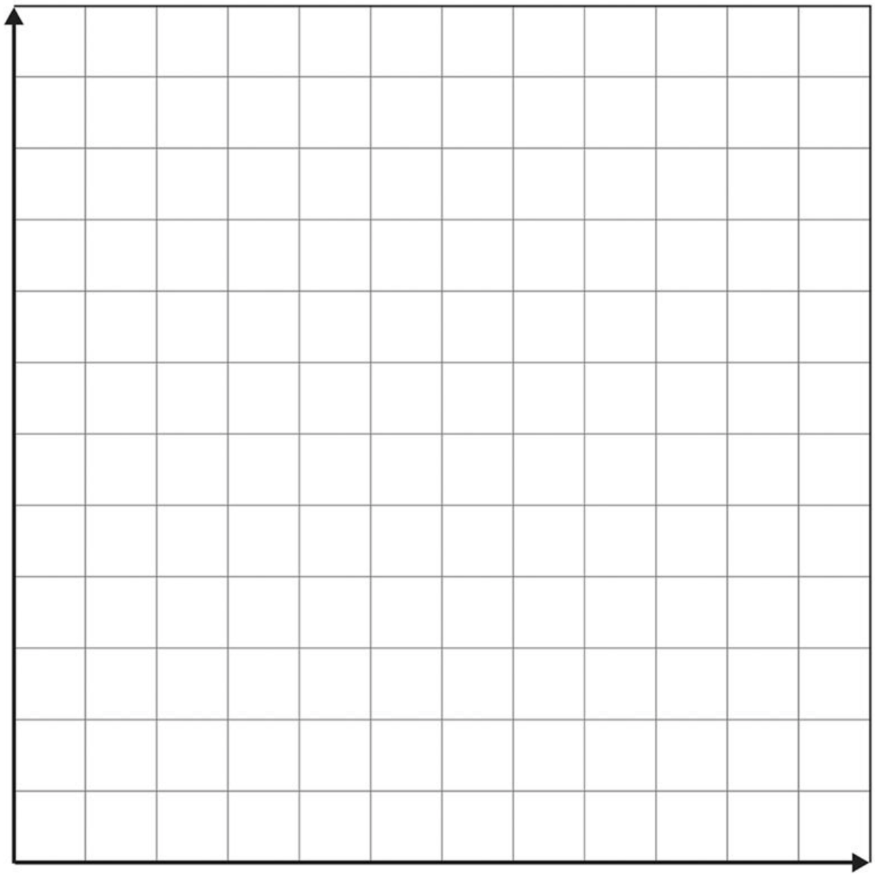
Appendix C. Table Handout

Case Number	Number of Squares

Case Number	Number of Squares

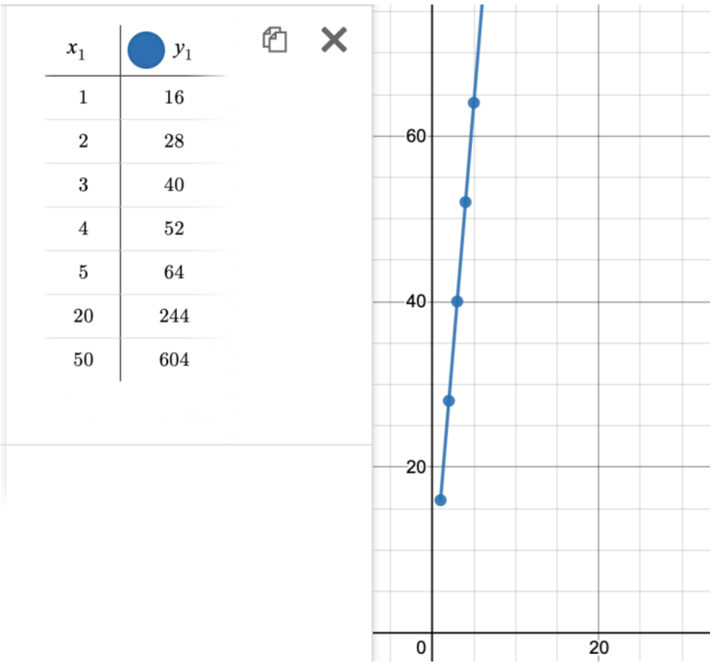
Appendix D. Graph Handout





Appendix E. Table/Graph Example for Bread in the Oven Quilt

Case Number	Number of Squares
1	16
2	28
3	40
4	52
5	64
N	16 + 12(N-1)
20	16+12(20-1) = 244
50	16+12(50-1) = 604



Citation

Gundlach, M., Tudor, M., & Osborne, M. (2023). According to Coded Quilts. In A. Bicer (Ed.), *Creativity-Directed Mathematical Tasks for 5th Grade Common Core Classrooms* (pp. 47-60). ISTES Organization.

Task 6 - Making Playgrounds

Aylin S. Carey, Fay Quiroz, Traci Jackson

Mathematical Content Standards

Focus Content Standard

CCSS.MATH.CONTENT.5.OA.B.3

Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns and graph the ordered pairs on a coordinate plane. For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.

Supporting Content Standards

CCSS.MATH.CONTENT.5.G.A.1

Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., *x*-axis and *x*- coordinate, *y*-axis and *y*-coordinate).

CCSS.MATH.CONTENT.5.G.A.2

Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane and interpret coordinate values of points in the context of the situation.

Mathematical Practice Standards

1) Construct viable arguments and critique the reasoning of others,
2) Look for and make use of structure, and
3) Make sense of problems and persevere in solving them.

Lesson Objective

The most important objective of this lesson is to have students engage in creative thinking and independent problem solving through algebraic patterns. Students will explore and extend pattern sequence by creating additional figures until they understand and be able to examine the pattern structure. Using visuals, tables, and graphs, students will be encouraged to predict the number of elements in the 5th, 10th, 50th, and later the 100th structure. Students will make connections among different representations by showing how mathematical knowledge/data/information represented with one form can also be represented with other forms (e.g., visuals, words, tables, and graphs). Later, students will compare how the two given visual patterns are similar and/or different by identifying an apparent relationship between the corresponding terms of each case in two patterns. This task will be completed in two-class time.

Engagement

(20 minutes) Share the following story of an elementary school:

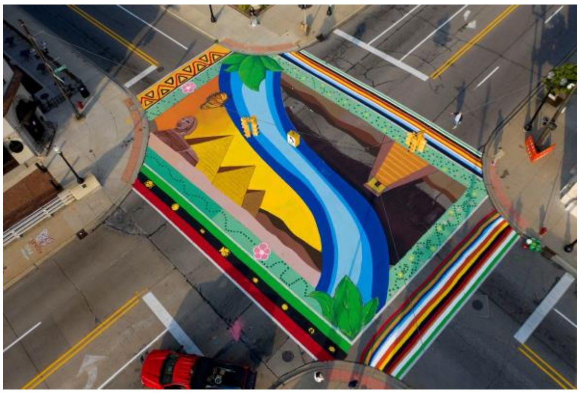
After a staff meeting, the principal and teachers have decided that the blacktop on the playground needs a makeover. The principal told teachers he is wanting suggestions from the students of what could be painted. Therefore, he has created a friendly competition inviting students to submit designs to be painted on the blacktop of the playground that are creative, show originality and go above and beyond the hopscotch, tetherball or four-square designs.

Ask students to visit with a shoulder partner for 1-2 minutes about what types of designs they could see being very interesting to paint on the playground. Bring the class back together to share some ideas. Then tell the students you did some research to hopefully spark creativity about different designs on playgrounds and share some playground designs (see Figure 1). Look at one image at a time and allow students to write a few comments about their notices and wonders. Give about 1 minute per picture to think about what they notice and 2-3 minutes for a few students to share their thinking. The goal is to create a discussion among the students about the types of designs. Use the following questions to help guide the discussion: “What do you notice?”, “What do you wonder?”, “Do you notice any patterns?”, and “Describe the details that make one design more creative than another.”

1.

2.

3.



4.

Figure 1. Playground Designs

Explore

(40 minutes) After discussing the pattern designs on the playground images, bring the class together and introduce the visual patterns (see Figure 2 & 3). Ask your students to think how they see the patterns growing in both playground designs, while providing students with handouts (see Appendix A and Appendix B). Have students work in groups of three or four. Since the essential lesson object is engaging in creative thinking and independent problem solving, each student individually should record the way of seeing the growth of the patterns and then compare with their group members. Challenge individuals and groups to find multiple ways that they can identify how the patterns grow (see examples in Figure 4a, 4b, and 5) and ask them to use colors to make their ways of seeing the growths visible to others. After students complete coloring and show at least two or three ways of seeing the growth of patterns, teachers can ask students to visit the other groups so that students can see how others identified the growths of the patterns. Then, encourage students to predict the 5th and 10th structure of each pattern and let them share their description by coloring, drawing, writing, and discussing with their group members. Teachers should provide grid papers to students if they like to draw additional cases of each pattern to predict the 5th and the 10th structure. It is expected and suggested that students can describe their predictions of the 10th structure orally or picture a rough draft without drawing the complete picture of the 10th structure. After students share their predictions, ask them to explore the structure number and the corresponding pattern that relates the structure number to the structure using numeric expressions. Although it is possible that some students attempt to draw all figures up to the 10th structure, some may generalize the growth they identified by expressing the 10th structure with words, pictures, or numbers.

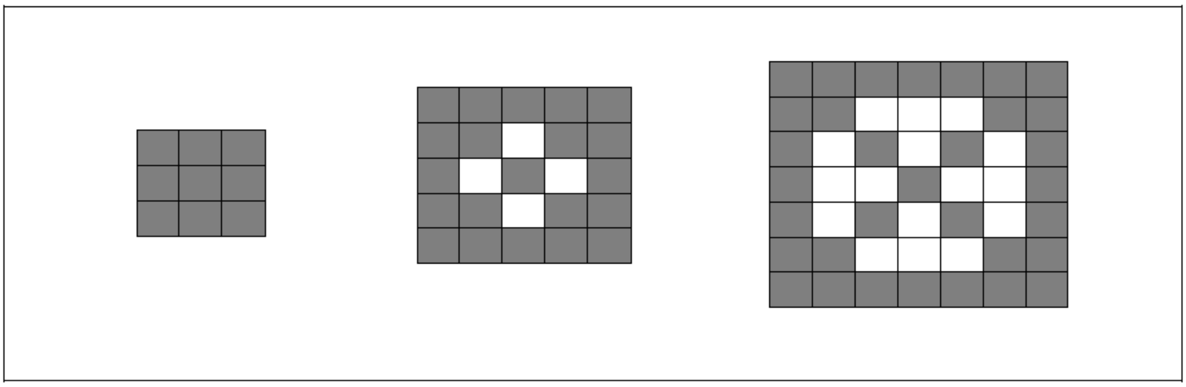


Figure 2. Pattern 1 on Playground 1

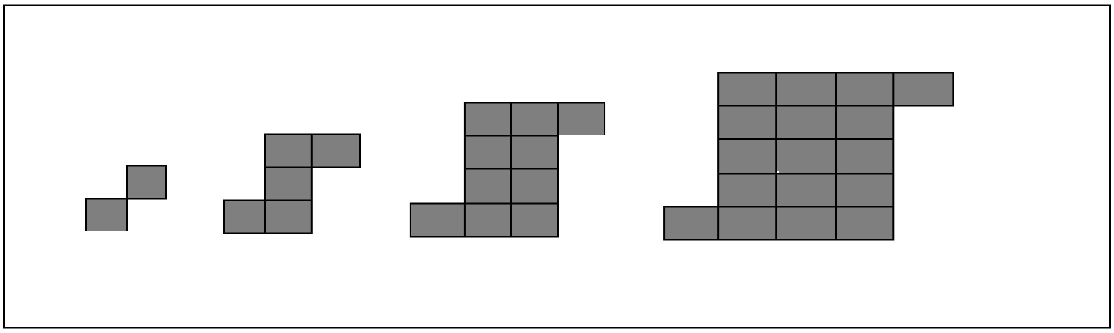


Figure 3. Pattern 2 on Playground 2

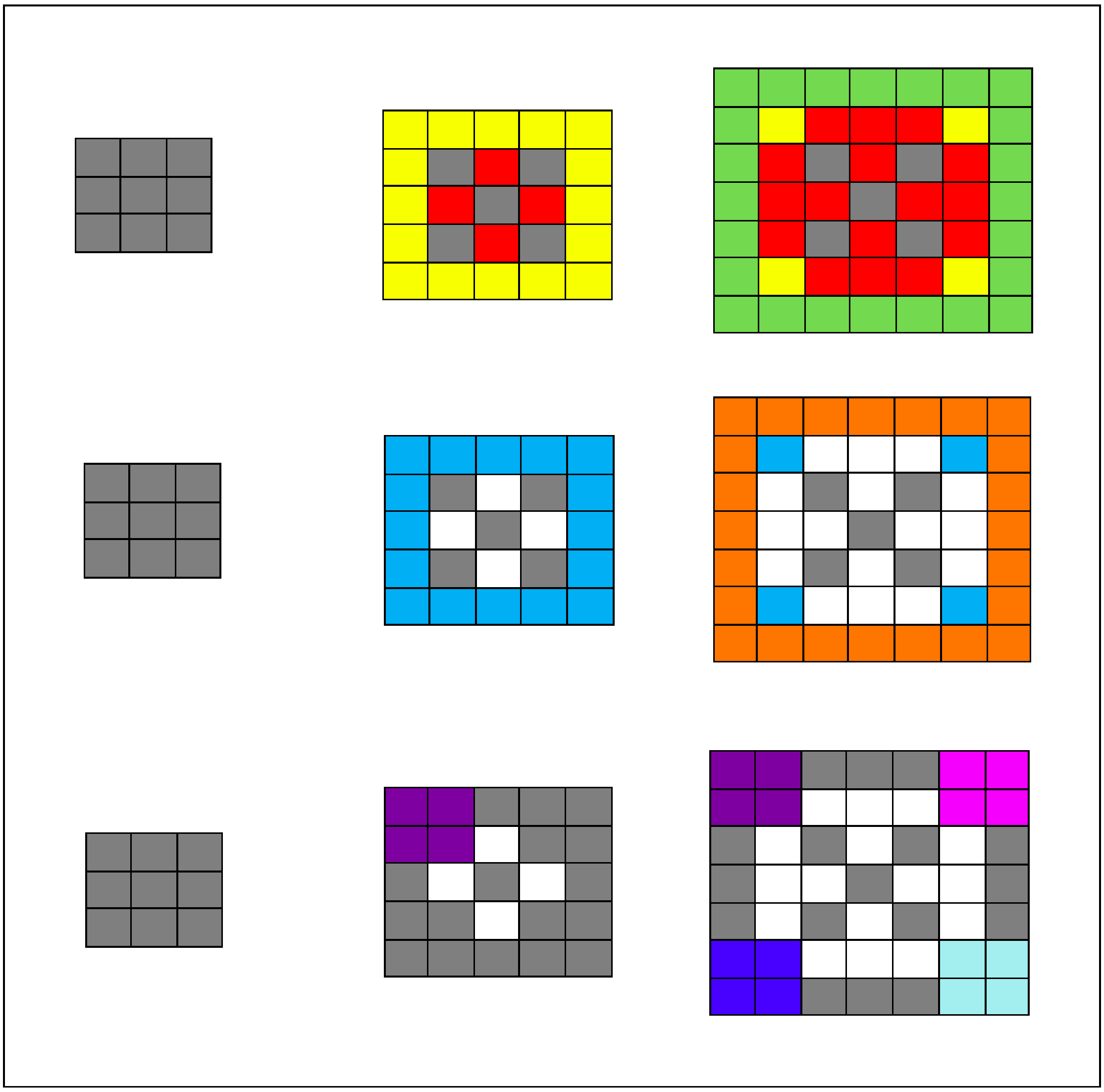


Figure 4a. Three Examples from Students' Work Pattern 1 on Playground 1

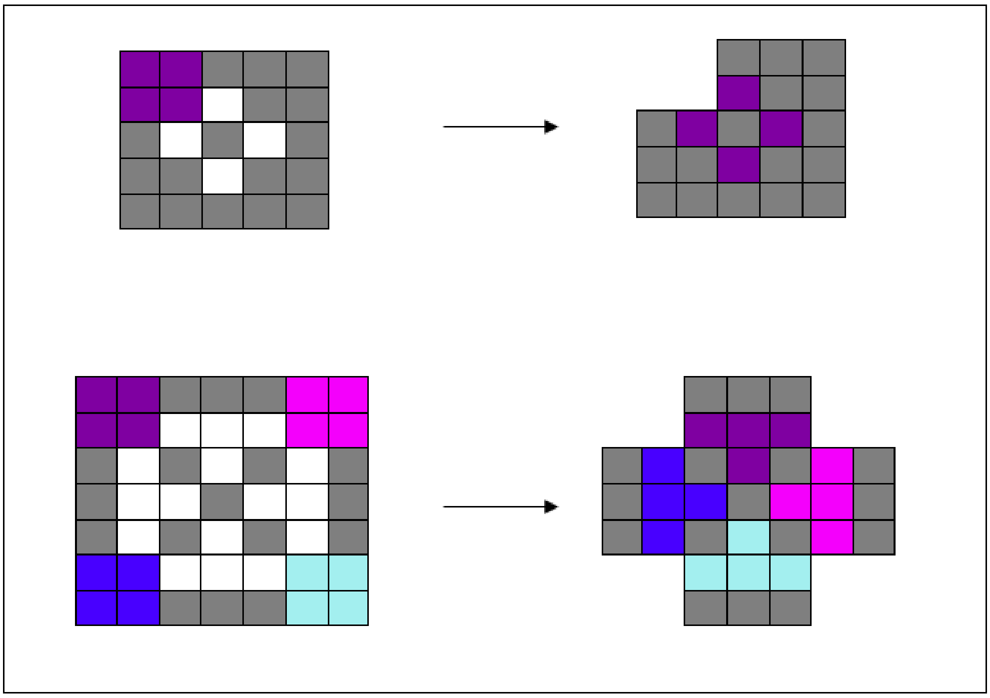


Figure 4b. Extension of the Third Example from the Student's Work Pattern 1 on Playground 1

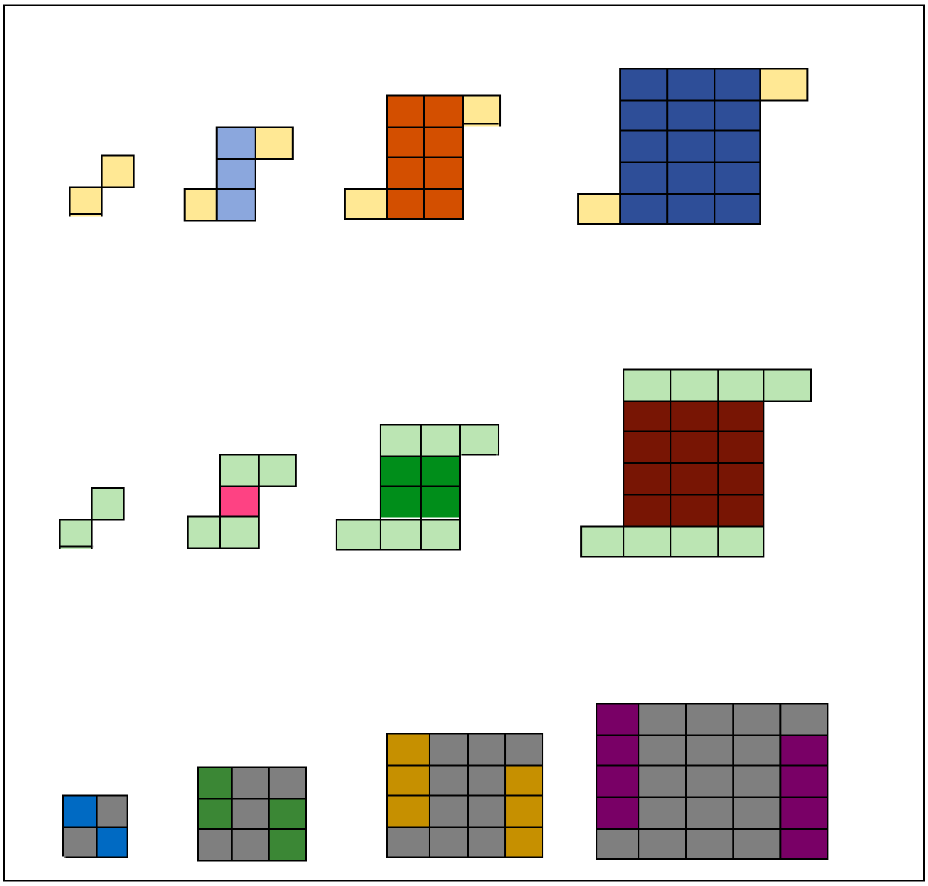


Figure 5. Three Examples from Students' Work Pattern 2 on Playground 2

Explain

After students attempt to draw the 10th case, ask them if it is possible to find the number of squares in the 50th and then 100th case for each pattern. Encourage each group to describe how they saw the number of tiles in the patterns and how they saw each pattern grow. Ask students if there is a better way to organize the information for the patterns. Listen for the suggestion of a table or chart. Hand out the tables for playground 1 and 2 (see Appendix C). Lead students through recording the tile numbers and growth for playground 1. Have students then try and fill out the table for playground 2 on their own. Lead a class discussion comparing the two patterns. They may say that playground 1 image has 3 figures and playground 2 image has 4 figures. Be sure to discuss how they add the same amount each time in the first playground image (add 12 tiles), but increasing amounts in the second playground image are as: add 3, add 5, add 7. Each pattern has square shape (with the square being in the middle of pattern 2), one pattern has missing pieces in the middle, both patterns grow.

Extend

Have students graph each pattern (see Appendix D) to connect with the standard CCSS.MATH.CONTENT.5.G.A.1 by using the x-axis as the figure number and the y-axis as the number of tiles. For example, the linear growth on the playground 1 can be shown as each time students move to the next figure on the x-axis, the number of tiles increases by 12 on the y-axis. After graphing each pattern, ask students how the table and graph are connected. Another option is to have students create a pattern from a given table (see table options in Appendix E and grid paper on Appendix F). After students have created their own pattern to fit the table, have them meet with a partner who has a different pattern to discuss similarities and differences.

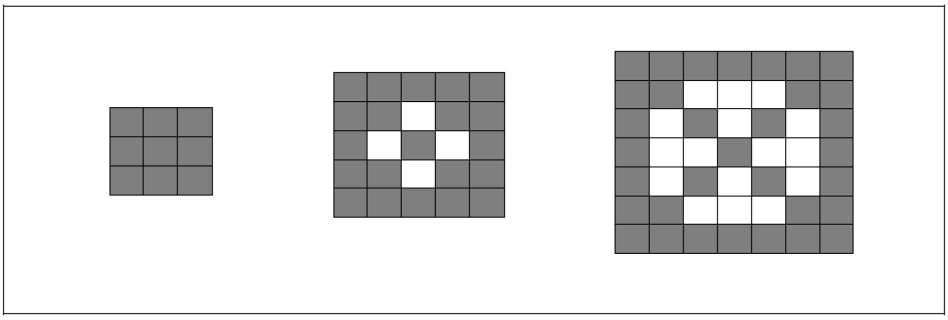
Evaluate

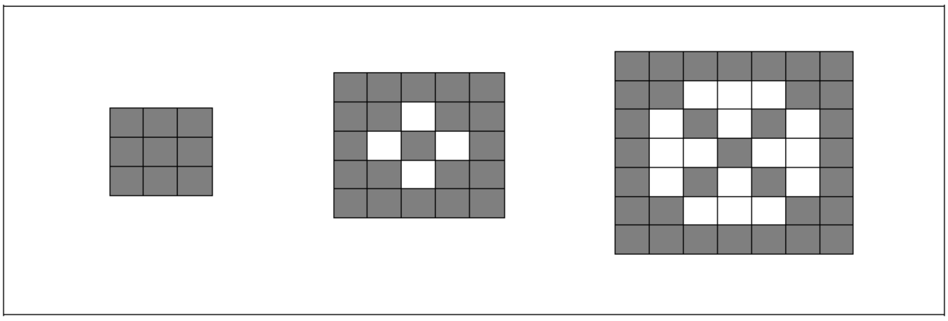
As students are showing multiple ways of representing their pattern in small groups, group members may only see one or two ways to represent the given growth pattern. To help students see multiple growth patterns, the teacher can suggest rotating the page or folding the paper in half to only view half of the pattern at one time. These suggestions can help the students expand their mathematical creativity.

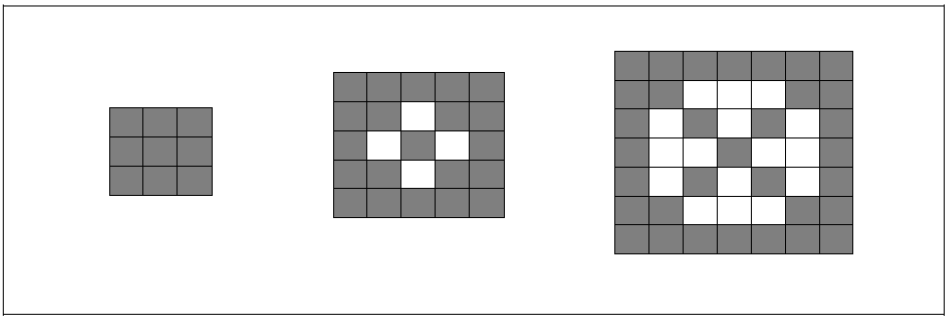
The student explanations about how they are counting tiles to see the pattern growth in playground image 1 can be used as an informal formative assessment piece to evaluate students' understanding. For example, in playground image 1, students might count the outside solid tiles and identify there are 3 in figure 1, 5 in figure 2, and 7 in figure 3. Moreover, students may decide to count the inside white tiles, and note there are 0 in playground 1, figure 1, 4 in figure 2 and 16 in figure 3. Teachers can help students understand the growth of plus 12 between each figure by putting the data into a table based on the discussion with the students. Teachers can do a formal assessment by using playground image 2 as students are asked to work independently to find the numeric pattern for figure 1, 2, 3 and 4. Teachers can observe students finding numerical patterns and how to put the numeric data into the table. For example, students identify a pattern of 2 in figure 1, 5 tiles in figure 2, 10 tiles in figure 3 and 17 tiles in figure 4. Additionally, the growth pattern would be +3, +5, +7, +9. As students finish making the table, if time allows teachers can prompt students to look for a second pattern growth of +2 in playground image 2. Teachers can see Appendix G to refer to as an answer key.

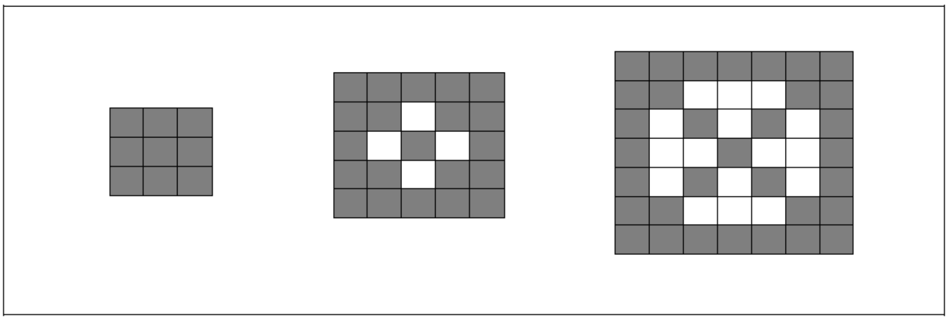
As students are counting the patterns in different ways, teachers can assess for students who can identify a growth pattern with or without drawing the figure to continue to scaffold the students' understanding. For example, the teacher may ask students to identify the numbers in the 5th pattern or the 10th pattern, and observe which students use the numerical numbers versus having to draw the pattern. Teacher feedback and guidance to help students build an understanding is an important step. If students are not able to see the numerical numbers, teachers might use the colors the students used and give a numerical value to the number of tiles that were shaded. As students understand the numerical growth pattern, teachers can help build students' understanding of the relationship between the case number and the corresponding number of tiles. Teaching students how to utilize a table and graphs (grid paper) to investigate the relationship between the two quantities is an important next step.

Appendix A

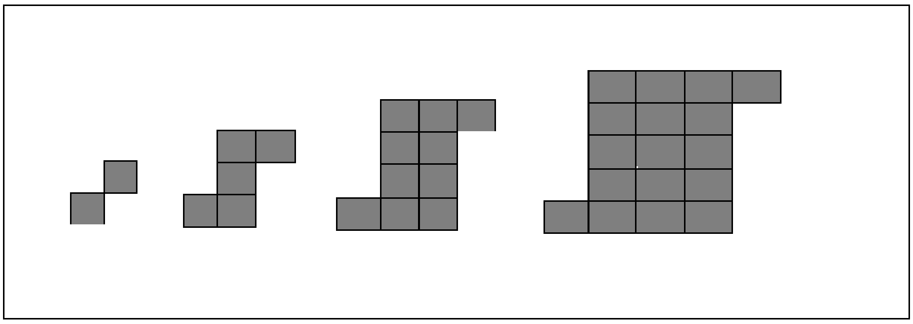


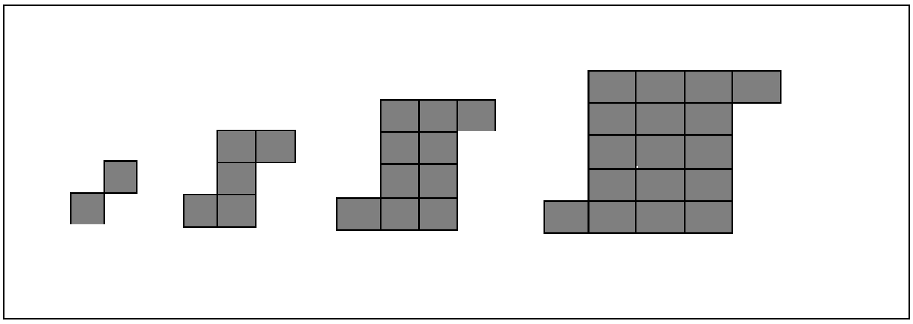


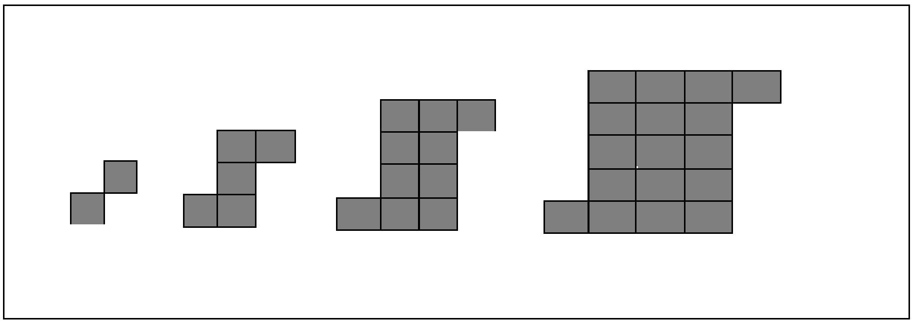


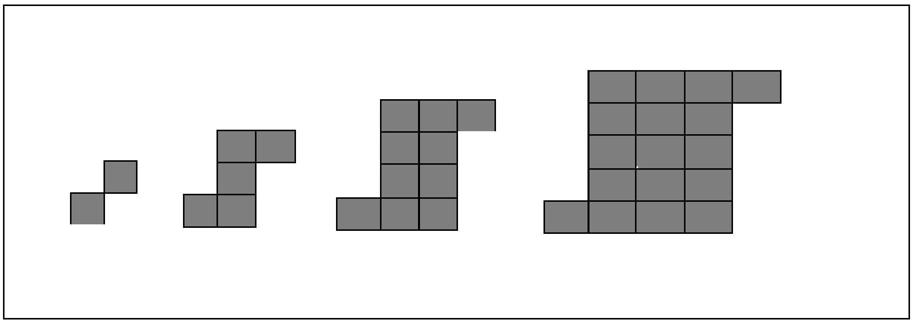


Appendix B









Appendix C

Playground Image 1

Figure Number	Number of Tiles	Tile Growth
1		
2		
3		
4		

Playground Image 2

Figure Number	Number of Tiles	Tile Growth
1		
2		
3		
4		

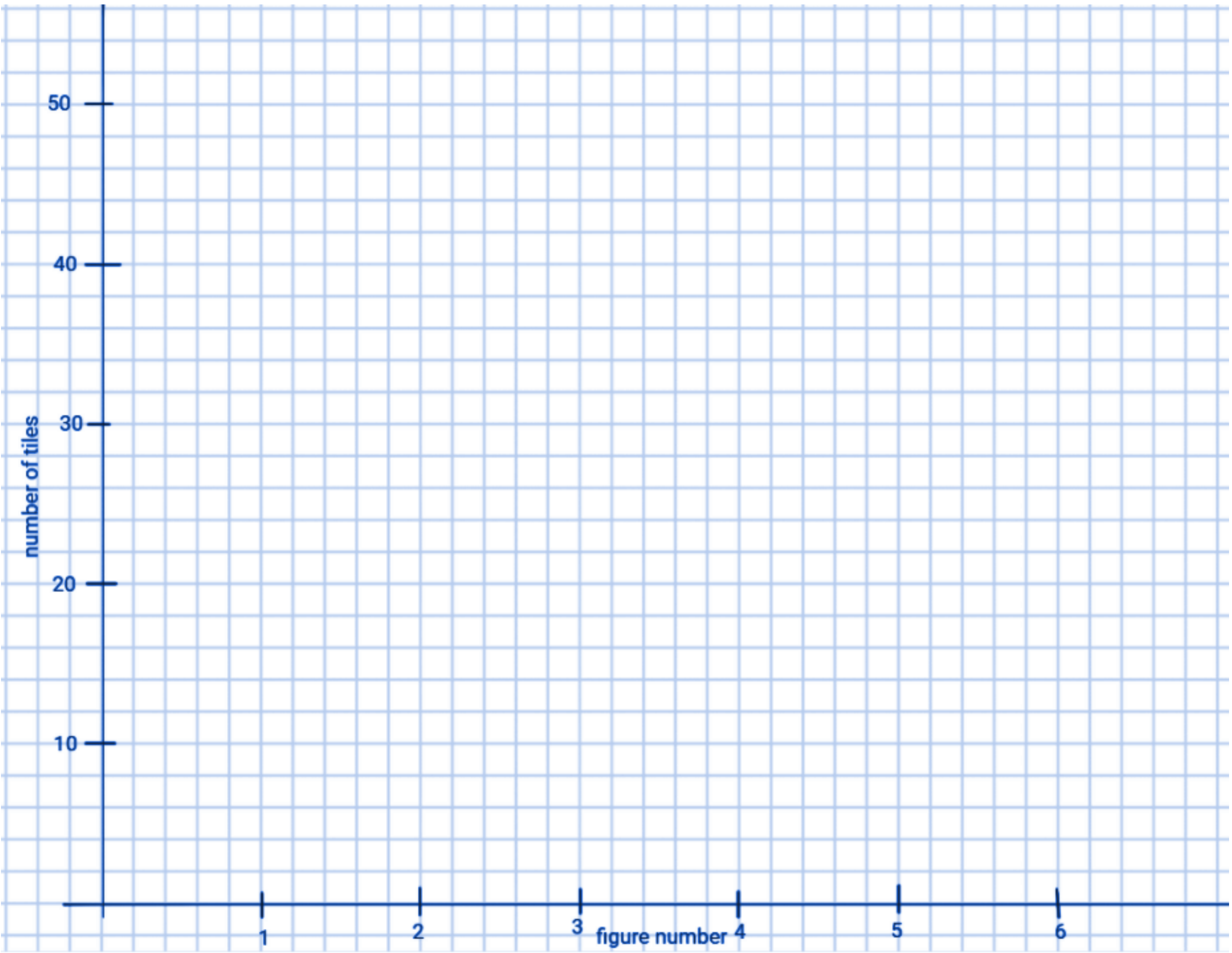
Playground Image 1

Figure Number	Number of Tiles	Tile Growth
1		
2		
3		
4		

Playground Image 2

Figure Number	Number of Tiles	Tile Growth
1		
2		
3		
4		

Appendix D



Appendix E

Figure Number	Number of Tiles
1	5
2	8
3	11
4	14

Figure Number	Number of Tiles
1	5
2	7
3	9
4	11

Figure Number	Number of Tiles
1	3
2	5
3	7
4	9

Figure Number	Number of Tiles
1	3
2	6
3	9
4	12

Appendix F

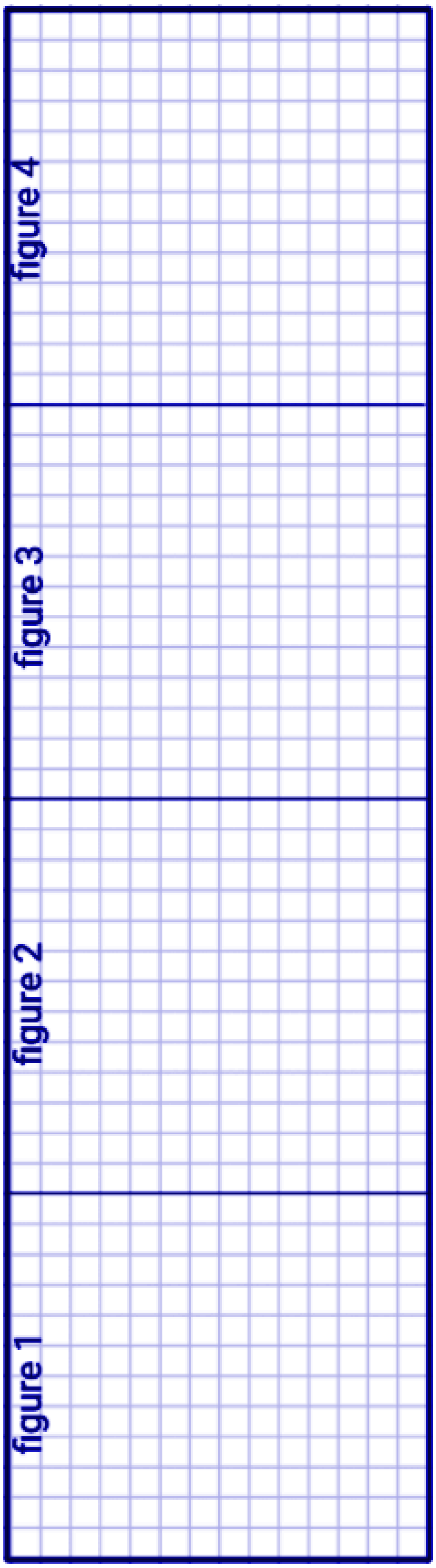


figure 4
figure 3
figure 2
figure 1

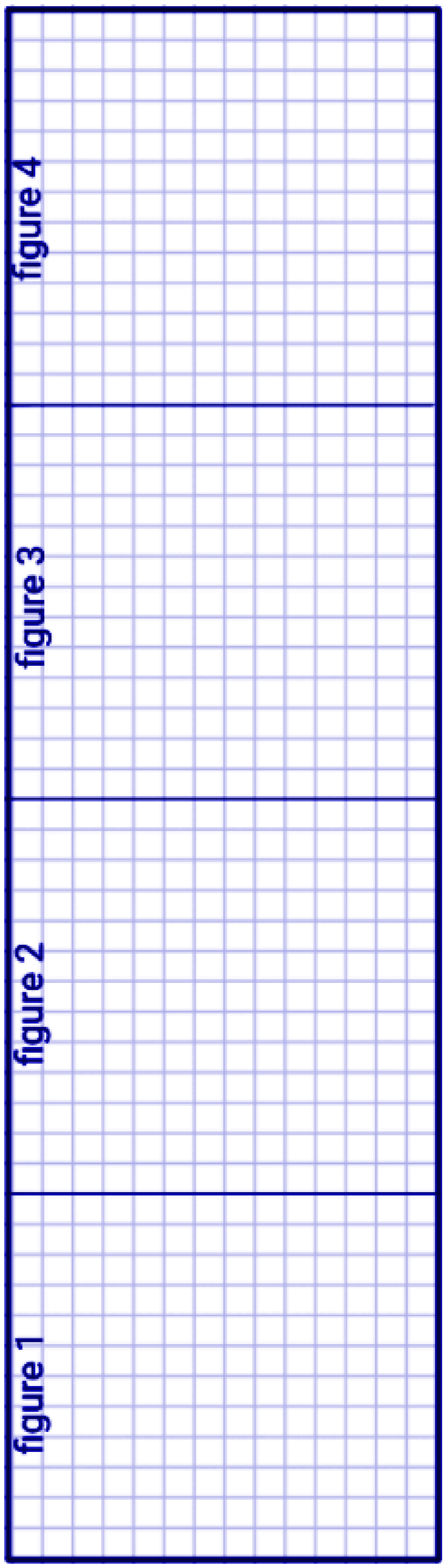


figure 4
figure 3
figure 2
figure 1

Appendix G

Playground Image 1

Figure Number	Number of Tiles	Tile Growth
1	9	+12
2	21	+12
3	33	+12
4	45	+12

Playground Image 2

Figure Number	Number of Tiles	Tile Growth
1	2	+3
2	5	+5
3	10	+7
4	15	+9

Citation

Carey, A. S., Quiroz, F., & Jackson, T. (2023). Making Playgrounds. In A. Bicer (Ed.), *Creativity-Directed Mathematical Tasks for 5th Grade Common Core Classrooms* (pp. 61-76). ISTES Organization.

Task 7 - Growing Squares

Jennifer Kellner, Amy Kassel, Chuck Butler

Math Content Standard

CCSS.MATH.CONTENT.5.OA.B.3

Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.

Supporting Content Standards

CCSS.MATH.CONTENT.5.G.A.1

Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate).

CCSS.MATH.CONTENT.5.G.A.2

Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.

Mathematical Practice Standards

CCSS.MATH.PRACTICE.MP1

Make sense of problems and persevere in solving them.

CCSS.MATH.PRACTICE.MP3

Construct viable arguments and critique the reasoning of others.

CCSS.MATH.PRACTICE.MP4

Model with mathematics.

CCSS.MATH.PRACTICE.MP7

Look for and make use of structure.

Lesson Objective

The objective of this lesson is to develop students' mathematical creativity by challenging them to identify multiple ways of seeing the growth of two patterns. Students will investigate how each pattern's extension to the 5^{th}, the 10^{th}, and the 100^{th} case using visual representations, verbal representations, numerical representations, and graphical representations. Students will make connections from the multiple representations by demonstrating how the representations are represented in different forms. Students will find similarities and differences between the patterns and the representations.

Engagement

(30-40 minutes) Show students some images of fractal patterns including Sierpiński's Triangle. Ask students what they notice. If necessary, the teacher should be prepared to prompt students for vocabulary words like patterns or repeating patterns. As a whole-class discussion, have students come up with their definition of a fractal. The definition of a fractal is "a never-ending pattern that repeats itself at different scales. This property is called 'Self-Similarity" (Fractal Foundation, n.d.). To further generate interest, teachers may choose from a variety of fractal experiences such as having your students create a Fractal Triangle or incorporate coding into your class by learning to code fractal generators (Fractal Foundation, n.d.).

After students have had an opportunity to explore fractals, the teacher will make the connection to this lesson by explaining that the students will also be exploring growing patterns. Give the students the following scenario: Three teachers in your school have each arranged the desks in their classrooms according to patterns 1, 2, and 3. Each week they add desks to their classroom because they have more students, creating the design shown in patterns 1, 2, and 3 respectively.

Put each of the patterns 1, 2, and 3 (see Appendix B, C, and D) on the classroom display and invite students to individually look at the patterns for 3-4 minutes to see how the patterns are changing. The teacher will then put students into groups of three or four and ask them to share how they see the patterns changing.

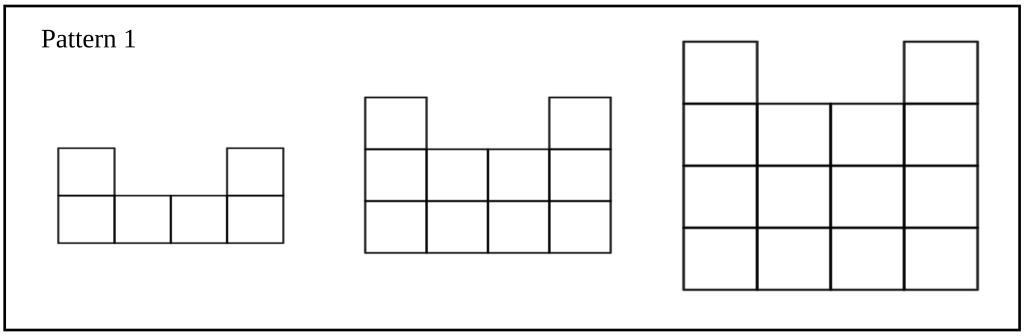


Figure 1. Three Cases of Pattern 1

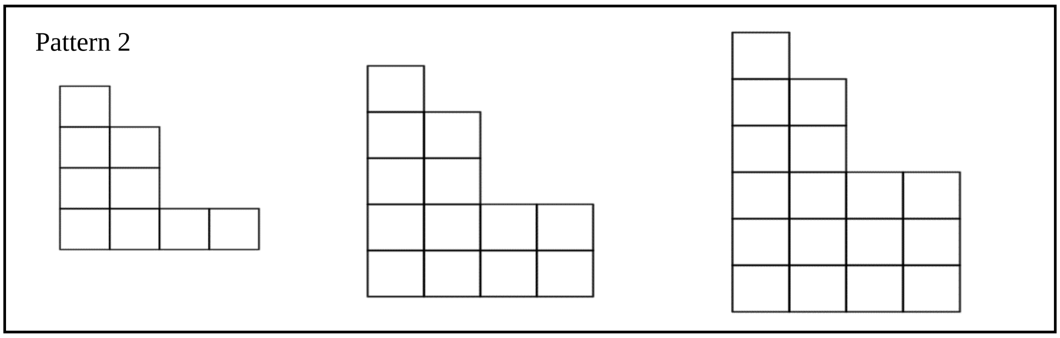


Figure 2. Three Cases of Pattern 2

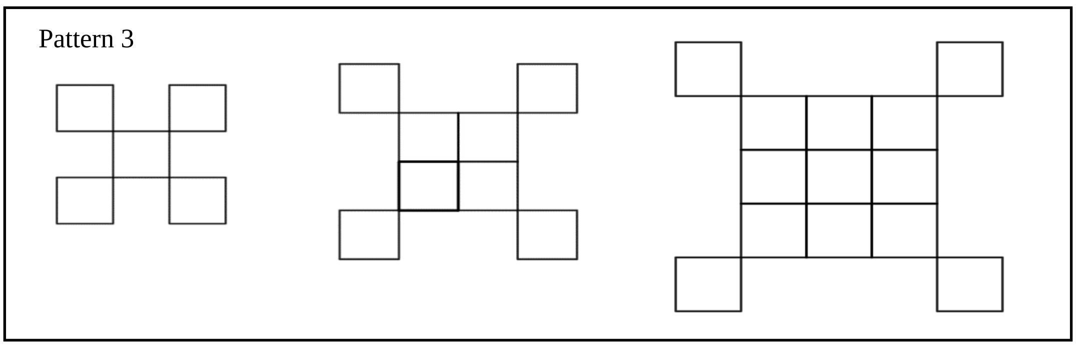


Figure 3. Three Cases of Pattern 3

Explore

(30 minutes) Part 1: Teachers provide students with handouts (see Appendix B, C, D, E) and colored pencils or markers (The first column of the tables in Appendix E is for case number. The second column is where students record the number of blocks in the pattern. The final column will be utilized in the Perimeter Extension.).

Students will work in teams of three to four. Each student should show how they saw the pattern changing by using color and explaining their way of seeing the pattern change to their teammates.

After each person has shared their way of seeing how the pattern changed, the team will move to the second part of the exploration (see Figures 4, 5, and 6 for potential ways students might see the pattern changing.)

Part 2: The team will select one of their ways of seeing how the pattern changed and express their thinking visually by using color, through writing by describing how the pattern is changing, and numerically. The team will then create the pattern for the 5th case using the grid paper provided.

Students will be asked to imagine the 10th case and describe how it might look or create a sketch of it or use numbers to describe it. Each team will post their work on poster paper for a gallery walk.

As teachers facilitate the exploration, they should be prepared to

- ask questions to prompt students to explain how they saw the pattern changing, encouraging the use of mathematical vocabulary.
- encourage students to make connections between the shapes, colors, written and verbal explanations.
- encourage written and visual representations of the growing pattern.
- look for and encourage multiple ways of seeing the pattern change.
- ask students how they might imagine the 10th pattern without drawing it out.

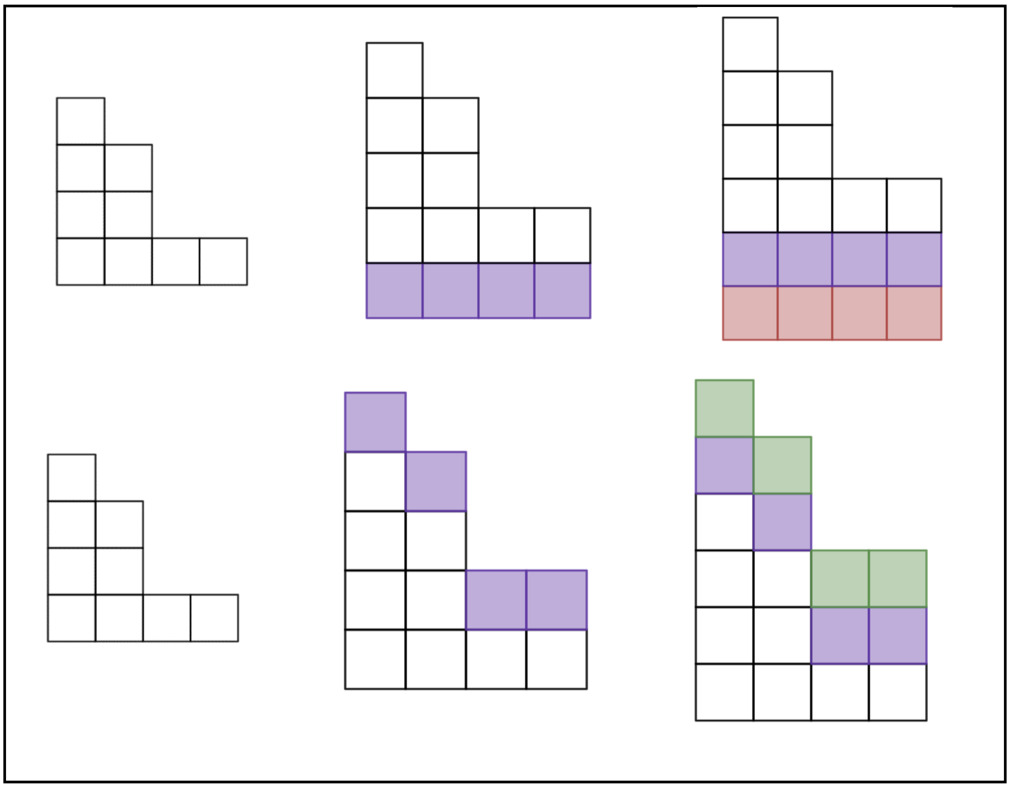


Figure 4. Two Examples of Student Work for Pattern 1

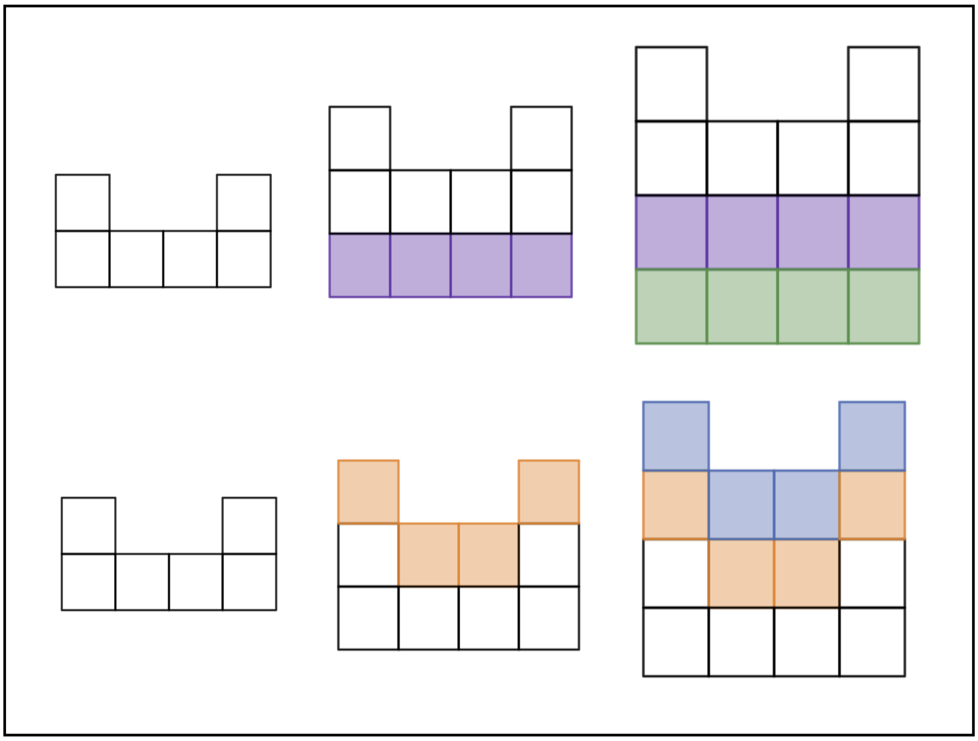


Figure 5. Two Examples of Student Work for Pattern 2

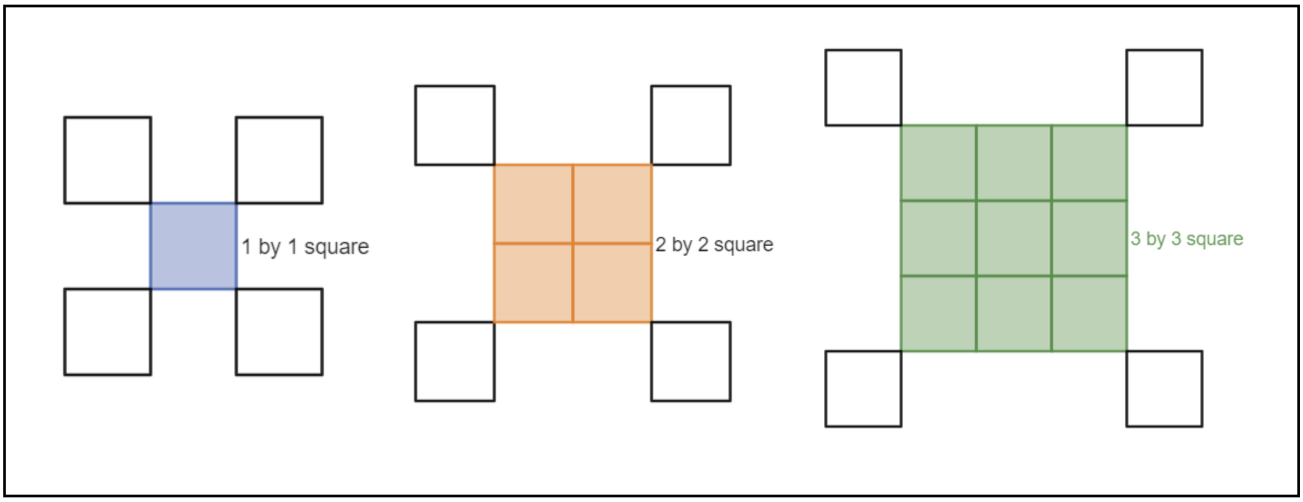


Figure 6. One Example of Student Work for Pattern 3

Explain

(20 minutes) A natural breaking point in this lesson is after students create their poster at the end of Part 2. The teacher and students should conduct a gallery walk. As students walk from poster to poster, the teacher should take time to consolidate the learning by asking someone not from the group to try to explain the thinking represented in the diagram (Liljedahl, 2021). This will allow students to learn from their peers and visualize other ways of seeing the patterns grow.

As the teacher structures the consolidation, posters representing ways of seeing the 10^{th} case for each pattern should be discussed. Ask students to generalize the ways in which their classmates were able to articulate the 10^{th} case. Then ask students if that would work for even bigger cases. Ask volunteers to explain how they might generate larger cases. If necessary, the teacher should be prepared to prompt students to look at the numerical values in each representation, perhaps even writing the values of the number of blocks in each case on a visual display of the patterns. The teacher can then demonstrate how to use a table to represent the values of the cases and the number of blocks.

As students are working on the patterns, they will most likely recognize that Patterns 1 and 2 are created by adding a constant amount to each figure but that Pattern 3 generates a pattern of side length × side length.

As students are finishing their tables for Part 2, the teacher may wish to extend their learning to creating ordered pairs (case number, number of tables). Fifth grade students are learning to plot ordered pairs by recognizing that the first value in the coordinate pair represents the distance traveled in the horizontal direction from the origin and the second value in the coordinate pair represents the distance traveled in the vertical direction. Students may practice plotting the ordered pairs for (case number, number of tables) using the coordinate plane on Appendix H. The teacher can incorporate technology by demonstrating the plotting of the points on the Desmos graphing calculator (see Figure 7). The teacher may expect students to show the pattern, table, written description of the pattern, and perhaps a graph for each pattern (see Figure 8).

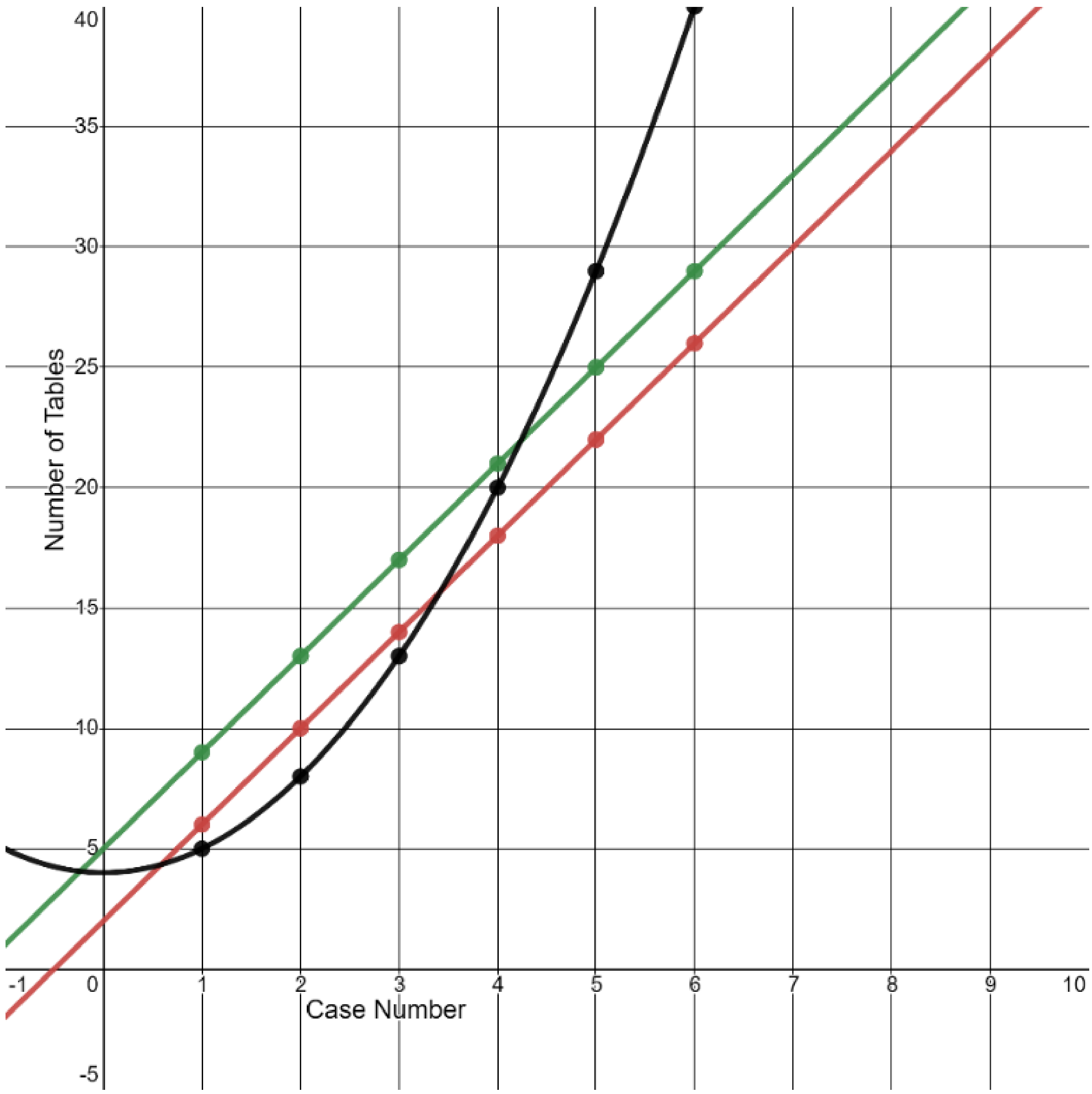
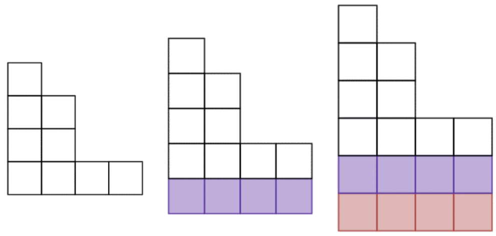


Figure 7. Plots of (Case Number, Number of Tables)



There are 9 tables in the first case. There are 4 tables added to make the next case.

n	t
1	9
2	13
3	17
4	21
5	25
6	29
7	33
8	37
9	41
10	45

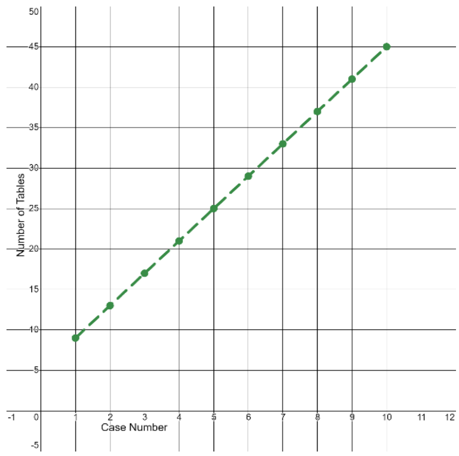


Figure 8. Example of Student Work showing Multiple Representations for Pattern 1

Extend

(15 minutes) The teacher may choose to assign one extension to the whole class or assign different extensions to different groups based on interest, ability, and readiness.

- **Sierpiński** - Students will investigate Sierpiński's Triangle to the 4th and the 5th using visual representations, verbal representations, numerical representations, and graphical representations. How many black triangles are in the 4th case, the 5th case, the 100th case, the nth case? How many white triangles are in the 4th case, the 5th case, the 100th case, the nth case? Hand out student version of Sierpiński's Triangle (see Appendix A).
- **Fourth Teacher** – After 10 weeks, a fourth teacher has 24 tables arranged creatively and quite differently than the other teachers. Students are asked to find the 5th and 10th cases and draw a representation for each case, showing how the pattern is growing. Hand out additional grid paper (see Appendix E).
- **Fifth Teacher -** For some strange reason, tables are leaving a fifth teacher's class. In the fifth week, the teacher had 34 desks. In the sixth week, the teacher had 28 desks. In the seventh week, the teacher had 22 desks. Students should create visual, verbal, numerical, and graphical representations of this situation and then find 10th and 11th cases. Hand out additional grid paper (see Appendix E).

- **Perimeter** - The teachers have decided that they are only seating students around the exposed edges of the tables. If each table can seat one student on a side which teacher's pattern can seat the most students with the fewest number of tables after 4 weeks? 5 weeks? 6 weeks? Hand out tables (see Appendix F) and additional grid paper (see Appendix E).
- **Quadratic Pattern** – As students recognize that Pattern 3 is created by side length x side length, the teacher should support this connection by facilitating a brief discussion on exponents and that $n \times n$ may be written as n^2.

Evaluate

(10 minutes)

- In part 1, formative assessment for creativity will occur as students explain through drawings and conversation how they see the shape growing. The teacher should give each student individual think time (2 minutes) in order to give time for the class to produce multiple representations. If the teacher notices that a group is going to have all the same representation, then the teacher can rearrange the student groupings or ask students to turn the shape and find a different representation. Students that finish early should try to find a more creative way to describe how the shapes are growing.
- In part 2, formative assessment for the standard will occur as the teacher asks how the numerical expression connects to the picture. For example, in pattern 1, the teacher could ask "how does the four in the expression connect to the visual?" Teachers can provide feedback about how each term in the unsimplified expression represents a part of the image.
- In Sierpiński's Triangle, formative assessment for creativity will occur as the students explain how the triangle is changing for each case. For example, in case 2, a student could see that a white triangle divided the black triangle into equal parts, resulting in 3 black triangles and one white triangle. For case 3, the student could see that a smaller white triangle divided each black triangle into equal parts, resulting in 3 black triangles in each part for a total of 9 black triangles.
- In Fourth Teacher, the teacher challenges the students to think of ways that this shape could be growing and can encourage students to use a different operation addition and subtraction.

- In Fifth Teacher, teachers can challenge students to visualize what the image would look like if the number of tables was below zero. This is an opportunity for students to creatively explore the idea.
- In Perimeter, the teacher should be prepared to support students in connecting the number of students to the perimeter of the shape through guided questioning. Ask students questions to build their understanding of the relationships between the inputs and outputs generated by the patterns. Place students in small groups to discuss how to answer the question "If each table can seat one student on a side which teacher's pattern can seat the most students with the fewest number of tables after 4 weeks? 5 weeks? 6 weeks?"
- The teacher should display the tables generated by the students and facilitate an analysis of the tables to decide which pattern might be best to seat the greatest number of students with the fewest tables.

References

Fractal Foundation. (n.d.). Retrieved January 28, 2022, from https://fractalfoundation.org/resources/what-are-fractals/

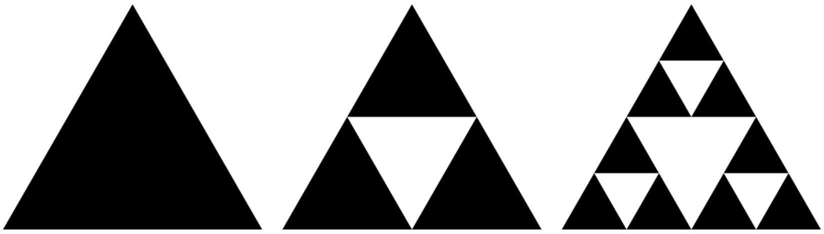
Liljedahl, P. (2021). *Building Thinking Classrooms in Mathematics.* Thousand Oaks: Corwin.

Sierpiński triangle. (2022). *Sierpiński triangle*. Retrieved from Wikipedia on February 2, 2022: https://en.wikipedia.org/wiki/Sierpi%C5%84ski_triangle

Appendix A.

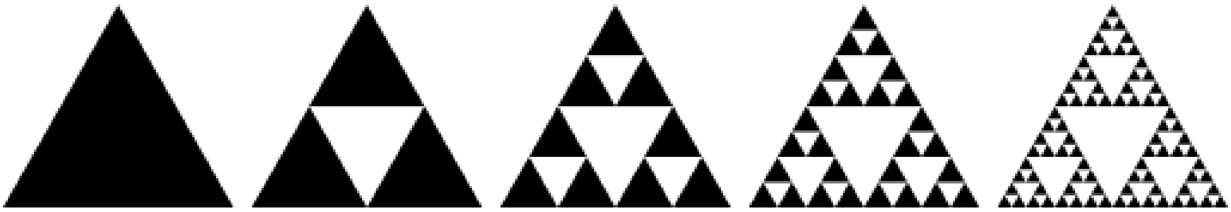
Sierpiński Triangle (Sierpiński triangle, 2022)

Student Version



Sierpiński Triangle		
Case Number	Number of Black Triangles	Number of White Triangles
1		
2		
3		
4		
5		
100		

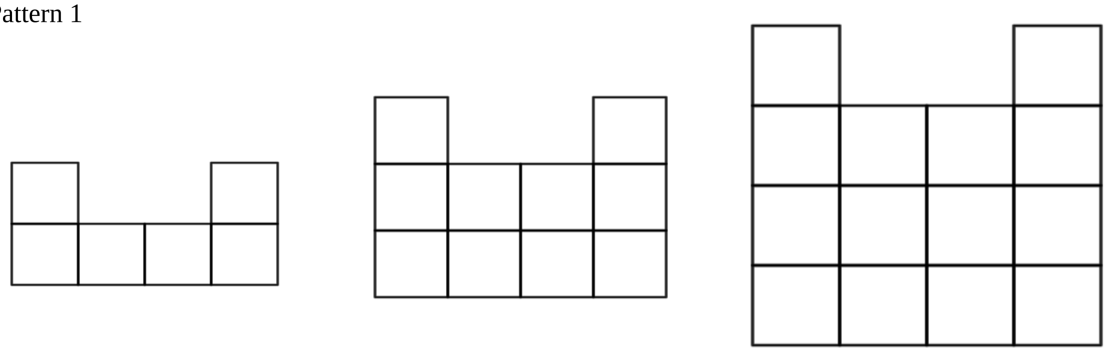
Teacher Version



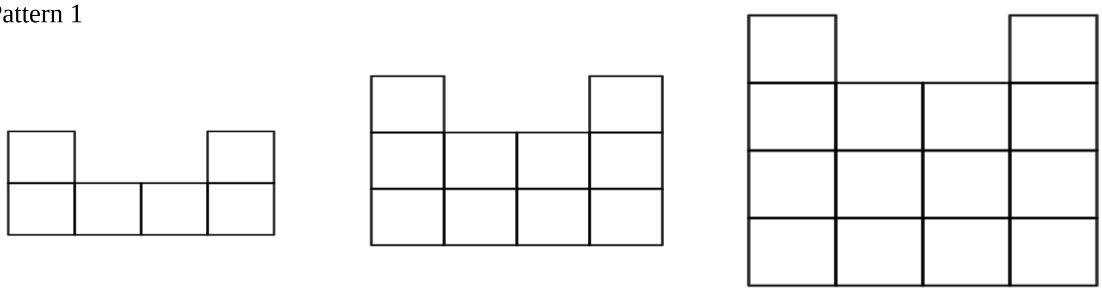
Sierpiński Triangle		
Case Number	Number of Black Triangles	Number of White Triangles
1	1	0
2	3	1
3	9	4
4	27	13
5	81	40
100	3^{99}	$3^{98} + 3^{97} + 3^{96} + \cdots$

Appendix B

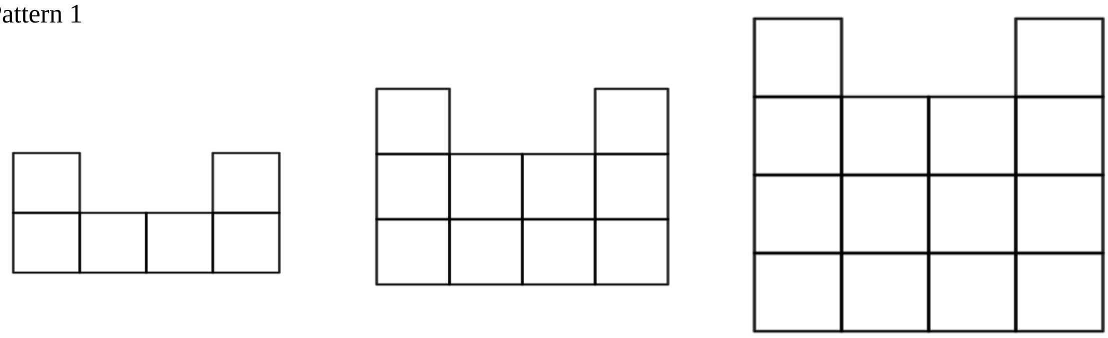
Pattern 1



Pattern 1

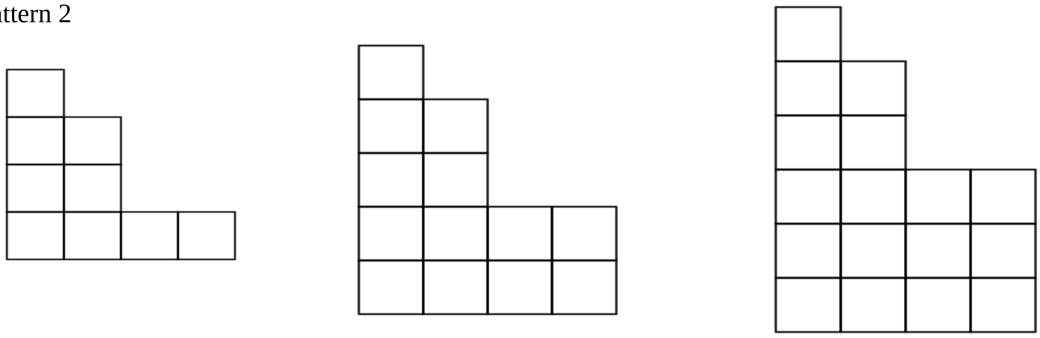


Pattern 1

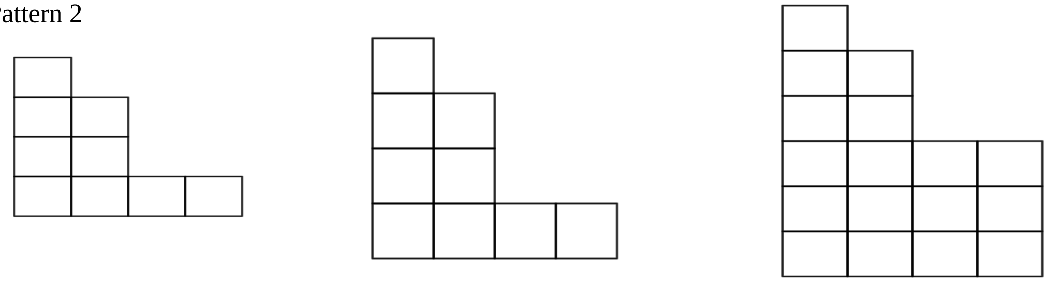


Appendix C

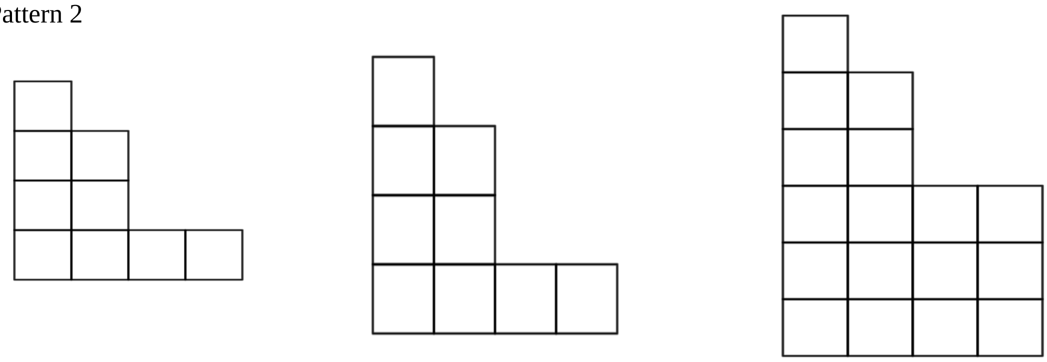
Pattern 2



Pattern 2

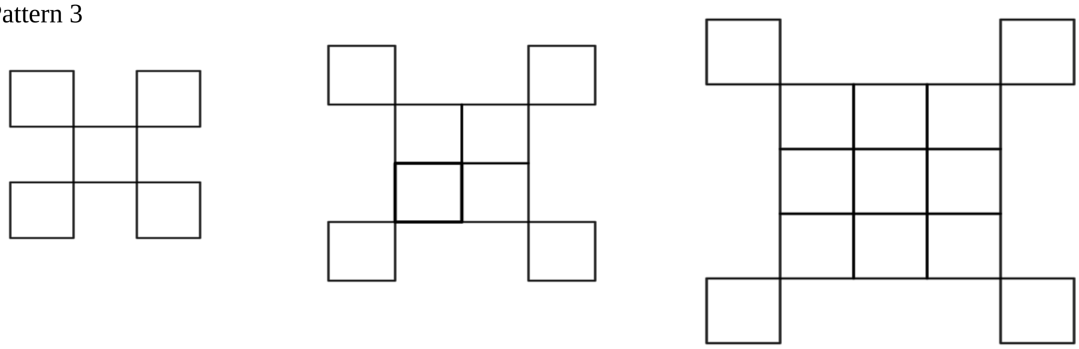


Pattern 2

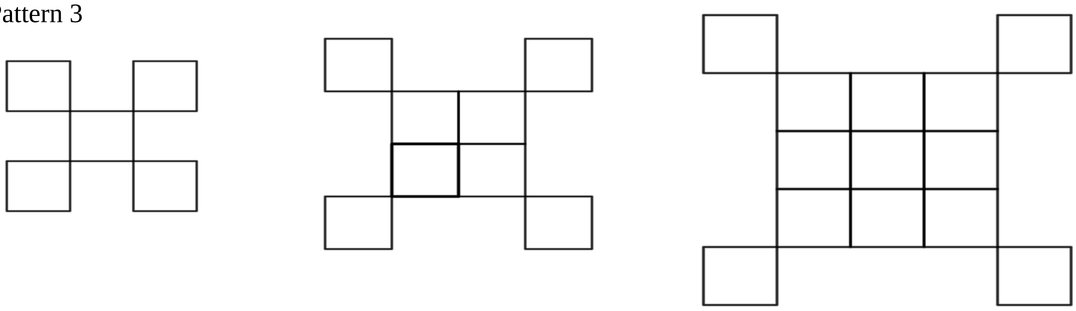


Appendix D

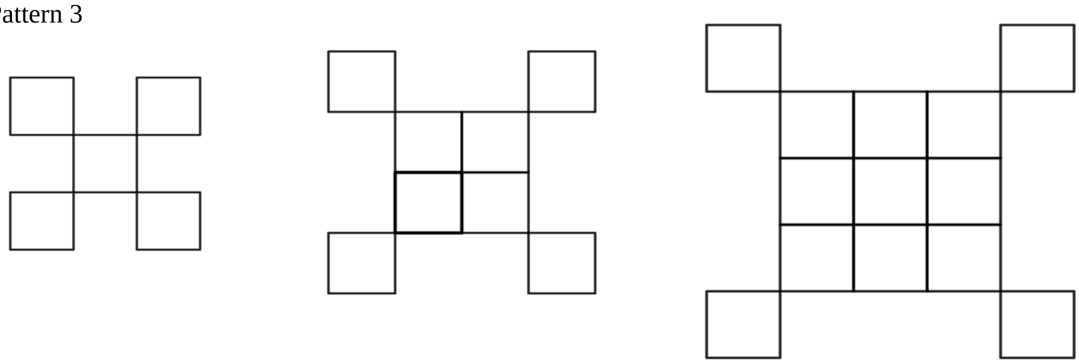
Pattern 3



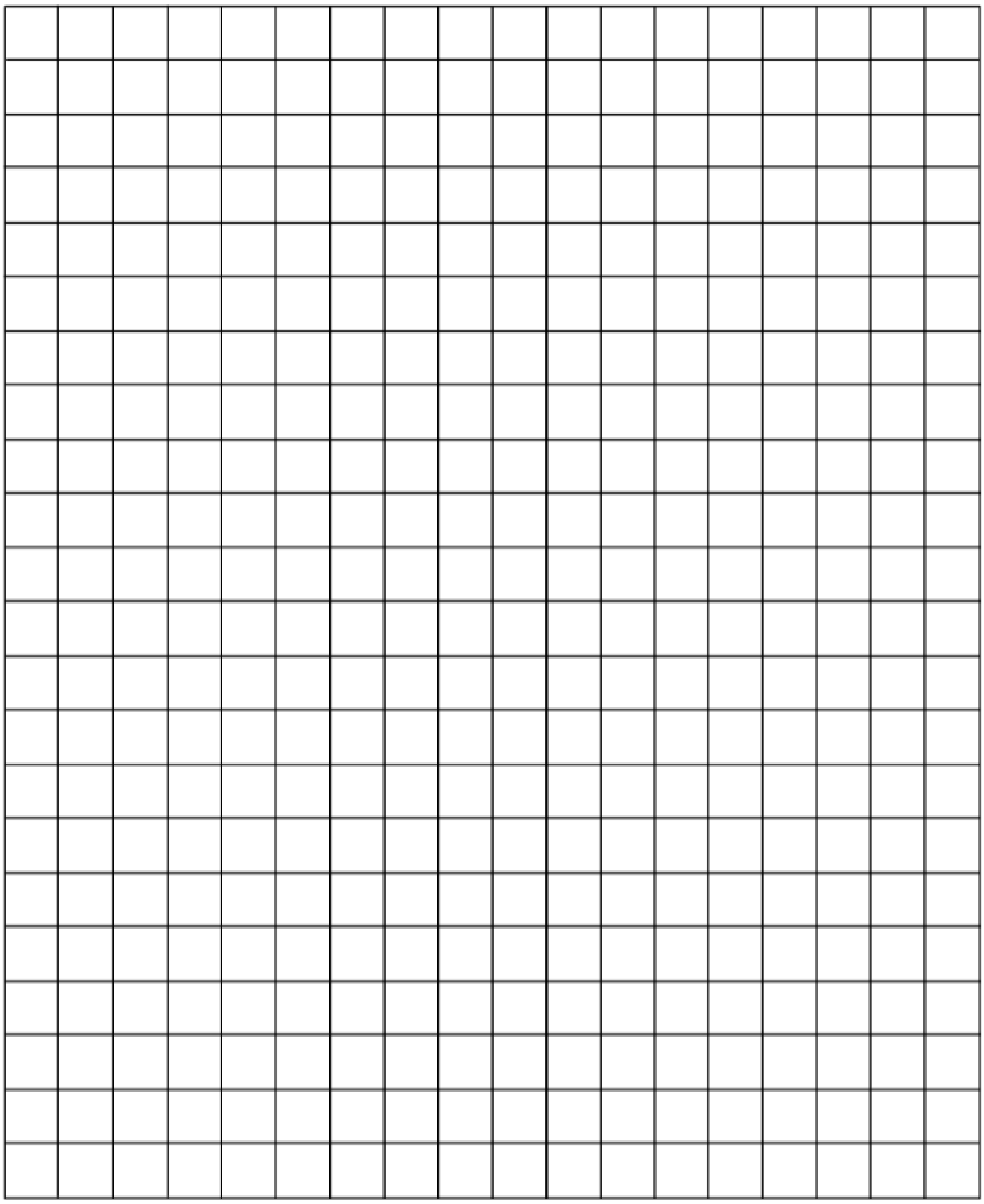
Pattern 3



Pattern 3



Appendix E



Appendix F

Pattern 1

Case Number	Number of Tables	Number of Seats for Students

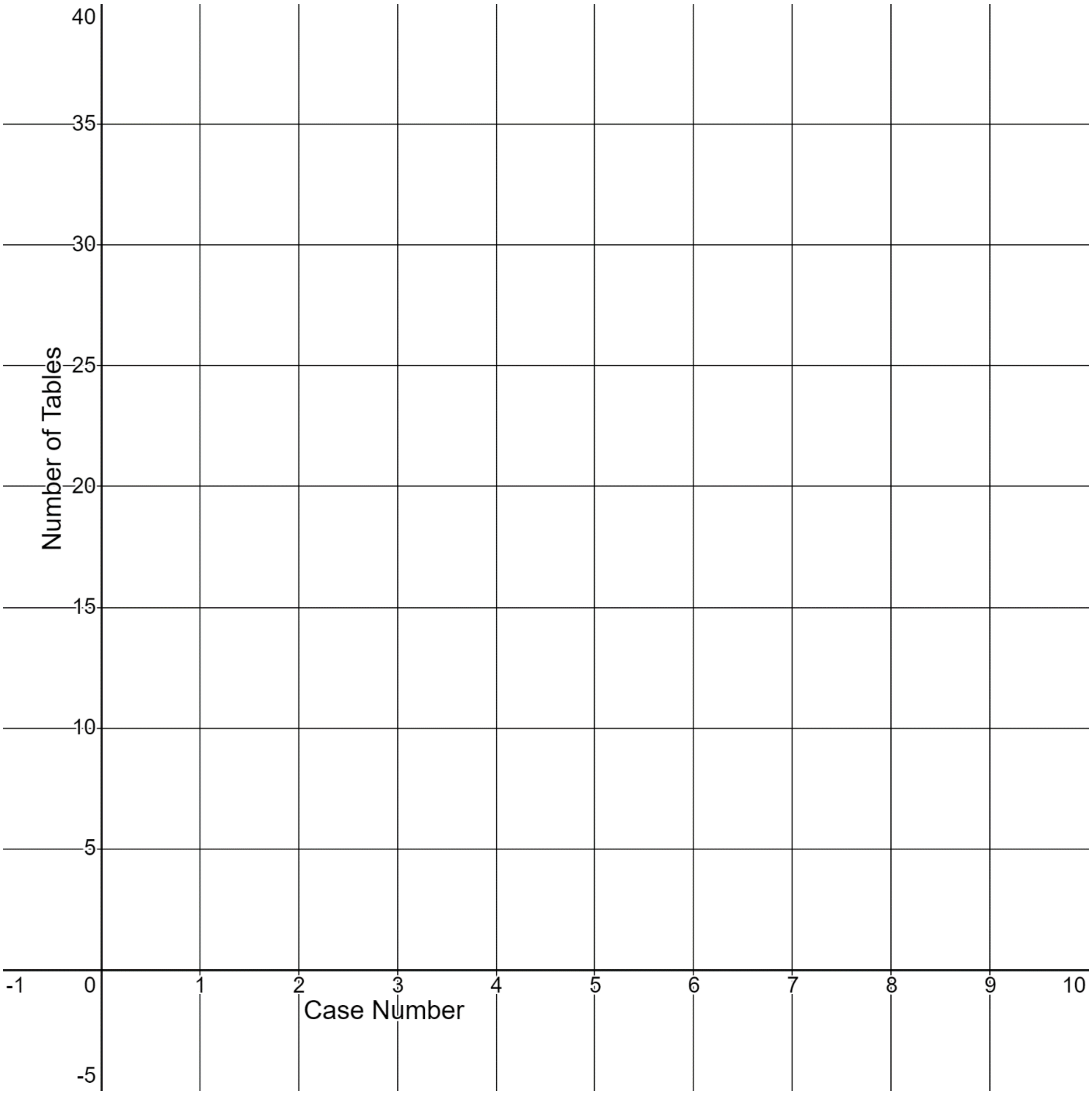
Pattern 2

Case Number	Number of Tables	Number of Seats for Students

Pattern 3

Case Number	Number of Tables	Number of Seats for Students

Appendix G


40
35
30
25
20
15
10
5
0
-5
-1
1
2
3
4
5
6
7
8
9
10
Number of Tables
Case Number

Citation

Kellner, J., Kassel, A., & Butler, C. (2023). Growing Squares. In A. Bicer (Ed.), *Creativity-Directed Mathematical Tasks for 5th Grade Common Core Classrooms* (pp. 77-96). ISTES Organization.

Task 8 - Patterns Everywhere!

Geoff Krall, Helen Aleksani

Mathematical Content Standards

Focus Content Standard

CCSS.MATH.CONTENT.5.OA.B.3

Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.

Mathematical Practice Standards

CCSS.MATH.PRACTICE.MP2

Reason abstractly and quantitatively.

CCSS.MATH.PRACTICE.MP5

Use appropriate tools strategically.

CCSS.MATH.PRACTICE.MP8

Look for and express regularity in repeated reasoning.

Lesson Objective

Students will explore patterns using tangram tiles and create conjectures on rule(s) that govern the pattern. Using visuals, words, expressions, tables, and physical or digital manipulatives, students will demonstrate mathematical creativity by crafting their own patterns. Students will see the connection between the multiple representations of the patterns.

Engagement

(15 minutes) Start the lesson by asking students where they see patterns in their everyday life. Some answers may include nature, architecture, or even patterns within the room. Tell students that patterns are everywhere and that we are going to play with patterns and create our own. First, let's look at some beautiful patterns inspired by Islamic art.

Show students this TEDed video on Islamic Patterns: https://www.youtube.com/watch?v=pg1NpMmPv48

During the video, the children are asked to take notes concerning what kind of mathematics they see. They might identify specific shapes or tiles. Show students additional pictures of Islamic art that uses patterns and tiles, such as this website: https://en.wikipedia.org/wiki/Tiled_Kiosk)

Pictures with Patterns and Tiles

Tell students, "In this lesson, you will learn to predict the next step of visual patterns and even create your own."

Teacher displays the following pattern (generated using Mathigon.org/polypad) and asks students to recreate the pattern using pattern blocks.

Figure 1

Students are given 3 to 4 minutes to think about creating the pattern individually, allowing the students to see the problem and patterns in the figure in their own way. After students are done working individually, have them share with a partner so they start developing additional ideas as a pair or assist each other in generalizing their strategy for the classroom discussion.

For students who don't know where to begin, it may be useful to ask some starter questions such as:

- How many different shapes do you notice?
- What do you notice about the different colors in each shape?

To help students prepare their response for the group and class discussion, teacher can prompt the following questions:

- What do you notice about this pattern?
- What stays the same? What is changing?
- Using words describe what the fourth figure in the pattern would look like.
- Ask students to create the fourth shape using pattern blocks.

Students now work with their groups and prepare a response on how they see the growth of the given pattern.

Explore

(30 minutes) Students are divided into groups of three and provided a handout to work as a group. On every handout, there are three to four copies of the given pattern so every student can mark and draw their discovery of the pattern on the handout (see Appendix A). Do not discourage those students who are making a similar conclusion to their group members. Instead, encourage these students to understand their group member's point of view. If a group finishes early, challenge them to look for multiple different ways to extend the pattern. To help with this step, provide students with different color pens or pencils to organize and color code their responses (see Figure 2 for some examples).

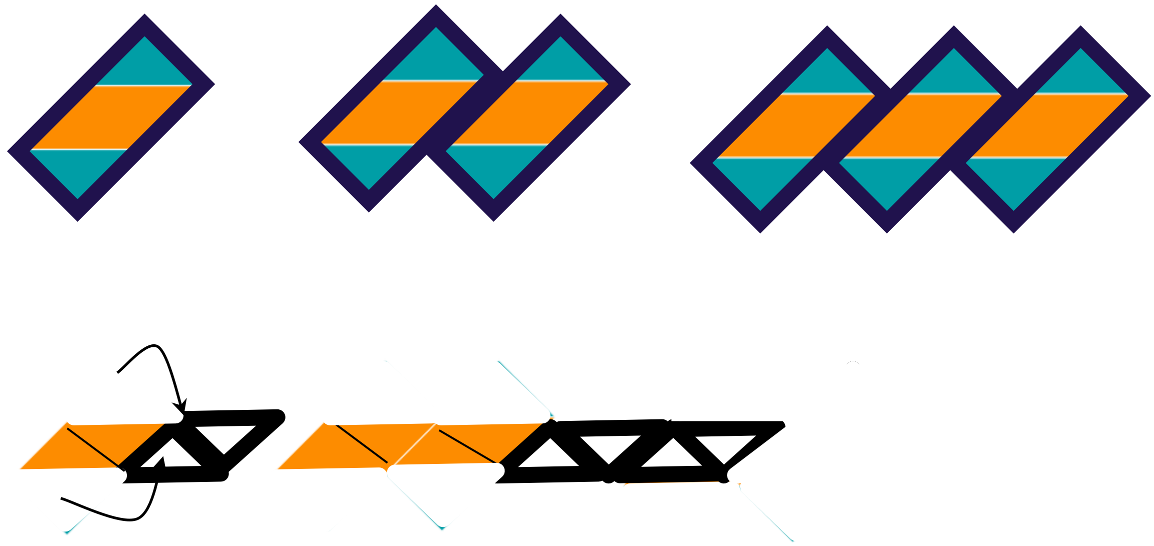


Figure 2. Two Examples Presented by Students on the given Pattern

Gallery walk. When groups are done by making their changes, have one member stay with the group and send the other two students to visit the different groups and learn about a different representation of the pattern so he/she can bring some ideas back to share with their group. At this stage, encourage students to think about the 10th or the 20th step. At this point, make it clear to the students that they don't need to draw a picture of these steps. Instead, they can use their imagination to predict the step and make a rough sketch of what they think the step should look like. After that, lead the discussion by encouraging students to explore a relationship between the step and the shapes in every step, and if they can start thinking about a strategy that would help predict the steps faster and easier. Provide the groups with 5 to 8 minutes to discuss their findings. Meanwhile, you can start walking around the class listening to their group discussions. Watch for students who find the pattern in different ways. Make note of their strategies. These different strategies developed by the students will be the focus of the discussion at the end, allowing students to connect to their prior work from previous mathematical experiences. If some groups struggle with noticing the relationship, provide the following facts and the way of thinking. The teacher asks students what's happening to the blue tiles, orange tiles, and total tiles.

- Blue tiles are growing by two each time.
- Orange tiles are growing by one each time.
- Total tiles are growing by 3 each time.

.

Teacher: These are the "rules" that govern this pattern. The teacher asks, "how many blue tiles, orange tiles, and total tiles will be in the 5th shape? The 10th shape?" Have students record their responses on the recording sheet (see Appendix B).

If extra practice is needed, teachers can also apply the steps discussed above to the following visual pattern.

Now let's look at another pattern. The teacher asks students to create the following pattern using pattern blocks.

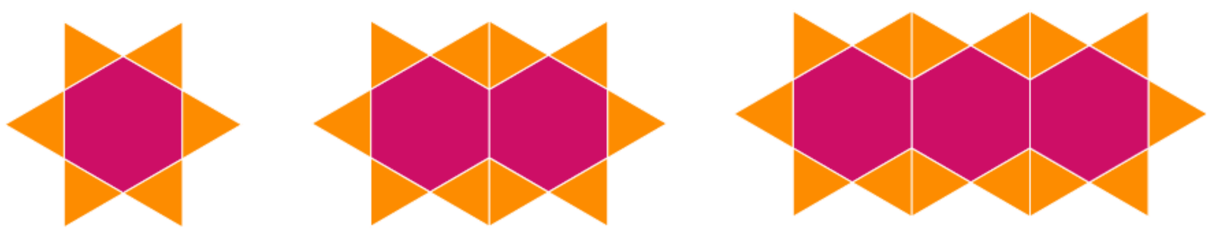


Figure 3

Ask the students what's the "rule" that governs this pattern.

- The hexagon increases by one each time.
- The pattern of triangles is 6, 10, 14

Have students predict the number of triangles in the fourth element of the pattern. Then have them create it using pattern blocks.

Ask students to predict the number of triangles in the 5th, 10th, and 20th shape. Have students record their responses on the recording sheet (see Appendix C).

Explain

(30 minutes) To help students see the value in developing mathematical rules to predict the pattern, ask students to find the number of rectangles, parallelogram, and the triangles of the first figure for the 100th step. For many students, this becomes a challenging task trying to draw or imagine the image for the 100th step. Once students understand the challenges involved in drawing an image for the 100th step, have students find the number of rectangles, triangles, and the parallelograms by developing a method. Provide the group with 5 to 7 minutes to share some ideas. Then, have one student stay with their group and ask the other two members to go to different groups to learn about their approach so they can go back to their group and share with their members to gain some ideas. At this point share some mathematical models such as creating tables with the students to help them learn to organize their thoughts and findings (see Appendix E). Remember to provide extra time for the groups to record their numbers in the provided table so they can start discussing the numerical patterns they were able to observe from one step to the next. Since the standard covered in this lesson also requires students to plot points on a graph, we can introduce the connection between drawing graphs and the tables. Students at this level can be given a graph paper so they can plot the points on three different graphs. Each graph can represent a shape from their pattern. For instance, one graph can focus on the number of rectangles while the other one can represent the number of rectangles. For example, in step one from the first visual pattern, there is one rectangle, two triangles and one parallelogram. Then, have students mark their x-axis representing the steps and the y-axis representing the number of different shapes in each step. To make sure all students know how to plot points, the teacher can model the first two steps showing the students how the coordinate point is presented here. For example, for the number of rectangles in step 1, we have (1,1). The first number in the set represents the step and the second number represents the number of rectangles in that step. The teacher then models how to plot the point on the graph in case some students are still struggling with

plotting points from their previous lesson (see Appendix F). After graphing the tables, students can connect the points on the graphs to help them compare the rate of change from each table. Direct the students to pay attention how fast the graphs are changing when comparing the number of rectangles, triangles, and the parallelograms from each step.

If the teacher decides to use the second pattern for practice in class as well, then the teacher can lead students in a discussion: what makes the pattern in Figure 2 different or more challenging than Figure 1? Students may notice that the triangles in the pattern in Figure 1 increase by the same number that was in the first element of the pattern (there are two triangles in the first shape and it increases by 2). In Figure 2, however, the triangles increase by four, which is different from the number of triangles in the first element (6). Ask students why that might be? One answer might be that the hexagons share a side, so that eliminates two triangles (one for each hexagon).

Once students are comfortable talking about rules for patterns, ask students to create their own patterns using pattern blocks. Have students use their mathematical creativity to create patterns using pattern blocks and/or drawings. Then students can challenge one another to find the next shape in the pattern as well as the rule that governs their pattern. Have students record their responses on the recording sheet (see Appendix D).

Extend

(20 minutes) Have students identify the missing pictures in the middle of a pattern (see Figure 3), and ask students to find the two missing steps in the middle of Figure 4.

Figure 3

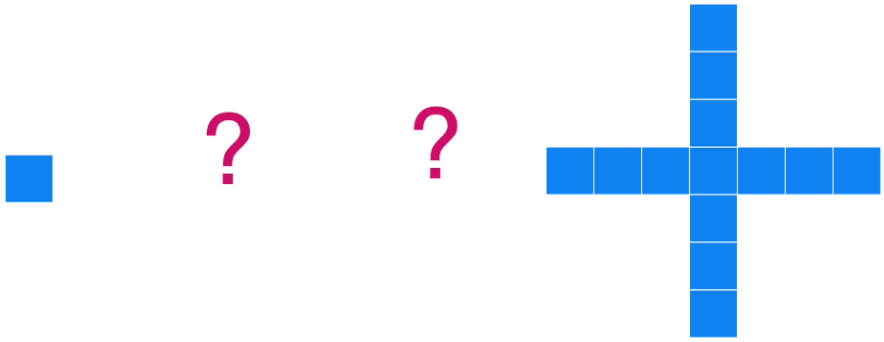


Figure 4

Require the students to create a table and a graph for figure 4. Ask the groups to identify different ways of seeing the pattern by their group members using color pencils. There is a great opportunity for teachers at this step to start introducing variables using words when modeling the pattern in Figure 4.

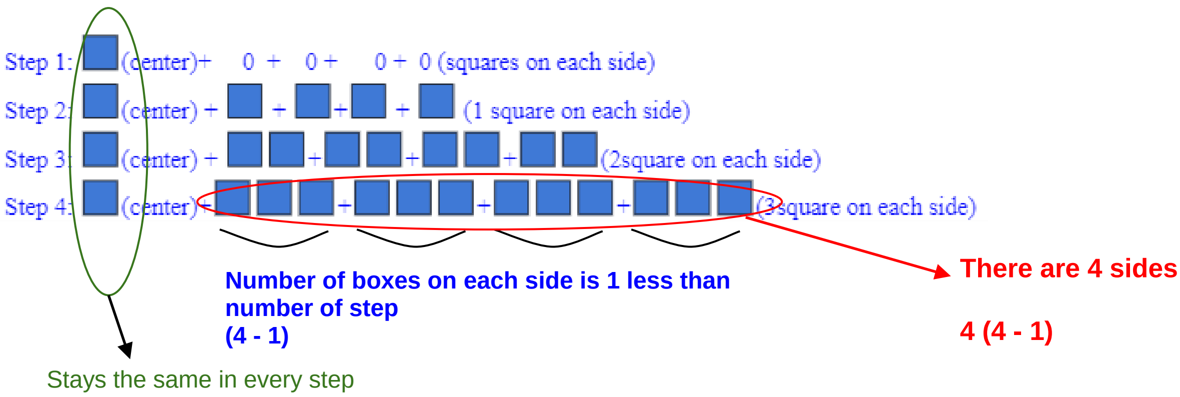
Step 1: (center)+ 0 + 0 + 0 + 0 (squares on each side)

Step 2: (center) + + + + (1 square on each side)

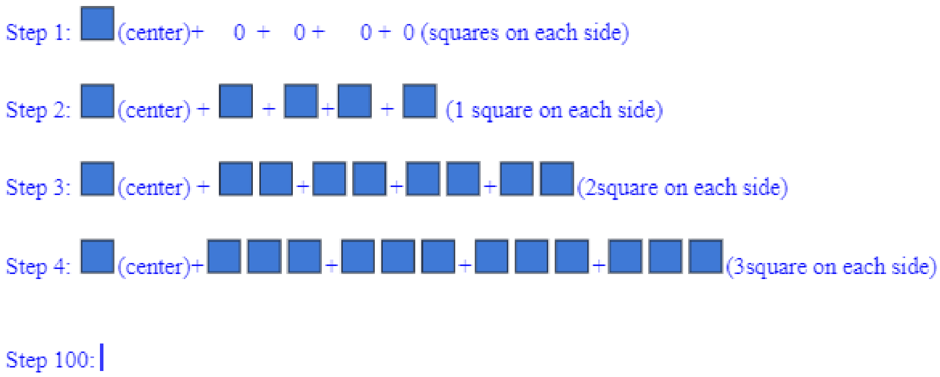
Step 3: (center) + + + + (2square on each side)

Step 4: (center)+ + + + (3square on each side)

Some teacher's notes can include the following:



Then, apply the mathematical pattern found to all the steps, including step 100, to help students make connections between numbers, words and number of boxes.



Although it is not expected of students to use variables representing the unknown at this level, it is critical that students start using words to help translate mathematical connections into expressions.

Evaluate

This lesson promotes creativity by encouraging students to share their way of discovering the visual pattern and apply it to the next step in the pattern. However, since students are working in groups, there is a possibility that some groups won't be able to develop multiple different strategies to discover the pattern. Therefore, it is critical for the teacher to prepare some direct questions or hints to help students discover a method other than the one they've come up with. Also, allowing students to visit other groups to learn about a different approach and have them go back to their group and describe it can be served as another strategy to help students gain skills in collaboration.

Appendix A

Appendix B

Recording sheet for Pattern 1

Shape Number	Draw the shape	Number of quadrilaterals	Number of triangles	Tot number of shapes
1				
2				
3				
4				
5				

Appendix C

Recording sheet for Pattern 2

Shape Number	Draw the shape	Number of quadrilaterals	Number of triangles	Tot number of shapes
1				
2				
3				
4				
5				

Appendix D

Recording sheet for Student created patterns

Whose pattern was this?

Shape Number	Draw the shape			
1				
2				
3				
4				
5				

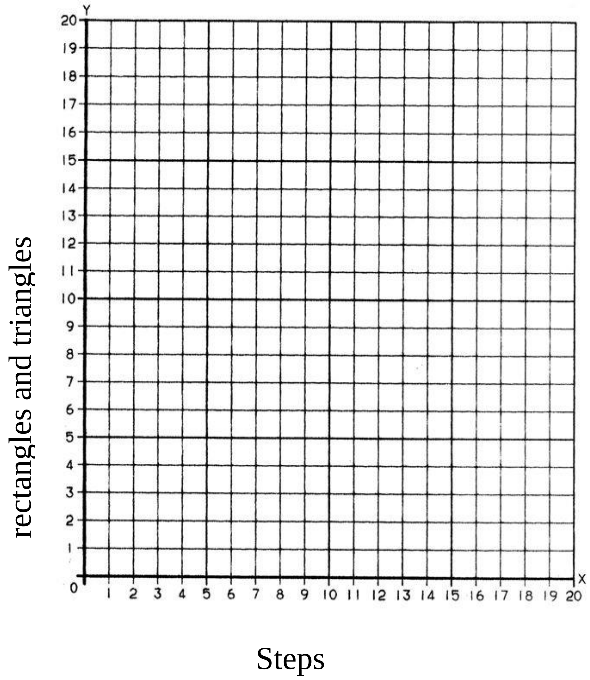
Appendix E

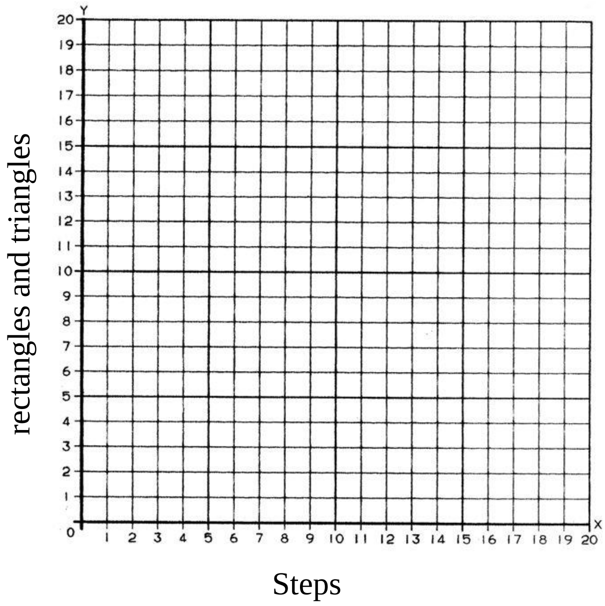
Table for the first visual pattern to use during the Explain section

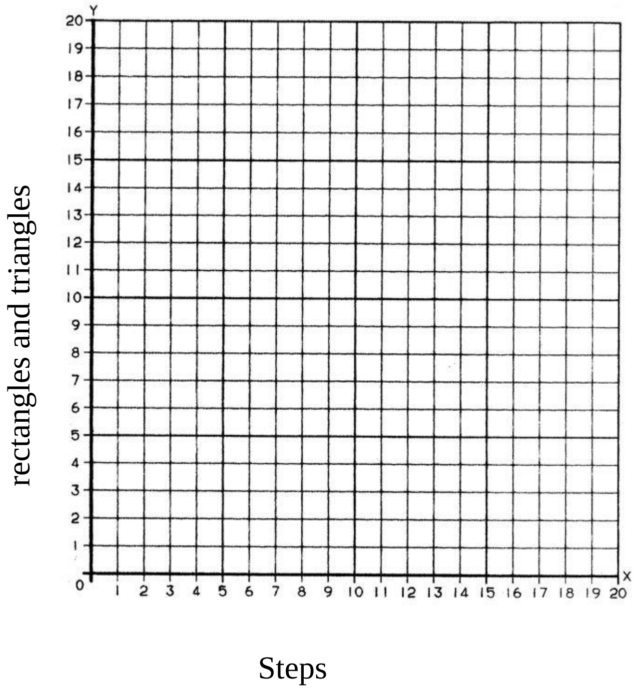
Step	Number of rectangles	Number of parallelograms	Number of rectangles
1			
2			
3			
4			
5			
6			

Appendix F

First quadrant coordinate plane for the first visual pattern






rectangles and triangles
Y
X
0
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
Steps

Citation

Krall, G., & Aleksani, H. (2023). Patterns Everywhere! In A. Bicer (Ed.), *Creativity-Directed Mathematical Tasks for 5th Grade Common Core Classrooms* (pp. 97-114). ISTES Organization.

SECTION 3 - UNDERSTAND THE PLACE VALUE SYSTEM

Task 9 - Smart Money

Melena Osborne, Michael Gundlach, Michelle Tudor

Mathematical Content Standards

CCCS.5.NBT.A.1

Recognize that in a multi-digit number, a digit in one place represents 10 times as much as the digit in the place to its right and 1/10 of what it represents in the place to the left.

Mathematical Practice Standards

1. MP 2 Reason abstractly and quantitatively.
2. MP 4 Model with mathematics
3. Look for and make use of structure.

Lesson Objective

Students will learn the power of ten through the use of money. Students' mathematical creativity will be supported by creating number lines and plotting tenths and hundredths by using their knowledge about the value of money. They will learn how units of money like penny, dime, dollar, ten dollars are all 10 times the one after the given unit or 1/10 of the one before the given unit. Students will be asked to use this information to increase or decrease a given amount of money. This is intended to be an introductory lesson for this standard and is intended to take 1-2 class periods.

Engagement

Read the Poem *Smart* by Shel Silverstein (Appendix A). Ask the students to keep track of what happens to the boy's money as the poem is read. Ask students: How much does the boy end up with? (Answer: 5 cents). What is his thinking process? Have students turn and talk with a partner and then discuss as a group. Have students turn and talk about the value of the money vs. the number of pieces of money. Ask students to share. Then ask: If you were to re-write the poem, what changes would you make? Would you rather have $1 or 9 dimes? How do you know $1 is more?

Explore

Give students a partially completed place value chart and a place value chart that has the amount of money that matches the place value. (Appendix B) Ask the students to fill in the missing numbers on the place value chart. Tell them to use the money as a clue if they need it. Tell them to look for a pattern and identify it (for example, x10 as you move to the left). Students may say that a zero is added each time the number moves to the left. Guide them to determine what operation is happening. They may not notice the pattern from left to right, so ask them if they can identify a pattern going both ways. Then ask students to label each place value.

Next, ask students to think about how they could represent the chart in Appendix B on a number line. Start with 0-3 counting by one's; the students will plot tenths first – using the idea of money (they know dimes and pennies are between $0 and $1). Ask student where they think .10 would go, .20, .30, .40, so on and so forth.

Once students have completed this, repeat the exercise using only 0-1 on the number line and have students plot hundredths. If you notice them struggle, tell students to think about the number line as "zooming in" between each number (i.e. zooming in between 0 and 1 creates tenths, and zooming in between 0 and .1 creates hundredths). As an additional discussion for students, ask them what they think will happen if they "zoom out" on the number line.

Explain

Lead a discussion with students about what they have discovered about numbers as they move across the place value chart. Make sure to emphasize the decimal fraction words for each value less than one (i.e. one penny is one hundredth of a dollar, a dime is one tenth of a dollar, etc.). To help students solidify their understanding of these ideas, have them write down the following numbers described below:

1. What is the value of 5 pennies, 7 dimes, and 8 dollar bills?
2. Write a number made up of 5 hundredths, 7 tenths, and 8 ones.

Have students compare the numbers they created for these two descriptions and talk about what they notice about the relationship between these two numbers. Then, have students write their own descriptions of numbers (along with the numerical representation) like the second one. Then, have several students share their descriptions and have the class write the corresponding numerical representations of the given descriptions.

Extend

Give students a handout with several copies of a marked number line (Appendix C). The number line will have tenths marked with unmarked dashes representing hundredths on the line. Have students first discuss with a partner what they think the unmarked dashes represent. It may be helpful to have students plot 0.05 on the number line to help develop this knowledge.

After the class understands that each dash represents a hundredth, have them order the following numbers from largest to smallest, using the number lines to justify their answer: 0.3, 0.13, 0.44. Once the students have finished plotting and ordering the three numbers, have them create a new number line.

Tell the students their number line must include whole numbers (0-3), tenths, and hundredths. Have each student give a dollar amount (anything $0.00 - $3.00), and write the amounts on the board. Now, have the students plot each of the dollar amounts on the number line they created.

Ask students what would happen if they “zoomed in” again in between the hundredths, thousandths, etc. Students should observe that there is always another number (decimal) between any two numbers.

After students have completed the task, place them in small groups and ask them to create a poem, song or chant that explains a digit in one place represents 10 times as much as the digit in the place to its right and 1/10 of what it represents in the place to the left. This will give students another connection to the concept and help solidify understanding.

Evaluation

During the Explore part of the lesson, listen for the patterns that the students discuss. Going from right to left, students might see that a zero is being added at the end of each number, some students may describe this as the decimal point moving one place to the right. Urge the students to think about *why* this would happen - what mathematical operation is being used? Some students may struggle a little more moving from left to right, but again, they may notice the decimal moving one place to the left this time. Prompt them once more as to *why* this would happen - what mathematical operation is being used here? The goal is to get the students thinking creatively here, really focusing on the *why*.

During the Explain part of the lesson, circulate the room and observe the different number descriptions the students are writing down. Check to see that their numerical representation corresponds to the number description. Some students may struggle with this part of the lesson. If they do, have them pair up for discussion. Then, have each pair come up with several different descriptions and numerical representations instead of just one.

During the Extend part of the lesson, circulate the classroom to check for understanding. Specifically, check to see if students are struggling with ordering the numbers. If they are, suggest plotting the numbers on the number line first, then ordering the number. Some students may also struggle with plotting. They may need additional explanation about the number line, and it may be helpful to plot the numbers between 0 and 0.1 as a whole class. Circulate the classroom, and check student answers (plotting and ordering).

Once students have ordered their numbers, plotting the numbers should be a little easier for them. Circulate the classroom while students are creating their number lines. If you notice students struggling, remind them to think about the number line as “zooming in” between each number (i.e. zooming in between 0 and 1 creates tenths, and zooming in between 0 and .1 creates hundredths), or you could have the students work in pairs to discuss and work through questions together. Once number lines are created, circulate the room to make sure students have plotted dollar values correctly. Creating a poem, song, or chant may be difficult for students at first; encourage students to be creative and have fun with this part of the “extend” section.

Appendix A: Poem

Smart

My dad gave me one dollar bill
'Cause I'm his smartest son,
And I swapped it for two shiny quarters
'Cause two is more than one!

And then I took the quarters
And traded them to Lou
For three dimes -- I guess he don't know
That three is more than two!

Just then, along came old blind Bates
And just 'cause he can't see
He gave me four nickels for my three dimes,
And four is more than three!

And I took the nickels to Hiram Coombs
Down at the seed-feed store,
And the fool gave me five pennies for them,
And five is more than four!

And then I went and showed my dad,
And he got red in the cheeks
And closed his eyes and shook his head--
Too proud of me to speak!

- Shel Silverstein

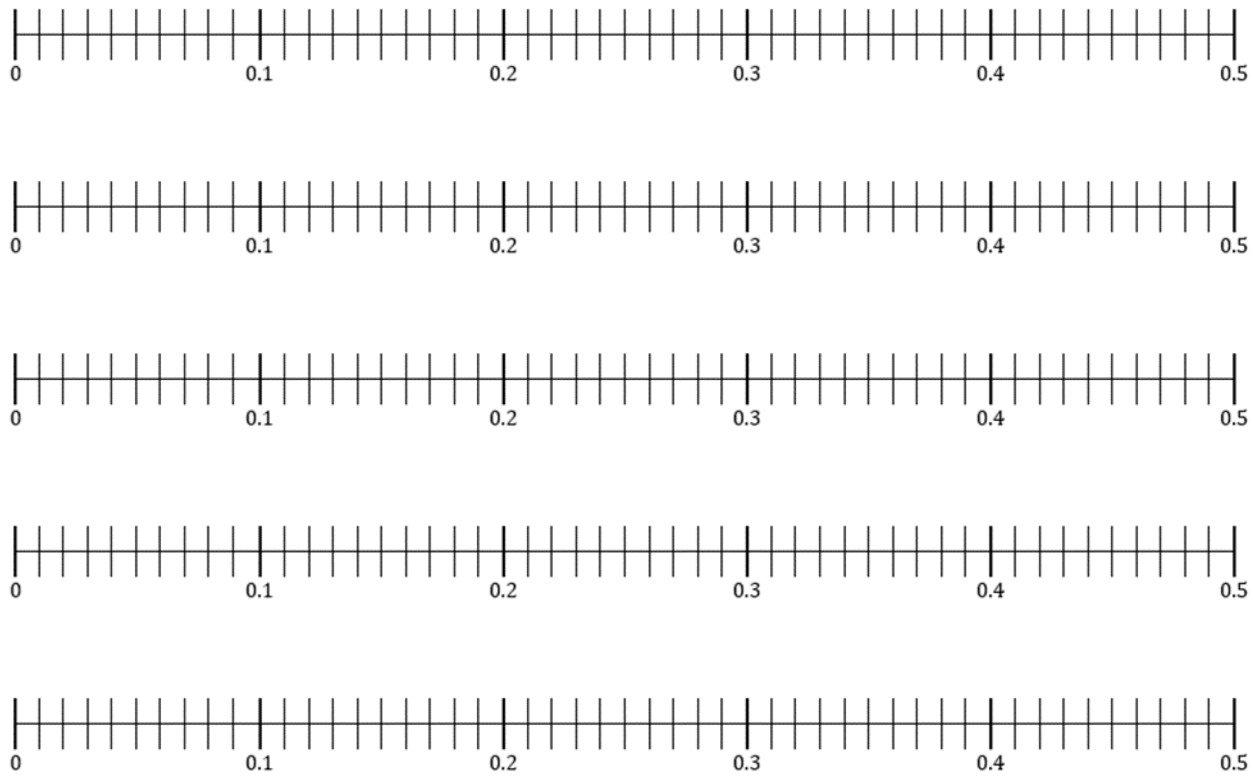
Appendix B. Explore Handout

	100,000	10,000		100	10		.10	.01	.001

What is the pattern as the numbers get larger? What is the pattern as the numbers get smaller?

Appendix C. Extend Handout



Citation

Osborne, M., Gundlach, M., & Tudor M. (2023). Smart Money. In A. Bicer (Ed.), *Creativity-Directed Mathematical Tasks for 5th Grade Common Core Classrooms* (pp. 115-122). ISTES Organization.

Task 10 - Money Problems

Fay Quiroz, Traci Jackson, Aylin S. Carey

Mathematical Content Standards

CCSS.MATH.CONTENT.5.NBT.A.2

Explain patterns in the number of zeros of the product when multiplying a number by Powers of 10.

Explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.

CCSS.MATH.CONTENT.5.NBT.A.3

Read, write, and compare decimals to thousandths.

CCSS.MATH.CONTENT.5.NBT.A.3.A

Read and write decimals to thousandths using base-ten numerals, number names, and expanded form,
e.g., $347.392 = 3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times (1/10) + 9 \times (1/100) + 2 \times (1/1000)$.

CCSS.MATH.CONTENT.5.NBT.A.3.B

Compare two decimals to thousandths based on the meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.

Mathematical Practice Standards

CCSS.MATH.PRACTICE.MP2

Reason abstractly and quantitatively.

CCSS.MATH.PRACTICE.MP3

Construct viable arguments and critique the reasoning of others.

CCSS.MATH.PRACTICE.MP4

Model with mathematics.

CCSS.MATH.PRACTICE.MP5

Use appropriate tools strategically.

Materials

Classroom money or paper money (Appendix A), base-ten materials for each student: (1 super square or cube), 5 hundreds (flats), 10 strips (rods), 10 tinies (ones), 10 circle hole punches

Vocabulary

Powers of 10, Exponents, Decimal Point, tenths, hundredths, thousandths, compare, base-ten numerals, number names, expanded form.

Lesson Objective

Students will investigate the relationship between reading, writing, and comparing decimals to the thousandths. The aim of this task is to develop students' mathematical creative thinking by challenging them to understand how money is representational of a decimal, and can also be represented with place-value blocks to further their understanding 1/10, 1/100, 1/1000. Students will read, write, compare and expand decimals using area and set models.

Students will create a new coin that is 1/1000 of the dollar or 1/10 of a penny. Then, students will build decimal number models using base-ten material to understand the relationship between the value of the whole numbers and decimals to the 1/1000. Students will represent their models in numeric form, word form and expanded form notation to demonstrate their understanding of the value of each number. This task will be completed in two-class times.

Engagement

(30 minutes) Note: Prior to this lesson, the teacher and students can have a brief conversation about coins: Where the first coins appeared and the concept of money. Then the teacher should ask students whether they have seen coins from other countries and can ask students to bring coins to the class from their background countries. Additionally, for those who would bring coins, the teacher can ask students to be prepared to share a story about the coin such as their history and usage. Teachers can look for conversations being made through economics, geography, history and mathematics, as students present their coins from other countries.

Start the class by having students share stories on international coins, if anyone brought coins from other countries. Ask students what they notice about the coins. Then show students the picture below. Ask "What do you notice? What do you wonder?" Sample responses, may include the words one tenth, the hole in the middle, if it is a real coin, it is not a US coin, the year or star shape.

Figure 1. 1/10 of a Penny Coin

Share with students that this is a coin issued by British South Africa under King George V and King George VI. It is worth 1/10 of a penny and the hole was for carrying coins around the neck because people did not have pockets. While the smallest denomination the United States minted was a half cent, many states created their own tokens called "mills" worth 1/10 of cent in order to pay the correct amount of tax.

Teachers can ask students if they can determine how many of these 1/10 penny “mills" would be needed to equal one dollar. Listen for understanding including using intermediate steps like 10 make 1 penny (see Figure 2) below, 10 pennies make 1 dime (so 100 mills would equal 1 dime) and 10 dimes equal 1 dollar (so 1000 mills would equal 1 dollar). As students share, the teacher can chart different ways of determining the total number of mills in 1 dollar. Teachers may choose to give students fake classroom money or cut out paper money (see Appendix A) to help students conceptualize their understanding of money.

Figure 2. Ten Mills Equivalent to One Penny

Ask students how they could represent 4 quarters, 1 dime, 3 pennies and 2 mills as a decimal amount of money ($1.132). They may have trouble with the mills. Reference the previous question to help them understand that a mill would be one thousandth of 1 dollar.

Then ask students to create a new coin with a value that is less than $1.00 and is not the same value as a quarter, dime, nickel, penny, or 1/10 penny. They draw their coin then write the value in cents then as a decimal fraction. For example, if they chose a coin worth 20 cents, they would write $0.20.

Have students use their coin and other traditional coins to draw an amount of money greater than $1.132 and less than $1.132 (see Appendix B). They may draw circles and label their coins with letters, like Q for quarter, or write the value as a decimal, like quarter would be

0.25 or 25¢. On the 20x50 grid, students should shade the total amount of money they made as well as the amount they are comparing.

In the first example, students creating a number greater than $1.132 they should shade two 1/1,000 of the smallest rectangle on the grid to represent 0.002. This will help students understand the relationship between the power of ten from 1/1,000 to 1.

Explore

(30 minutes) Students will use base-ten materials to explore whole numbers and decimals. Pair students up and give each group the following materials: fake classroom money, or cut out paper money from (Appendix A), base-ten materials including one super square (known as a cube), five squares (known as a flat), ten strips (known as a long or a rod), and ten tinies (known as singles), and ten “circle hole-punch” (to represent the 1,000ths) (see Figure 3).

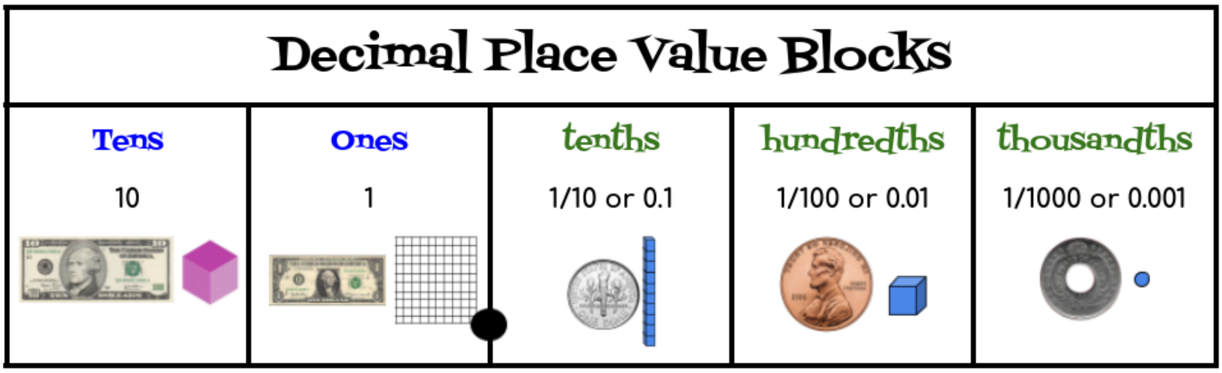


Figure 3. Example of Decimal Place Value Blocks and Money

The teacher should connect students’ understanding from the engagement task where they made their own coin, and explore the knowledge built about how money is represented in an area model and as a decimal. The teacher can begin by asking students to represent $1.28 using their money or 1 dollar bill, 2 dimes, and 8 pennies. Ask students to then create $1.28 using place value blocks or 1 flat, 2 rods, and 8 tinies (see Figure 4).

Students are familiar with how to use place value blocks with whole numbers, but may not be proficient in using place value blocks with decimals. The teacher can observe which students are comfortable connecting money to the base-ten materials to see the value of the number.

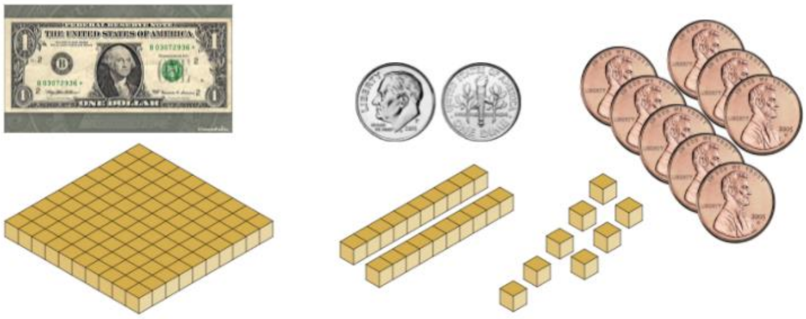


Figure 4. Example of Money and Base-ten Materials for $1.28

Students will continue to explore how money can be represented with base-ten materials to further their understanding of 1/10, 1/100, 1/1000. Ask students to explore ways they can represent different coins using base-ten materials using the following examples (see Figure 5): 1.032, 3.45, 2.526, 13.79, 5.219. Teachers should listen for conversations and connections that emerge about the relationship between base-ten materials and decimals. This may include ways students connect one dollar to the whole, or how students are saying the numbers such as "one and thirty-two thousandths." Students should also connect, and ask for, the "mill" coin as well as the "circle hole-punch" to represent 1/1000ths for the money and base-ten materials. The teacher can ask the students if they can make the numbers in a different way? For example, students could use ten dimes for a dollar, ten pennies for a dime or ten mills for a penny.

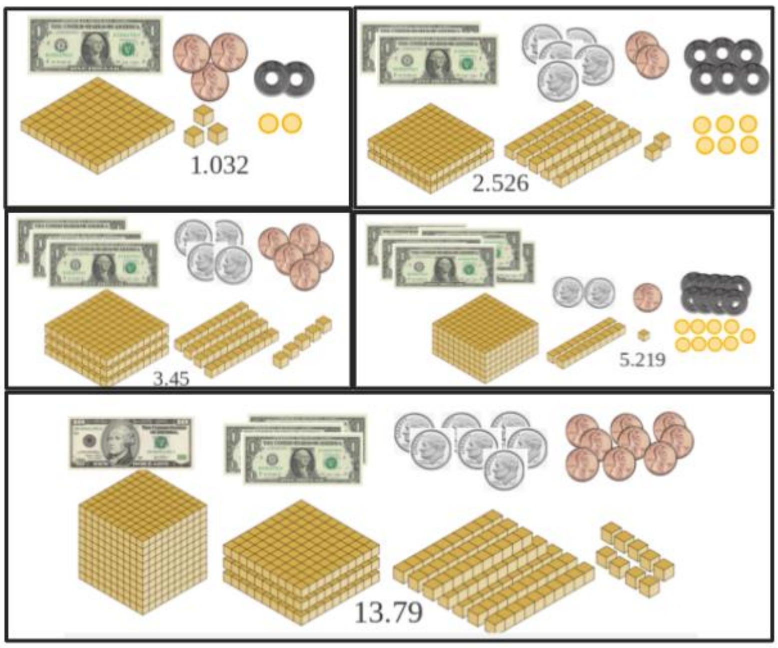


Figure 5. Examples of Possible Student Solutions using Money and Base-ten Materials

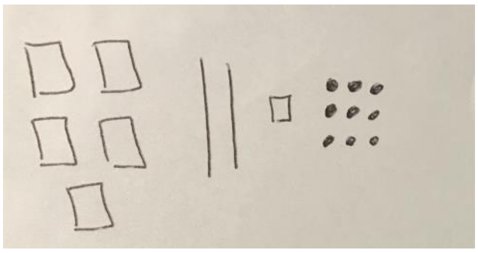
Explain

(30 minutes) Now that students have an understanding of how money relates to base-ten materials, teachers transition into *only using base-ten materials for the next section*. Teachers can continue to develop students' conceptual understanding of building with the base-ten materials and connect the base-ten materials to draw and write in expanded form. Begin a whole class discussion with students about how the examples from the previous activity could help us write numbers in standard or numeric form and expanded form to the thousandths place. For example, the teacher can ask students to reflect on a previous number from the explore activity like 5.219, by only saying the number, not writing it on the board. Teachers can ask students to think about and discuss how they would write in numeric form, word form, draw and expand that number. It may be helpful for students to share their initial ideas with a shoulder partner before sharing their thoughts with the whole class. Share the Write, Build, Draw and Expand Table (see Table 1) with students as a way to record their thinking with words, numbers and drawings. As students share out, we expect students to write the number in numeric form as 5.219, word form as five and two-hundred and nineteen thousandths, and expand the number like (5x1) + (2 x 1/10) + (1x 1/100)+ (9x 1/1000). Model how students can record their thinking referring to each box in the table. For example, after the number was written in numeric form, writing the number in word form, then after building the figure with base-ten materials students would check off that box, and then draw their base-ten materials in the draw area with flats, longs, tinies and "circle-hole punch" (see Table 2).

Table 1. Write, Build, Draw and Expand Table

Numerical Notation	Word Form	Build	Draw
		X	
Expand:			

Table 2. Student Example using the Write, Build, Draw and Expand Table

Numerical Notation	Word Form	Build	Draw
5.219	five and two-hundred nineteen thousandths		
Expand: (5x1) + (2x1/10) + (1 x 1/100) + (9x 1/1000)			

After the whole class discussion, give each student a copy of Appendix C and tell students they will continue to practice writing, building, drawing and expanding decimal numbers but as part of a game. Pair students up and review the directions for the game. As students are playing the game, observe if they are recording the number correctly. For example, if they roll a 5, 6, 2, 1 teachers should look for students writing the greatest number like, 6.521 in numeric form.

Teachers should also listen for the connection to how students are saying and writing the number. For example, students should read 6.521 as six *and* five-hundred twenty-one thousandths and noting the “and” represents where the decimal point is located. Teachers can also pay attention to how students are expanding numbers. We want to see students using a multiplication sign to expand the digit and the value of that number like (6 x 1), and also understanding to add the values together by using an addition sign. The number 6.521 would be written in expanded form (6 x 1) + (5 x 1/10) + (2 x 1/100) + (1 x 1/1000). If students are struggling writing the expanded form teachers can refer back to their drawing or the base-ten materials to help connect the value of the number to the digit. The teacher can ask questions about what student’s notice about the base-ten materials and their drawings. Or teachers can ask students what they notice about how their drawings connect to the value of the numbers in expanded form.

When students have finished playing three rounds of the game and have determined a winner, teachers can debrief with students, for a short time, to explain how they know who won the game. Students' responses should include a description about how they looked at the first digit and then each consecutive digit to determine who had the greatest number. For example, if partner A wrote 6.521 and partner B wrote 6.311, an explanation that they both had a six in the ones place, but because partner A had a 5 in the tenths place that five tenths is greater than three tenths, partner A won that round. As students finish the game, teachers should give students Appendix D, to complete independently. The independent Write, Build, Draw and Expand Tables focus on students being able to read the decimal number in word form and translate this to numeric form, and then building and drawing the number. The teacher can use this activity as a formative assessment to check student understanding.

Once students finish their independent work, have them bring their game boards and independent work to a whole class discussion. Ask multiple students to share numbers they rolled during the game, and carefully listen to how students say each number. Teachers should use this time to evaluate students' understanding of how they read the numbers aloud. At the end of the whole class discussion students can see how to correctly say decimal numbers to the thousandths.

Extend

(30 minutes) Once students have an understanding of four ways to represent numbers and place value—standard form, base ten form, expanded form, and word form—have students discover whole and decimal numbers that are used in the real world and share with the class. While they share, ask students to write the expanded form of the numbers. Pay attention to the way they read the numbers before writing them down. For example, assuming that the average bank interest rate for checking accounts in the United States is 0.03%. Ensure that students read the percentage as 0 and 3 hundredths. Students will then write this first in a place value chart and then in the expanded form (see Table 3). Students should also practice numbers to the thousandths place, and the teacher can refer to Table 4 for an additional example.

Table 3. Place Value Chart showing the Average Bank Interest Rate

Number	Ones		Tenths	Hundredths
0.03	0	.	0	3

0.03=(3 x 1/100)

Table 4. Place Value Chart showing Numbers to the Thousandths

Number	Ones		Tenths	Hundredths	Thousandths
0.007	0	.	0	0	7

0.007 = (7 x 1/1000)

By now, students should be familiar with large numbers being used in daily life and came across them in newspapers, on digital sources or even in other school subjects. This would be a good place to write large numbers in a place value table to help make sense of it and show how to use powers of ten to express large numbers. Students should have an understanding of how the power indicates the number of times 10 is multiplied by itself. For example, 10^2 means 10 x 10. The table below (see Table 5) is a list of the powers of ten that can be used as a visual.

Table 5. List of the Powers of Ten that can be used as a Visual

Name	**Number**	**Powers of Ten**
Ten	10	$10 = 10^1$
Hundred	100	$10 \times 10 = 10^2$
Thousand	1,000	$10 \times 10 \times 10 = 10^3$
Ten thousand	10,000	$10 \times 10 \times 10\ 10 = 10^4$
Hundred thousand	100,000	$10 \times 10 \times 10 \times 10 \times 10 = 10^5$

Million	1,000,000	10 x 10 x 10 x 10 x 10 x 10 = 10^6
Ten million	10,000,000	10 x 10 x 10 x 10 x 10 x 10 x 10 = 10^7
Hundred million	100,000,000	10 x 10 x 10 x 10 x 10 x 10 x 10 x 10 = 10^8
Billion	1,000,000,000	10 x 10 x 10 x 10 x 10 x 10 x 10 x 10 x 10 = 10^9
Ten billion	10,000,000,000	10 x 10 x 10 x 10 x 10 x 10 x 10 x 10 x 10 x 10 = 10^{10}
Hundred billion	100,000,000,000	10 x 10 x 10 x 10 x 10 x 10 x 10 x 10 x 10 x 10 x 10 = 10^{11}
Trillion	1,000,000,000,000	10 x 10 x 10 x 10 x 10 x 10 x 10 x 10 x 10 x 10 x 10 x 10 = 10^{12}

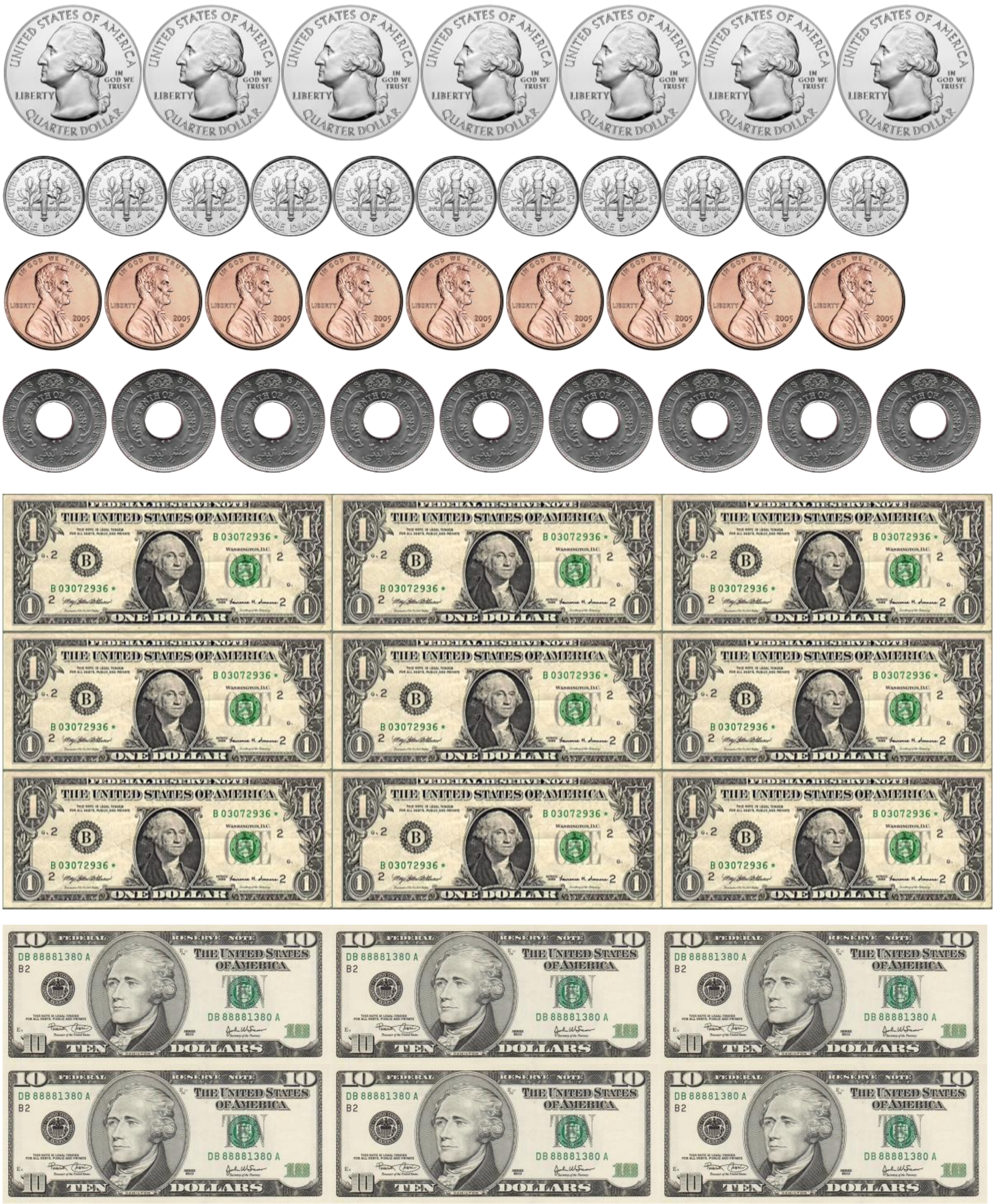
The teacher can use the list above to create a place value table as a reminder of working with large numbers easily (see Appendix E). Bring class together with a discussion on where and when a small or large magnitude of numbers be used. During the discussion, it would be ideal to bring back the interest problem and ask students whether there is a different way of writing such as $0.03 = (3 \times \frac{1}{10 \times 10}) = (3 \times \frac{1}{10^2})$.

To improve students' use of mathematics vocabulary and help them understand better, it is also critical to mention that power indicates repeated multiplication of the same factor and has two components: base and exponent. For example, in the interest problem, 10^2 is the power where 10 is the base and 2 is the exponent. Providing additional examples with whole numbers helps students understand the powers of 10. For instance, using a real-world example telling that 60,000 people attended a concert and ask whether there is a different way of writing this number with a power of ten. We want students to share 60,000 = (6 x 10,000) = (6 x 10 x 10 x 10 x 10) = (6 x 10^4). Continuing to reinforce the use of mathematics vocabulary such as 10 is the base and 6 is the exponent is important to enhance students' understanding.

Evaluate

While students are working on the activities, observe what they do and verbally express, whether they are able to read the numbers in standard and expanded form, or they struggle and make errors. When they struggle and make errors, look for students' ability to put numbers in correct form. Record how they use the base-ten materials to form a number in expanded form. Remember the aim of the lesson: Building strong number sense through creativity-directed tasks before introducing decimals. Since there is an interdisciplinary component with the money concept, look for connections being made through economics, geography, history and mathematics, when students present coins from other countries.

Appendix A


UNITED STATES OF AMERICA
IN GOD WE TRUST
LIBERTY
QUARTER DOLLAR
ONE DIME
FEDERAL RESERVE NOTE
THE UNITED STATES OF AMERICA
ONE DOLLAR
TEN
DOLLARS

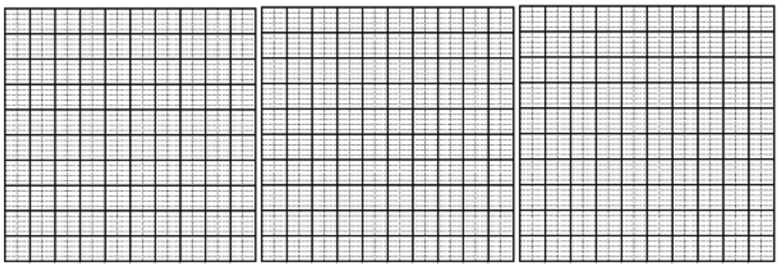
Appendix B

My coin is worth ______ cents or $0.______

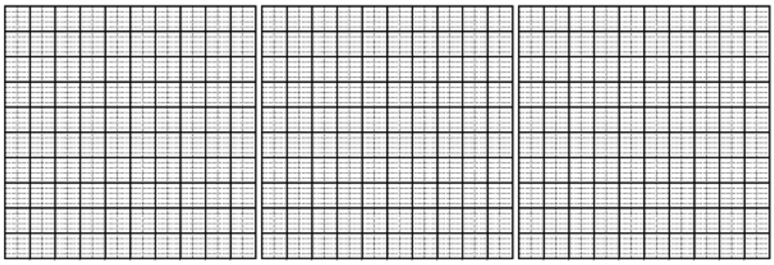
Drawing of my coin

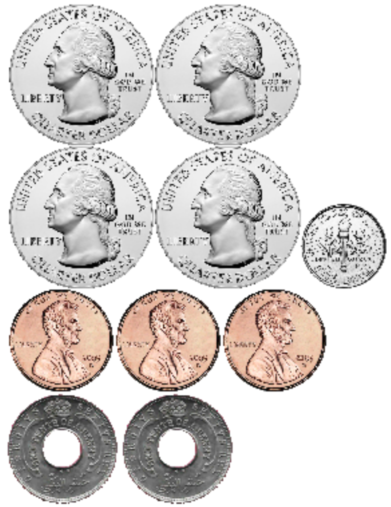
Coin Drawing

Area Model



Area Model



Coin Drawing	Area Model		Area Model	
				

Appendix C

Directions for the game:

- Roll 4 dice and arrange the dice in any order to create the largest number possible.

**REMEMBER: One digit must be in the ones place and the rest in the tenths, hundredths and thousandths places.

- Write your number in word form.
- Build your number using base-ten materials
- Draw what you built using flats, longs, tinies and "circle-hole punches"
- Expand your number.
- Compare your number with your partner. If your number is greater, YOU WIN!

Write "YES!" if not, write "NO"

WINNER WINS BEST two out of three rounds!

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E</td><td>Numerical Notation</td><td>Word Form</td><td>Build</td><td>Draw</td><td>I won</td></tr>
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Appendix D

<table>
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Appendix E

Trillion	Hundred billion	Ten billion	Billion	Hundred million	Ten million	Million	Hundred thousand	Ten thousand	Thousand	Hundred	Ten	Unit

Appendix F

References for Images

1/10 of Penny

https://en.numista.com/catalogue/pieces10696.html

One Dollar Bill

https://www.etsy.com/listing/591263327/one1-real-one-dollar-uncirculated-united?gpla=1&gao=1&&utm_source=google&utm_medium=cpc&utm_campaign=shopping_us_e-paper_and_party_supplies-other&utm_custom1=_k_CjwKCAiA6Y2QBhAtEiwAGHybPfzRkNKiOCJGaGBPv_vPsSbYkVj5QfpZX0s9V524szvSR4_yRc9XdRoCYGAQAvD_BwE_k_&utm_content=go_12573081319_122305196769_507799197306_aud-1184785539978:pla-314261241267_c__591263327_129132885&utm_custom2=12573081319&gclid=CjwKCAiA6Y2QBhAtEiwAGHybPfzRkNKiOCJGaGBPv_vPsSbYkVj5QfpZX0s9V524szvSR4_yRc9XdRoCYGAQAvD_BwE

Dime

https://www.walmart.com/ip/1950-Roosevelt-Dime-BU/461531366?wmlspartner=wlpa&selectedSellerId=1118&&adid=22222222227000000000&wl0=&wl1=g&wl2=c&wl3=42423897272&wl4=pla-51320962143&wl5=9029355&wl6=&wl7=&wl8=&wl9=pla&wl10=118771194&wl11=online&wl12=461531366&veh=sem&gclid=Cj0KCQiAxoiQBhCRARIsAPsvo-x_oHzc3LreCguwmWFuZbTBt0vOCPwCX9w1KIgtYnKuWswVMHSfVy4aAj_tEALw_wcB&gclsrc=aw.ds

Ten Dollar Bill

https://www.ebay.com/itm/303999311326?chn=ps&norover=1&mkevt=1&mkrid=711-117182-37290-

0&mkcid=2&itemid=303999311326&targetid=1262749490102&device=c&mktype=&googleloc=9029355&poi=&campaignid=15428034462&mkgroupid=133947154481&rlsatarget=pla-1262749490102&abcId=9300763&merchantid=6296724&gclid=CjwKCAiA6Y2QBhAtEiwAGHybPXaFQL1-hJYio95lEExhgr4HuKYxTGxhDto5MULYNGffY84YEIJkfRoC1DIQAvD_BwE

Base-Ten Materials

https://www.matholia.com/au/apps/tools/mt_isamb_w6626?cid=1353

Penny, Dime, Quarter

https://www.theresawills.com/mathurdays

Citation

Quiroz, F., Jackson, T., & Carey, A. S. (2023). Money Problems. In A. Bicer (Ed.), *Creativity-Directed Mathematical Tasks for 5th Grade Common Core Classrooms* (pp. 123-144). ISTES Organization.

Task 11 - The Power of Magnification

Amy Kassel, Chuck Butler, Jennifer Kellner

Mathematical Content Standards

CCSS.MATH.CONTENT.5.NBT.A.1

Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.

CCSS.MATH.CONTENT.5.NBT.A.3.A

Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., $347.392 = 3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times (1/10) + 9 \times (1/100) + 2 \times (1/1000)$.

Supporting Standard

CCSS.MATH.CONTENT.5.NBT.B.6

Add, subtract, multiply, and divide decimals to hundredths, using concrete models, or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Mathematical Practice Standards

CCSS.MATH.PRACTICE.MP2

Reason abstractly and quantitatively.

CCSS.MATH.PRACTICE.MP4

Model with mathematics.

CCSS.MATH.PRACTICE.MP7

Look for and make use of structure.

Vocabulary

Standard form, expanded form, powers of 10, mosaic

Materials

Device capable of taking pictures and zooming, individual whiteboards, dry erase markers, color pencils or crayons or markers, grid paper, copies of the charts in the Appendices

Lesson Objective

Students will be able to represent numbers in expanded and standard forms making connections between the relationships developed from multiplying by powers of ten. Students will expand their mathematical creative thinking by using different representations to show the relationship between values multiplied by powers of 10. They will use their creativity by, first, taking pictures and using their zoom feature to visually see what happens to a picture. Second, students will use multiple representations to make connections to place value, expanded form, and standard form of values multiplied by powers of 10.

Engagement

(20-30 minutes) To engage students, show students a few images from the website Readers' Digest: Can You Identify Everyday Objects by these Close-Up Pictures (Reader's Digest, 2021) and have them guess what the objects are. Using their device, i.e., iPad, Chromebook or phone, have the students each take pictures of two objects. Using the zoom feature, make one object larger (1000%) and one smaller (10%). Pair students up and have them identify objects in the photos. Discuss how multiplication can change an image by making it larger or smaller.

The teacher may conclude the engagement by showing students images of a broken tile mosaic zoomed in and zoomed out. See Appendix A for an example. Then the teacher may answer questions about how a broken tile mosaic is created.

Explore

(30 minutes) Part 1: On the board, the teacher should start with an example of an expanded form of a decimal, multiplying each digit by its place value and adding them together. $43.875 = (4 \times 10) + (3 \times 1) + (8 \times 1/10) + (7 \times 1/100) + (5 \times 1/1000)$. On personal whiteboards, allow students to explore other expanded decimals such as: 31.018, 7.239, and 11.702. Next, ask the students what they would do with the expanded form if they wanted to translate it to the decimal such as:

$(6 \times 100) + (9 \times 10) + (1 \times 1) + (2 \times 1/10) + (0 \times 1/100) + (8 \times 1/1000) =$

______. Next, have the students come up with 2 examples on their own whiteboards and share with one other student.

Part 2: Using the chart from Appendix B, have the students write the decimal 745.269. In the next rows, have them represent 745.26×10 and then 745.26×100.

10,000's	1,000's	100's	10's	1's	.	1/10's	1/100's	1/1000's	1/10,000's
7									
	7								
		7	4	5	.	2	6		

Sample student response:

10,000's	1,000's	100's	10's	1's	.	1/10's	1/100's	1/1000's	1/10,000's
7	4	5	2	6	.	0			
	7	4	5	2	.	6			
		7	4	5	.	2	6		

Now have the students represent $745.26 \times 1/10$ and $745.26 \times 1/100$.

10,000's	1,000's	100's	10's	1's	.	1/10's	1/100's	1/1000's	1/10,000's
		7	4	5	.	2	6		
			7						
				7					

Sample student response:

10,000's	1,000's	100's	10's	1's	.	1/10's	1/100's	1/1000's	1/10,000's
		7	4	5	.	2	6		
			7	4	.	5	2	6	
				7	.	4	5	2	6

Part 3: Put students into small groups. Pose this problem: You are creating a mosaic for several projects using fragments of broken tiles. One unit requires 24.6 broken tiles. Your team has been hired to complete several projects. The projects to complete include a wall that is 100 times the unit, a table that is 10 times the unit, a serving platter that requires 1/10 of the unit, and a hot pad (trivet) that is 1/100 of the unit. How many tiles will you need to complete the full project you have been hired for? Explain your reasoning visually using a place value chart (Appendix C). Draw a picture representing each project (Appendix D).

Sample student response:

10,000's	1,000's	100's	10's	1's	.	1/10's	1/100's	1/1000's	1/10,000's
			2	4	.	6			
	2	4	6	0	.	0			
		2	4	6	.	0			
				2	.	4	6		
				0	.	2	4	6	

Explain

(30 minutes) Teachers may choose to complete each part of the lesson with an explanation before moving on to the remaining parts.

Part 1: The teacher will monitor student work by observing their whiteboards and asking questions to clear of misrepresentations. Some fifth graders may not have been introduced to the thousandth's place value. The teacher should help students predict that notation based on notation for tenths and hundredths.

The teacher may ask students to explore the patterns that they see when a number has a 0 in it. For example, ask students to explore values such as 302.1, 50.23, 45.06, 38.70 multiplied by powers of 10. Facilitate the class discussion to explore what happens when the place value contains a 0. Teachers may have students complete the pattern through a whole class discussion.

$$302.1 \times \frac{1}{100} = 3.021$$

$$302.1 \times \frac{1}{10} = 30.21$$

$$302.1 \times 1 = 302.1$$

$$302.1 \times 10 = 3{,}210.0$$

$$302.1 \times 100 = 30{,}210.0$$

If students are struggling, ask them if it is necessary to add the 0 to the expanded representation, i.e., $\left(0 \times \frac{1}{100}\right)$. and would it be beneficial to include it to focus on the $\times 10$ relationship. As an example, ask students to write 45.06 and 450.6 in expanded form:

$$45.06 = (4 \times 10) + (5 \times 1) + \left(0 \times \frac{1}{10}\right) + \left(6 \times \frac{1}{100}\right)$$

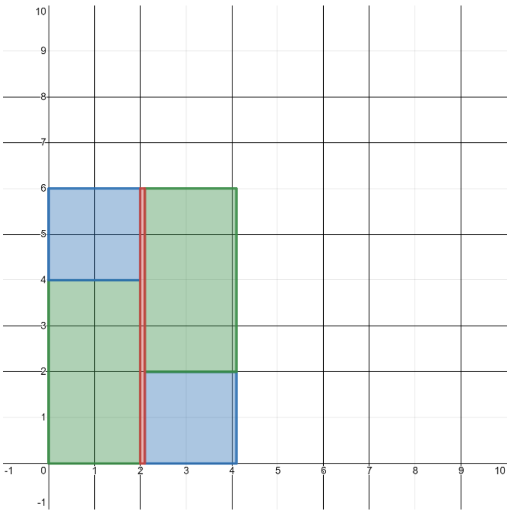
$$450.6 = (4 \times 100) + (5 \times 10) + (0 \times 1) + \left(6 \times \frac{1}{10}\right)$$

This may support students in recognizing that the 0 must be represented in its appropriate place value as the number is multiplied by powers of 10.

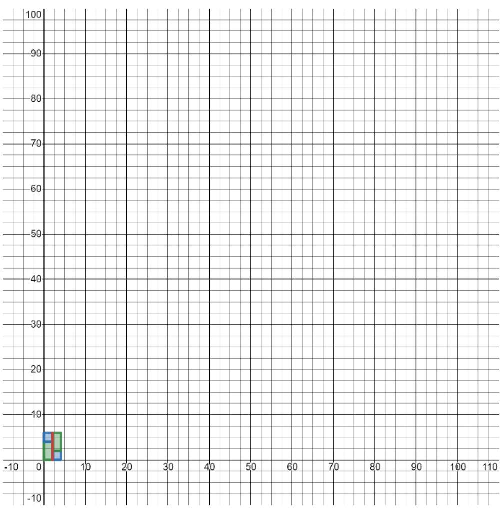
Part 2: The teacher should be listening for students who state that the decimal is moving right or left. The teacher may ask questions to help the student recognize that it is the value of the number that is changing, and the placement of the decimal point shows the new value.

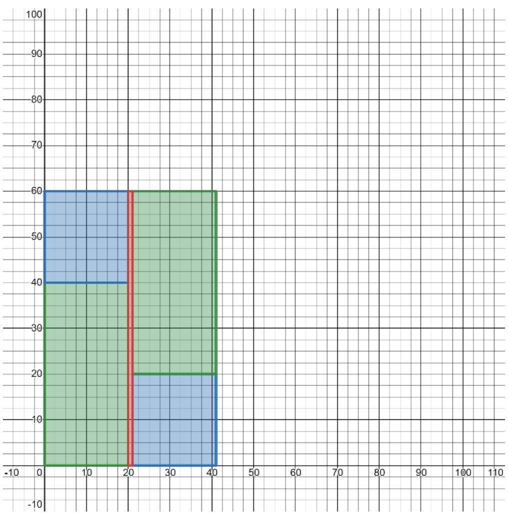
Part 3: The teacher should facilitate a class discussion examining how powers of ten represent the visual representation of the mosaic projects. The teacher should make sure students are visually representing the patterns of $\times 10$, $\times 100$, $\times 1/10$, and $\times 1/100$ accurately on the grid paper. For example, students may initially represent 24.6 by creating a pattern on grid paper

Representing 24.6 on 1×1

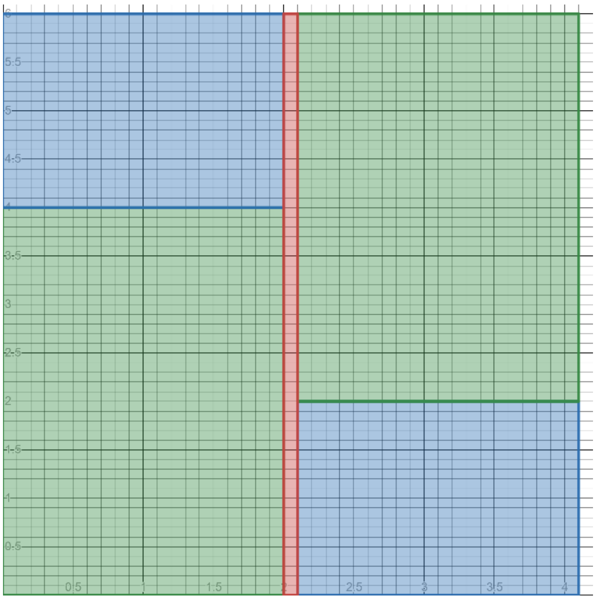


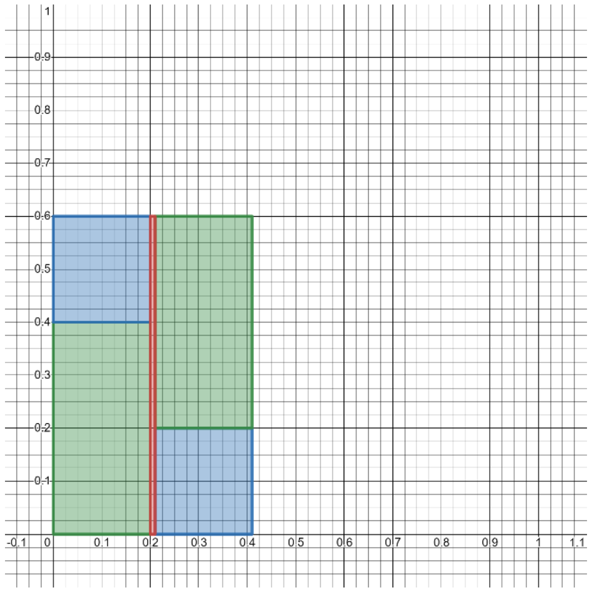
Representing 24.6 on the 10×10 and representing 24.6×10 on the 10×10





Representing 24.6 on the $\frac{1}{10} \times \frac{1}{10}$ and representing $24.6 \times \frac{1}{10}$ on the $\frac{1}{10} \times \frac{1}{10}$





As students are deciding how to represent the initial unit, the teacher should be prepared to ask questions about how students might be planning to represent their visual diagram when they multiply by the powers of 10. For example, if the student chooses to design their unit design as a 1×1 square on the grid paper, the teacher might ask the student, "How will you represent the design that $\times 1/10$?" The student may suggest dividing the grid paper into smaller units or may recognize that it might be easier to change the initial unit to a 10×10 design or possibly a 100×100 design on grid paper. If the student makes a larger unit design, the teacher might inquire how the student might represent the design $\times 10$ or $\times 100$. The student may suggest taping together multiple sheets of grid paper. The teacher should be prepared to discuss how multiplying by powers of 10 changes the magnitude of the visual representation. The students may select which grid paper would be best to represent their mosaic (Appendix D, Appendix G).

The final question of the lesson asks students to write the total number of tiles needed to complete all three projects. Students will be adding the number of tiles for each project together. Students may not have learned to add decimals yet. To introduce the addition of decimals, the teacher may choose to lead a whole-class discussion by having students represent the values of the number of tiles from their projects in expanded form, thus, making the connection between addition and place value to facilitate learning to add decimals.

Extend

The teacher may choose to assign one extension to the whole class or assign different extensions to different groups based on interest, ability, and readiness.

- **Visual –** Students will express the unit from part one (24.6) visually using base-ten blocks.
- **Who am I** – Given the visual for 24.6 (Appendix E), students will find three other numbers that could be represented by the same image.
- **Big or Small –** Students will select a few items in the classroom and determine their weight or height or length to the hundredths place. Using the table in Appendix F or creating a poster with a similar table, students will determine the weight/height/length of the $\times 10$, $\times 100$, $\times 1000$, and $\times 10^n$ representations of these shapes. Students will also determine the weight/height/length of the $\times 1/10$, $\times 1/100$, $\times 1/1000$, and $\times 1/10^n$ representations. The $\times 10^n$ and $\times 1/10^n$ representations can be expressed verbally or with a written description of the pattern the students observe.

Evaluate

- In part 1, formative assessment for the standard will occur as students explain why they wrote the decimal expansion in that form.
- In part 2, formative assessment for the standard will occur as the teacher asks students to explain how the original unit connects to the $\times 10^n$ and $\times 1/10^n$ representations of the number. The student needs to verbally explain that the value of the number in a specific place is 10 times as large as the same number immediately to the right and 10 times smaller than the same number immediately to the left.
- In part 3, formative assessment of the students' numerical work can be assessed by examining the values from the students' tables. The teacher may find more insight

into students' conceptual understanding of place value and the relationships by observing their visual representations of their patterns and listening to their explanations of the decision-making process students used. The verbal and/or written dialogue about unit and the relationships by $\times 10^n$ and $\times 1/10^n$ will provide formative assessment for students' understanding.

- In the Visual, formative assessment for creativity will occur as students are asked to express the unit from the mosaic project using base ten blocks.
- In Who Am I, formative assessment for creativity will occur as students realize they can assign any unit value to a block. This will also help teachers assess students' understanding of number sense conceptual through place value and the relationships between operations.
- In Big or Small, students will see the relationship been the weight/height/length of an item and the $\times 10$,$\times 100$, and $\times 1000$ representations, as well as $\times 1/10$, $\times 1/100$, and $\times 1/1000$ representations. Students can use these patterns to generalize for the $\times 10^n$ and $\times 1/10^n$ representations by writing or verbally expressing the pattern. Teachers can formatively assess students' understanding of number sense and pattern building by observing students' work in groups or reading their responses.

References

Reader's Digest. (2021, October 4). *Can You Identify Everyday Objects By These Close-Up Pictures?* Retrieved February 6, 2022, from rd.com/list/everyday-objects-close-up/

Wikipedia. (2022, January 7). *Mosaic*. Retrieved February 7, 2022, from https://en.wikipedia.org/wiki/Mosaic

Appendix A

Copacabana (Rio de Janerio) broken tile image (Wikipedia, 2022)

Appendix B

10,000's	1,000's	100's	10's	1's	.	1/10's	1/100's	1/1000's	1/10,000's
7									
	7								
		7	4	5	.	2	6		

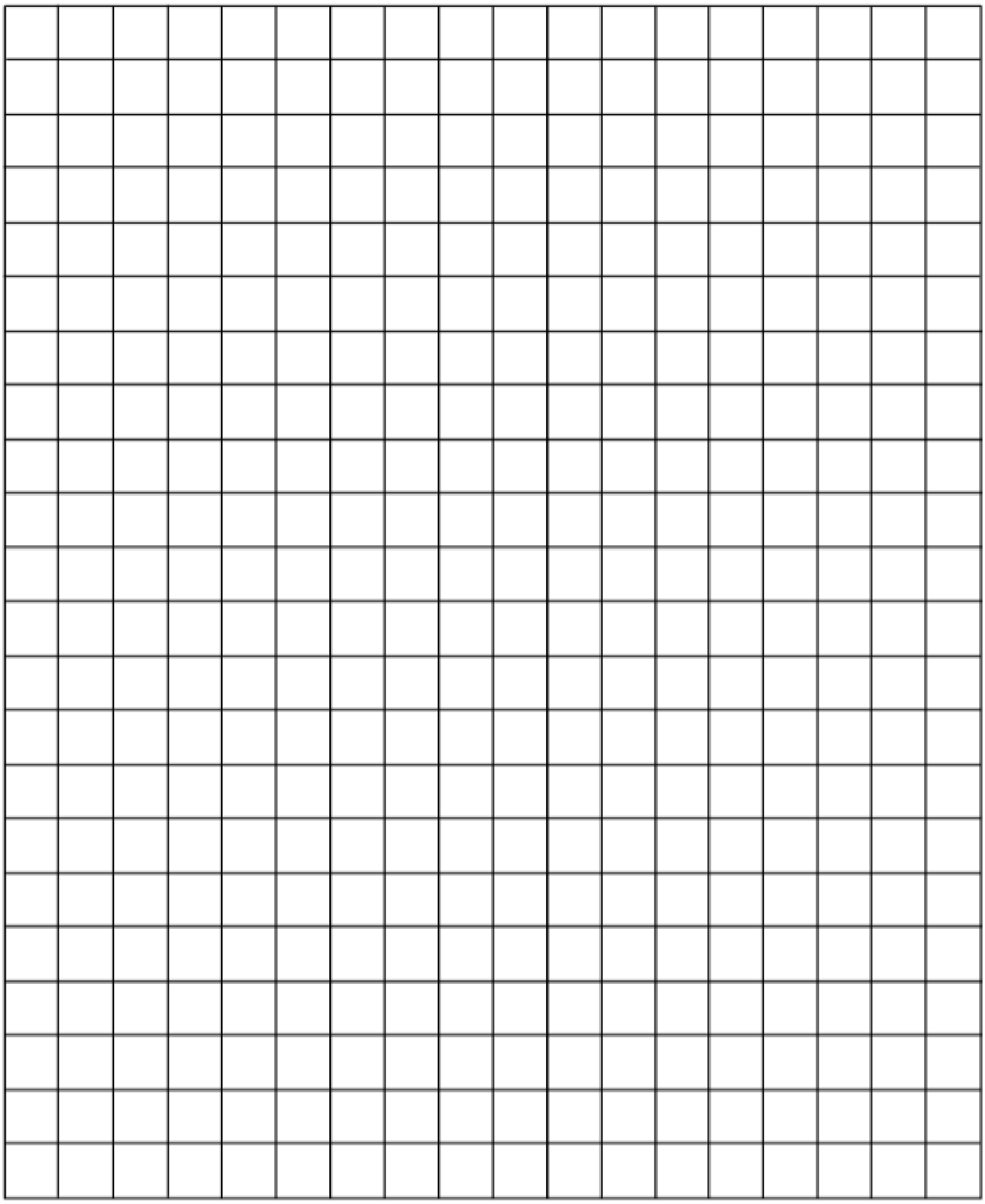
10,000's	1,000's	100's	10's	1's	.	1/10's	1/100's	1/1000's	1/10,000's
		7	4	5	.	2	6		
			7						
				7					

Appendix C

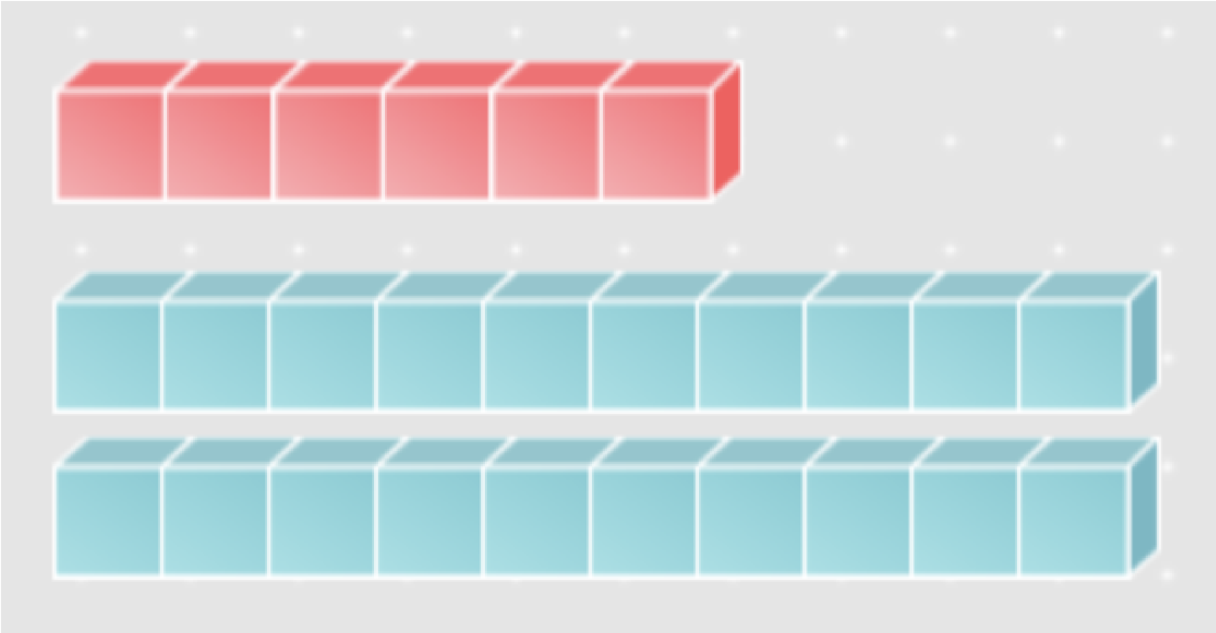
10,000’s	1,000’s	100’s	10’s	1’s	.	1/10’s	1/100’s	1/1000’s	1/10,000’s

10,000’s	1,000’s	100’s	10’s	1’s	.	1/10’s	1/100’s	1/1000’s	1/10,000’s

Appendix D



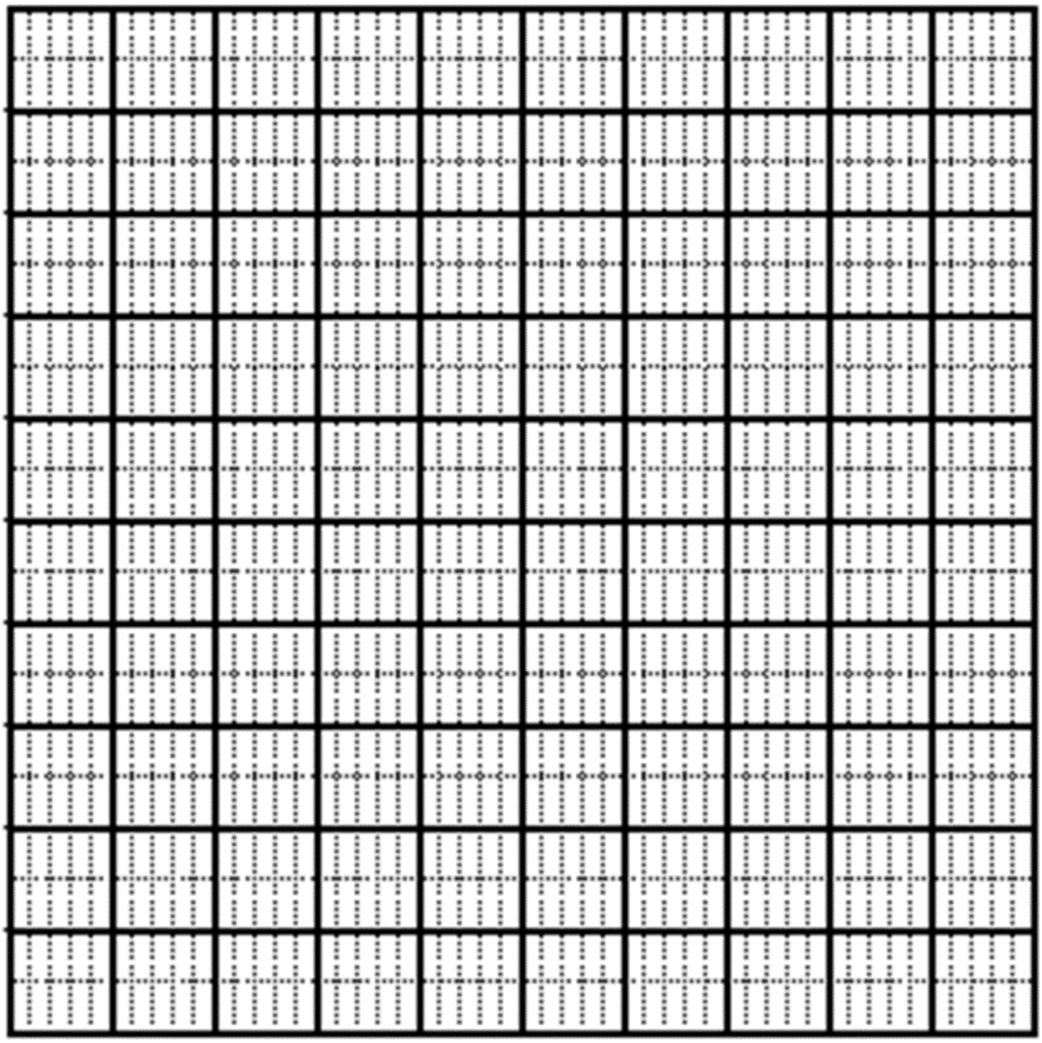
Appendix E



Appendix F

Item Name	$\times 10^n$	$\times 1000$	$\times 100$	$\times 10$	Weight /Height /Length	$\times 1/10$	$\times 1/100$	$\times 1/1000$	$\times 1/10^n$

Appendix G



Citation

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Task 12 - There is More to Base Ten!

Helen Aleksani & Geoff Krall

Mathematical Content Standards

Focus Content Standard

CCSS.MATH.CONTENT.5.NBT.A.3

Read, write, and compare decimals to thousandths.

Mathematical Practice Standards

CCSS.MATH.PRACTICE.MP2

Reason abstractly and quantitatively.

CCSS.MATH.PRACTICE.MP4

Model with mathematics.

CCSS.MATH.PRACTICE.MP5

Use appropriate tools strategically.

Lesson Objective

Students will investigate how to read and write decimals to the thousandths place in word form, base-ten form, and expanded form. Represent decimals with base ten models and number lines. Estimate decimals to a near benchmark. Compare two decimals to the thousandths using benchmark comparisons or place value. To enhance mathematical creativity, students will be using the base ten blocks, area model and the number line to demonstrate their understanding of the concept and apply it to real life application.

Engagement

(15minutes) Begin by asking the following question to students: "How long is 0.01 (or one hundredth) of a second?" Students may choose to represent one hundredth of a second by snapping their fingers or holding up two fingers very close.

Continue by saying: “What if I told you that one hundredth of a second can mean the difference between a gold medal in the Olympics and a silver medal in the Olympics?

Show students this video and ask students to pay close attention to the seconds. https://www.nbcsports.com/video/2008-olympics-michael-phelps-earns-dramatic-win-100m-butterfly

Show students the official results (see Table 1) from this website: https://en.wikipedia.org/wiki/Swimming_at_the_2008_Summer_Olympics_%E2%80%93_Men%27s_100_metre_butterfly

Table 1. Results from the 2008 Summer Olympics Men’s 100 Meter Butterfly Event

Rank	Lane	Name	Nationality	Time	Notes
1	5	Michael Phelps	United States	50.58	OR
2	4	Milorad Cavic	Serbia	50.59	EU
3	3	Andrew Lauterstein	Australia	51.12	OC
4	6	Ian Crocker	United States	51.13	
5	2	Jason Dunford	Kenya	51.47	
6	1	Takuro Fuji	Japan	51.50	AS
7	7	Andriy Serdinov	Ukraine	51.59	
8	8	Ryan Pini	Papua New Guinea	51.86	

Ask students: “How much faster was Michael Phelps than the second-place finisher, Milorad Čavić? What is the difference between finishing with a Gold medal and *not even getting a medal at all*? How big of a time difference is that?” Students may not know how to subtract with decimals, but that’s okay. It’s sufficient if students see the difference between first place and fourth place is less than one second.

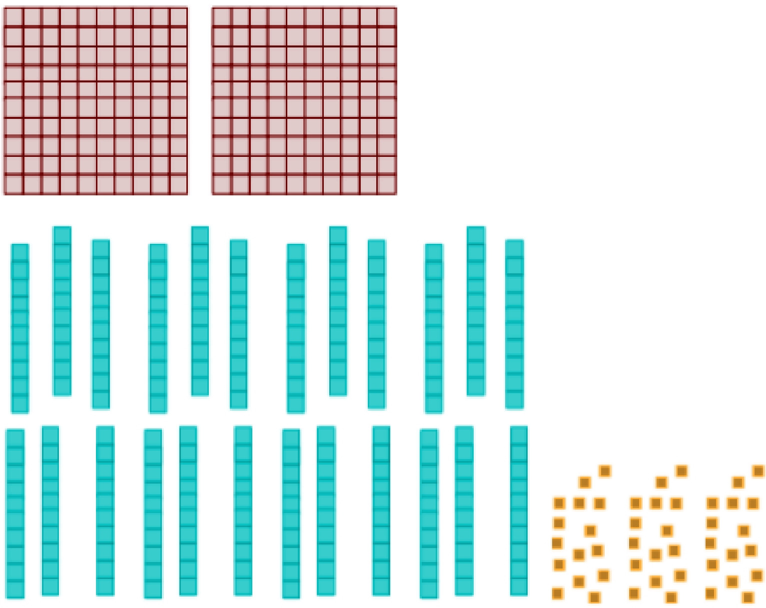
Tell students, “today we are going to explore the mathematics behind very small numbers, less than one.”

Explore

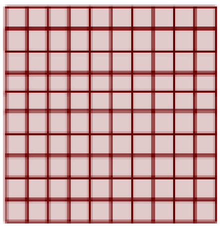
(25minutes) Start this section by reviewing what students learned about decimals from their last class since it is the critical piece of this lesson. Then, ask students to count their fingers. Once they finished counting, call on a student to share with the class how many fingers did he/she count. Once we all agree that there are ten fingers, show students one rod  and ask them to count how many ones  exist in a rod? Allow students a few seconds, about 15 to 20 seconds, to talk to their elbow partner and share their answer. Call on a student to share their answer with the class that there are 10 ones in a rod. After that ask students if they see a connection between the rods, ones and their fingers. Provide students with 1 to 2 minutes to discuss their response with their elbow partner. During this time, walk the class and listen to their responses. Have a student share their response with the class. The goal is to have students see the connection between one rod representing both their hands and the ones representing the fingers.

Next, put students in groups of three and drop a container of base ten blocks on the floor and ask the following questions:

- What do you wonder and what do you notice?
- Is there more of a certain block?
- Is there less of a certain block?
- What do you think each item dropped on the floor represents? How do you know?
- Can you estimate the total value? How would you do that?



After listening to students' conversations within their groups and having some students share their responses, share with the class that we will be looking at these blocks a little differently today. First discuss the base ten system and remind the students of what they have learned in their previous lessons. Then, tell the students that if we decide for a flat to equal 1, then what is the value of a rod?

If  (flat)=1 , then  (rod) =?

Give students about a minute to talk to their group and have their answer ready. Have one or two groups share their responses. The primary goal is for students to see that 10 of the green rods will fill one whole. Therefore, the green rod is one-tenth of the whole.

After helping students see the connection between the rods and the flat that a rod is 1/10th of the flat, bring students' attention back to the container that was dropped on the floor. Share with the class that one of their classmates believes that we can't write one number to represent all these blocks because there are more than 10 tenths and more than 10 hundreds. Do you think your classmate is correct?

Provide each group an identical container as the one dropped on the floor and allow the groups to explore playing with the blocks for a minute or two. Before releasing students to work with their group to answer this question, project the following questions on the screen:

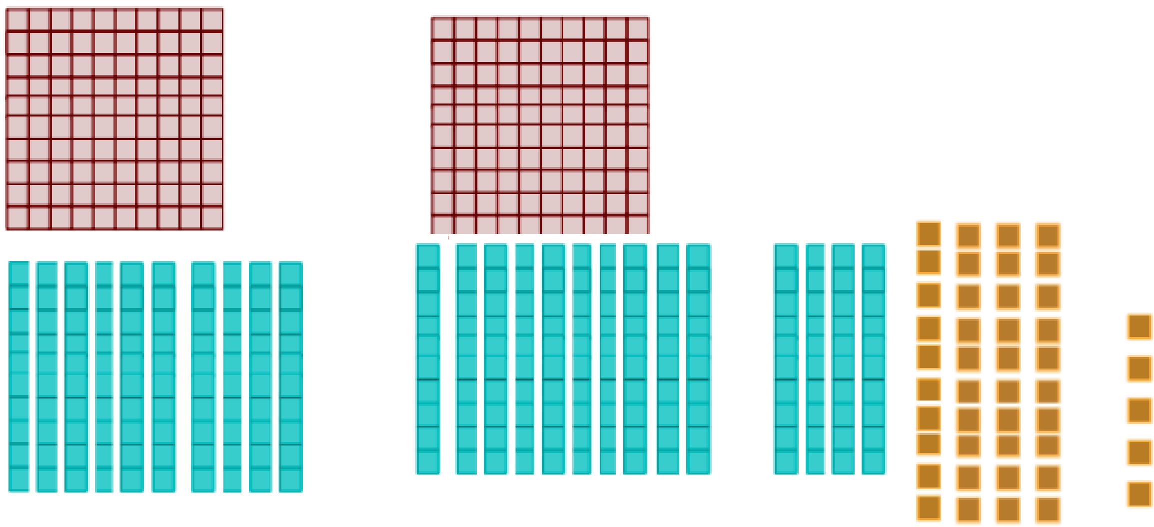
- How many hundredths, tenths, and ones do you see on the floor?
- How would you write this number?
- What happens when you have more than 10 tenths and more than 10 hundredths?
- Is it possible to write an equation to show this number?
- If yes, would that equation be the only possible one?
- Is there another way to show this number with base ten blocks?

As you are walking the class look for the followings in groups**:**

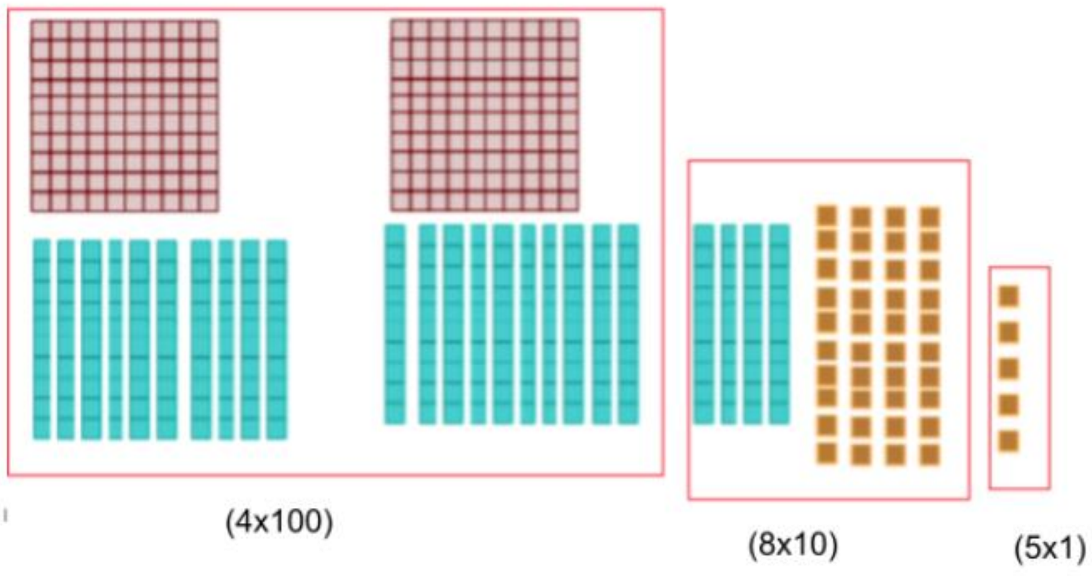
- Students who can make groups with hundredths and tenths
- Students who can calculate the total accurately

- Students who can say and write the total accurately
- Students who can write at least one equation to show standard form
- Students who can identify the number of hundredths, tenths and ones
- Students who can decompose and recompose numbers

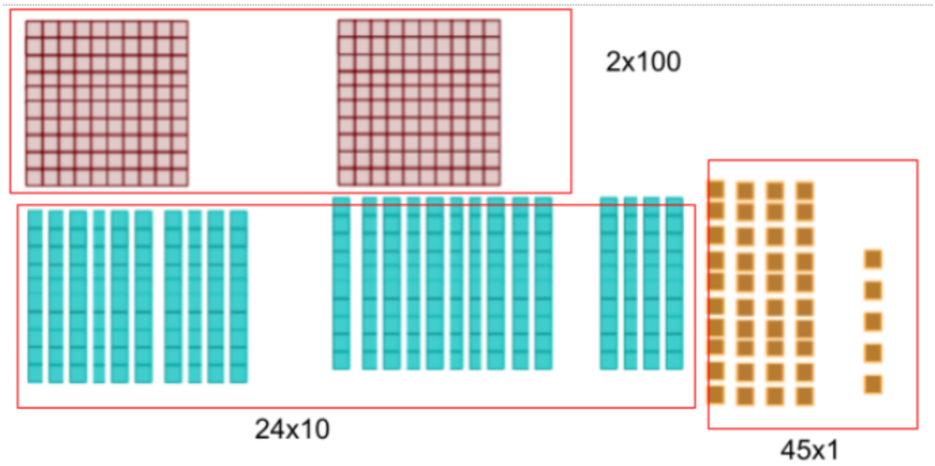
The following could be possible groupings of the blocks done by the students:



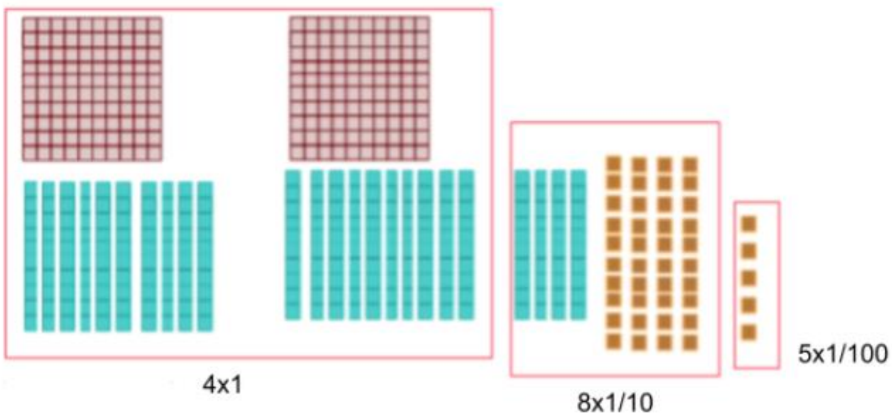
This group used the rods to build flats and uses units to build rods.

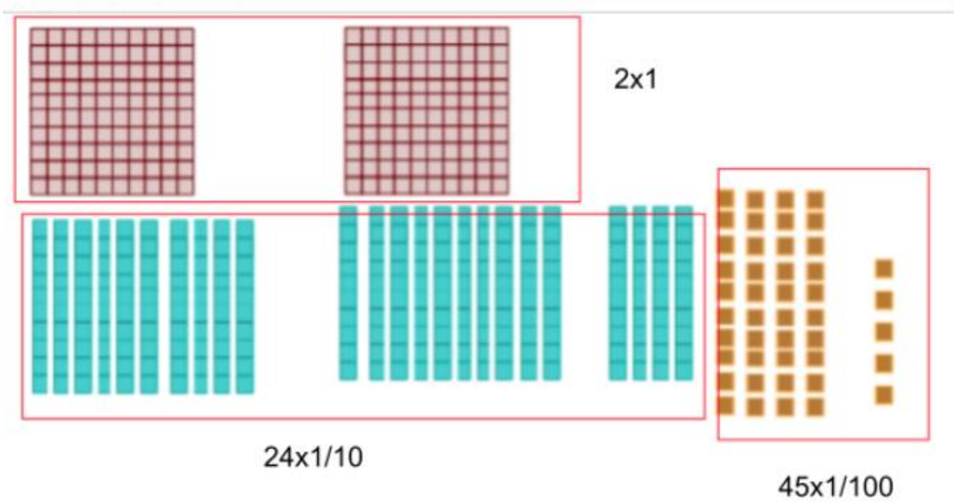


This group mainly focuses on the number of units in flats and rods. With this group, encourage the students to see if they can take the rods and make them into flats and do the same with the units.

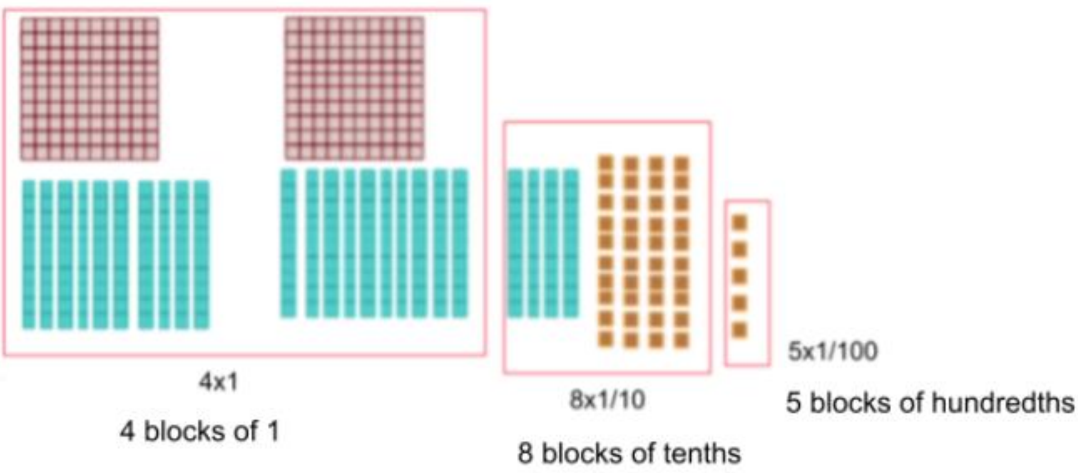


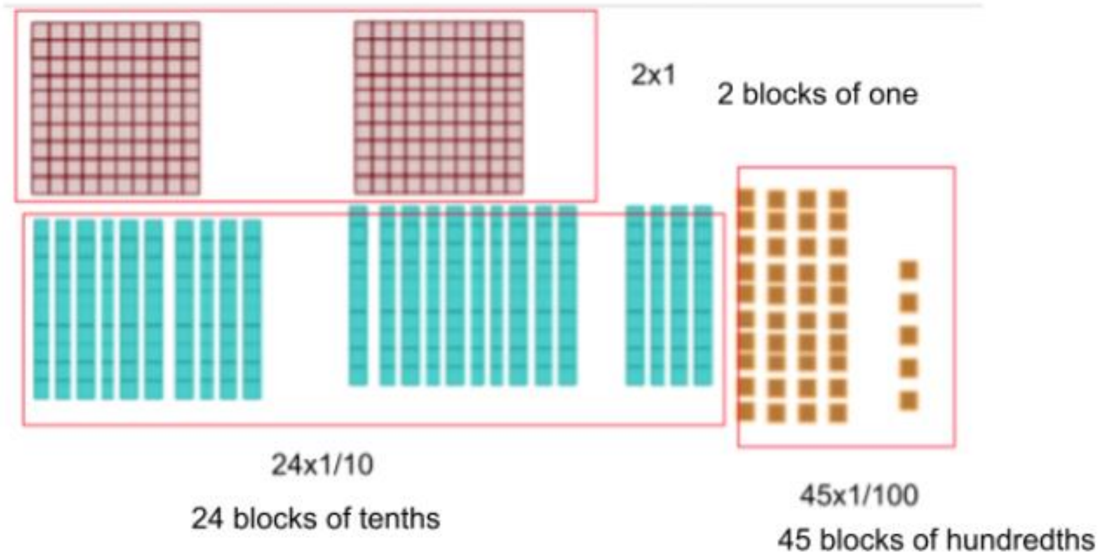
After sharing a few of the students' examples with the class, remind the students of what we have decided at the beginning of the class. The decision was made that a flat will represent a unit (1), the rod is one-tenth of the unit and the small units are one-hundredth of the unit. Then, ask students to modify their response and make the necessary changes.



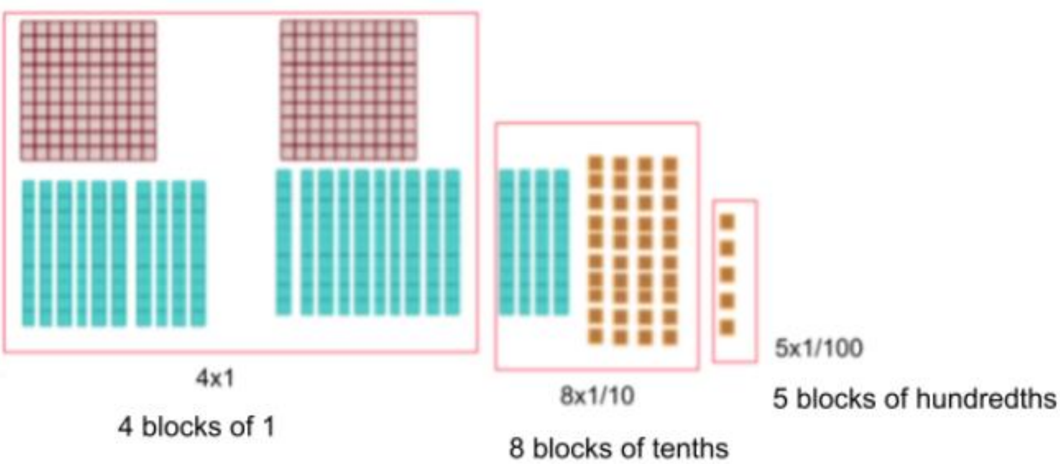


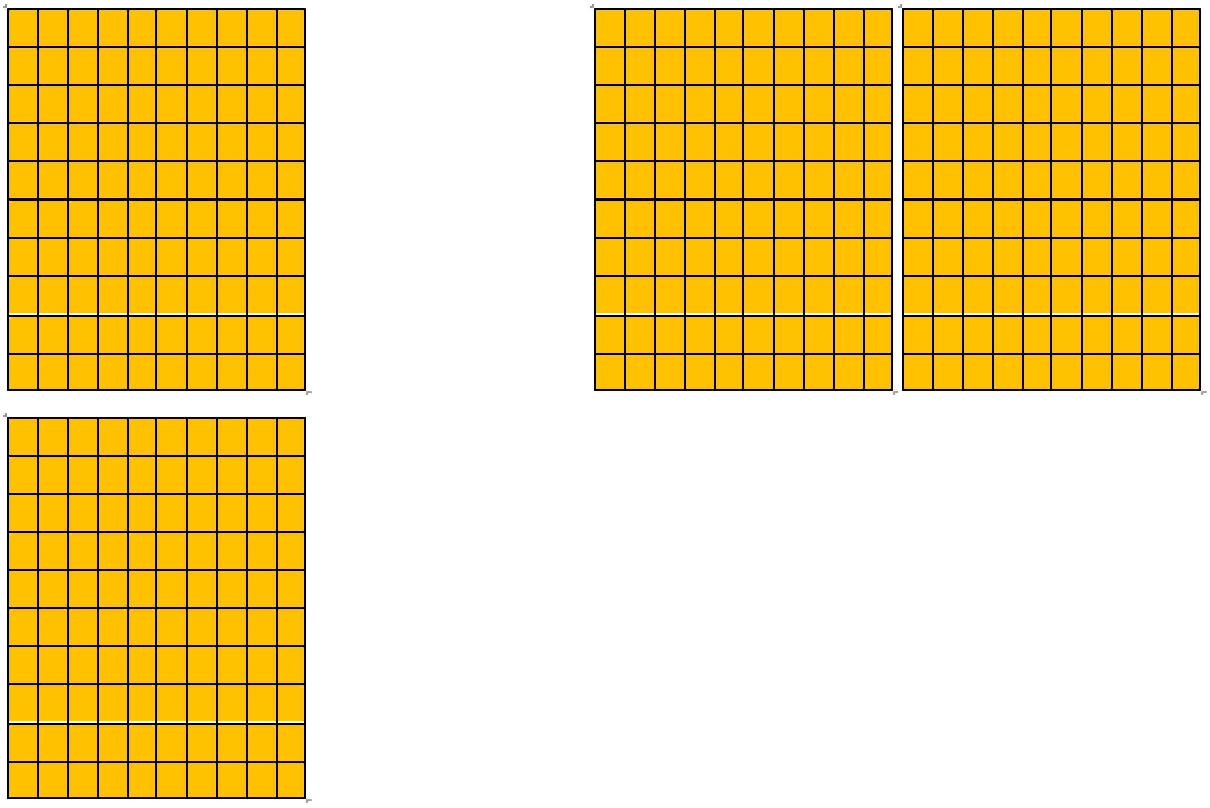
Then encourage students to also write their answers in words



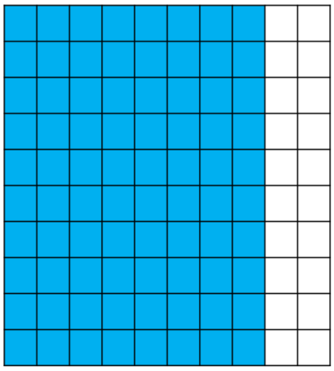


Then, ask students to build an area model for their representation of the blocks since they have practiced building area blocks in the previous lessons. If students seem to be struggling with the area model, revisit the concept before starting the task. Here is an example of what to expect of students work.

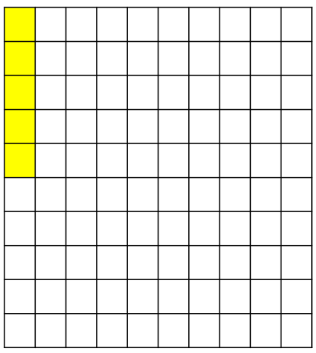




4 blocks of (1x1) = 4



8 blocks of 1/10 = 8x1/10=8/10= 0.8



5 blocks of 1/100 = 5x1/100 = 5/100 = 0.05

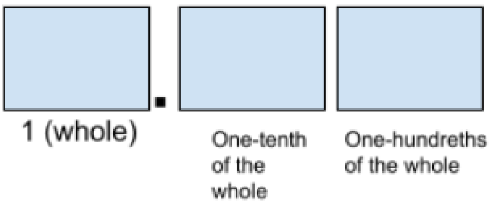
4 + 0.8 + 0.05 = 4.85

At the end, students are required to show their answer on a number line. For example,

4 + 0.8 + 0.05 = 4.85

Explain

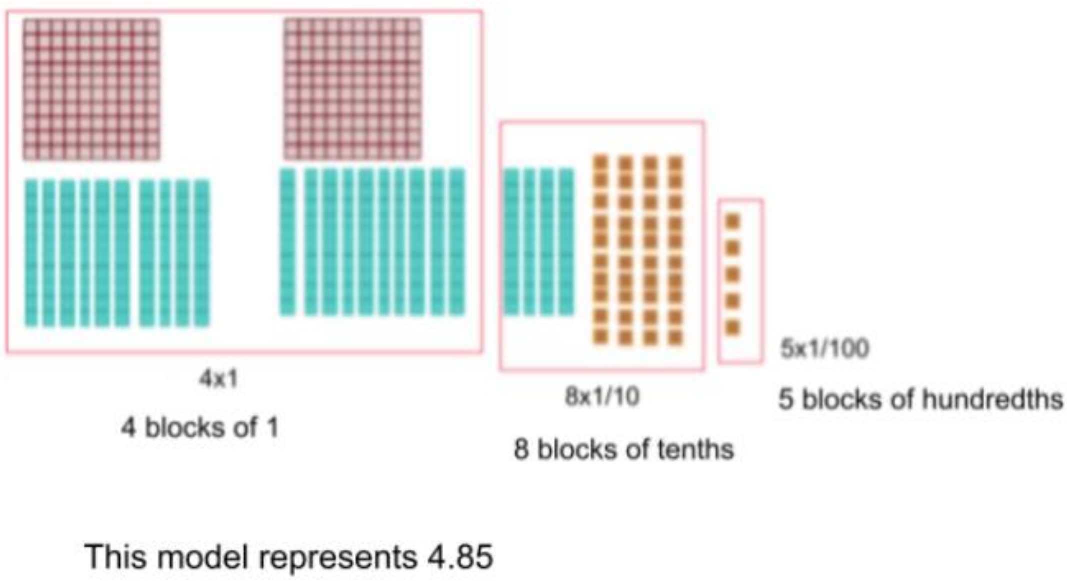
(15 minutes) In this section, start with showing the students the place value of a decimal number. For instance, we can introduce the concept by using the following model:



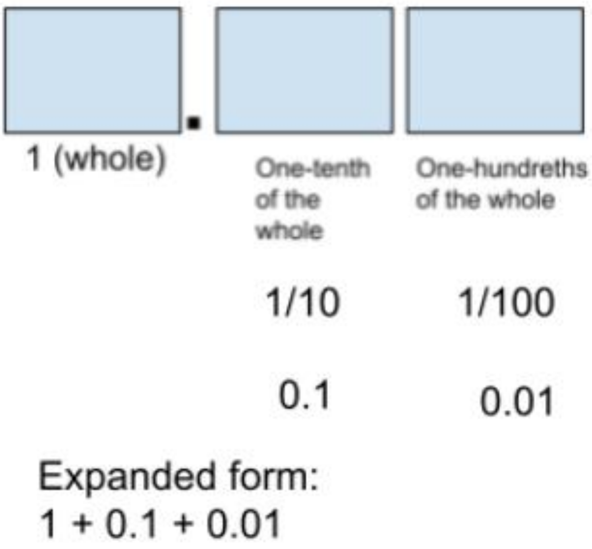
Then, provide students with 1 to 2 minutes to apply the concept to their model and decide on the value that represents their model.

The expectation is that students are able to use the template provided and apply it to their model by counting their number of whole, tenths and hundredths.

Possible students' solution could be as follows:



After that, introduce the following model and have students apply it to their representations



Then, ask students if they can group their model differently and record their examples in the given table (see Appendix A)

Number of 1(whole)	Number of tenths	Number of hundredths	Number of thousandths	Expanded form
4	8	5	0	4+8/10+5/100 4+0.8+0.05=4.85

Mathematics ideas to highlight:

- Understand that a number can be represented in base ten, by name, in words, and in expanded form
- Understand that expanded form is the additive value of each place
- Understand that numbers are read by naming the period of each group
- Base ten materials represent the expanded form of a number
- A number can be represented in a variety of ways with hundreds, tens, and ones

Extend

(20 minutes) Have students reflect back to the beginning of the lesson, where we looked at the results of the Olympic swim meet (see Figure 1). Have students replicate the work they

just completed with the times in the Olympics. Have students develop the chart in Appendix B, showing the swimmer and the expanded form of the final swim times. Note that in this scenario we also have to deal with the 10's digit.

Have students give a brief presentation on their favorite Olympic sport that uses decimals. Ask students to find their favorite Olympic sport from the most recent Olympics (Summer or Winter) that uses decimals and create a chart for the top three finishers (see Appendix C). Different sports might have different need for place value. For example, a marathon probably won't need to get to the 100th of a second. But the 100m dash will.

You may also wish to see if students can apply the same concepts of decimals and place value to distance, rather than time.

If students are unfamiliar with timed or distanced Olympic sports, consider the following possibilities.

Timed Summer Olympic Sports	Timed Winter Olympic Sports
100 meter dash	Ice skating
100 meter hurdles	Bobsled
Cycling	Downhill skiing
Distanced Summer Olympic Sports	Distance Winter Olympic Sports
Long Jump	Ski Jump
Javelin	
Shot put	

This website has the results for all Olympic events. https://en.wikipedia.org/wiki/Olympic_results_index Students may need some help navigating to the correct event and most recent year.

Evaluate

When working with the results of the Olympic swim meet, check the accuracy of their responses and ask students to share with one another their favorite sports and the decimals involved once students have created their chart. Students may give a formal presentation in front of the class or share with one another in small groups. If students struggle using the blocks learning about this standard, model some basic examples using the base 10 blocks. For example, demonstrate how to use the base 10 blocks to represent a decimal number

Appendix A

Number of 1(whole)	Number of tenths	Number of hundredths	Number of thousandths	Expanded form

Appendix B

Swimmer	Number of 10s	Number of 1(whole)	Number of tenths	Number of hundredths
Michael Phelps				
Milorad Čavić				
Andrew Lauterstein				
Ian Crocker				
Jason Dunford				
Takuro Fujii				
Andriy Serdinov				
Ryan Pini				

Appendix C

Name:______________________________

My Favorite Olympic Sport Is _________________

Here is how the sport uses decimals:

In the most recent Olympics, here were the results for the top 3 finishers.

Year:

Gold:

Silver:

Bronze:

Here are the results in expanded form:

Citation

Aleksani, H., & Krall, G. (2023). There is More to Base Ten! In A. Bicer (Ed.), *Creativity-Directed Mathematical Tasks for 5th Grade Common Core Classrooms* (pp. 163-178). ISTES Organization.

SECTION 4 - PERFORM OPERATIONS WITH MULTI-DIGIT WHOLE NUMBERS AND WITH DECIMALS TO HUNDREDTHS

Task 13 - Planning a Party

Michelle Tudor, Melana Osborne, Michael Gundlach

Mathematical Content Standard

CCSS.MATH.CONTENT.5.NBT.B.7

Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Mathematical Practice Standards

1. MP 2 Reason abstractly and quantitatively.
2. MP 7 Look for and make use of structure.
3. MP 8 Look for and express regularity in repeated reasoning.

Lesson Objective

Students will learn how to add and subtract decimals to the hundredths by helping the teacher plan a family weekend trip and then extending this knowledge to creating and planning a family/friend vacation without a prescribed budget. Students' creativity will be fostered through enabling them to connect their previous knowledge of whole number addition and subtraction to addition and subtraction of decimals. Students will learn how the general strategy for adding and subtracting decimals is an extension of our whole number addition and subtraction algorithms by using regrouping and place value strategies. Creativity will be encouraged with multiple representations for these algorithms. This lesson is intended to take 2-3 days.

Materials List

Base ten blocks, play money, grid paper, colored pencils, poster paper, copies of Mr. Moore's budget.

Engagement

(10 minutes) Watch a video about a popular tourist attraction. A suggested video can be found at the link: https://youtu.be/ILynYG975-o (This is a 4-minute video about Mammoth Cave, a tourist attraction in Kentucky.)

We recommend choosing a video about a tourist attraction near your location students may be interested in visiting. As students watch the video have them jot down a couple of things they would like to do if they were at that tourist attraction.

After showing the video, have students discuss in pairs or small groups their "dream vacation." Let a few students share their dream vacation with the whole class.

Explore A

Situation: Mr. Moore took his wife and 2 kids on a weekend trip to Mammoth Cave National Park. Pause here and ask "What will Mr. Moore have to pay for while they are on the trip? Allow students to brainstorm costs that Mr. Moore will have to account for. Then present the following information: They stayed 2 nights at a hotel with free breakfast, toured the cave, bought 2 souvenirs, and ate at a restaurant 4 times. Mr. Moore isn't sure what the total cost of the trip was, but he wrote down what everything cost. Mr. Moore would like us to calculate the total amount he spent on the trip because he is hoping he didn't go over the $350 he planned on spending. His expenses are listed below. Did Mr. Moore stay within his budget?

Hotel	$109.99/night			
Cave Tour	$8.25/adult	$6.50/kid		
Souvenirs	Stuffed Bear $10.99	T-shirt $12.50		
Friday Dinner	$10.25	$7.25	$6.99	$4.99
Saturday Lunch	$12.99	$6.99	$8.50	$15.00
Saturday Dinner	$15.99	$10.99	$11.50	$6.50
Sunday Lunch	$7.99	$5.99	$6.99	$5.50
Gas	Mr. Moore lives 100 miles away from Mammoth Cave. His car gets 25 miles per gallon. Gas costs $3.19 per gallon.			

Explain 1

Before students work on calculating the cost of the trip, have them come together to discuss different ways they could solve the problem. What operations are they going to be using? Is there more than one operation that can be used to calculate an expense? (For example, if you multiply, is there also a way to add?) Use the hotel cost and the cave tour cost (both require doubling. Ask questions to lead students to notice this) to model using place value, sets, or number lines to calculate the totals. Allow students time to discuss after each model is shown. Tell students the gas price is a challenge and allow them to try it on their own first.

Explore B

Allow students time to work with a partner or group to calculate Mr. Moore's expenses. Students should model their answers in at least two different ways. Circulate as students are working to help with questions or misconceptions.

Explain 2

Call students back together to discuss the costs they have calculated and to answer the question if Mr. Moore stayed within his budget. Allow students to share different ways they modeled the cost of different items. Compare answers and discuss any differences. Clear up any misconceptions or errors and allow the students to correct it. Students may make regrouping errors or decimal placement errors. Explicitly model how to calculate the cost of the gas and allow students to compare their strategies and answers.

Engage Part 2

Show students the video that shows the top ten places to travel with kids. https://www.youtube.com/watch?v=ogTN8la5gaU

After students have watched the video, ask them to brainstorm places they would like to go. Tell them they will be planning a vacation for their family. Have some suggestions to make in case students struggle to think of a place to go. Allow students to find 1 or 2 other people who want to plan the same vacation and begin to research the costs. Alternatively, you could assign groups and destinations.

Extend

After students have selected a group or been placed in one, they can begin planning. They will have a budget for activities, lodging, and food. The students will need to plan activities they want to do, how many nights they will stay, and their meals. Students will model the computation of the costs using place value, sets or number lines as well as a numerical representation of the budget.

The students will have $1500 to spend on their vacation. They must create a budget and a detailed plan of their vacation. Students should plan accordingly for how many people will attend their vacation. Give students time to look up realistic prices for the place they choose to go, such as lodging costs, activity costs, food/grocery costs, etc. Students will then divide up the $1500 to create their budget for each expense. Once students have created their budget and vacation plan, have each group create a poster that shows their vacation plan, budget, and models. Allow students to present their work to the class. Have students vote on which vacation they think is the best budgeted and planned based on the trip, the activities, staying in the budget and the model used.

Extend Challenge Task (Optional)

For this part of the Extend, the students will need to add a gas bill into their budgets. Tell the students that the car they are taking on the vacation gets 25 mpg. The students will need to calculate the mileage they will travel (have them pick a destination and use Google Maps for mileage; give them a maximum number of miles that is realistic) to calculate how much money they will spend on gas. Students will need to look up current gas prices prior to doing this calculation. At this point, the students have not been formally introduced to multiplication of decimals. However, they can use what they have learned about multiplication of whole numbers, addition and subtraction of decimals, and their number sense to try and figure out how to multiply decimals.

Evaluate

During the Explore (A/B) and Explain (1/2), listen to student discussion about strategies of decimal addition and subtraction they can use to find if Mr. Moore stayed within his budget.

Students should be discussing addition of costs, then subtraction of the costs from the $300 budget. If this is not in the students' discussion or they seem to be struggling, give students a simpler example such as, your parents gave you $20 to go to the movies. If your movie ticket is $8.00 and your drink and popcorn is $6, how much money do you have left? This will help them to think through the operations they are using without being overwhelmed with so much information.

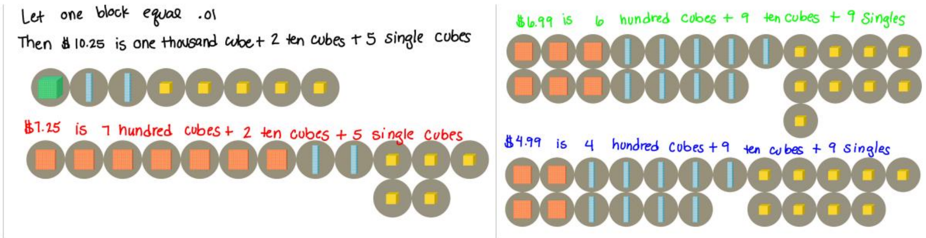
Circulate and look at the students' models. During this part of the lesson, it is important to check for multiple representations of the students' work. Some students may struggle creating multiple models, if they do, give some additional examples of adding and subtracting decimals using different models; such as, use the play money to group the ones, dimes, nickels, and pennies and show the students again how regrouping works when we add/subtract decimals.

During the Extend part of the lesson, circulate the room and check to see if the students are dividing up the $1500 budget realistically. To do this, the students will need to research current and accurate prices of the different categories listed (lodging, activities, food, and gas). Circulate and briefly check each groups' progress throughout the vacation planning; check for obvious mistakes such as prices that are too high or too low. Errors could also include things like forgetting to include all the meals for each person or finding the price of the hotel for the total number of nights. Encourage each group to use a different model from a neighbor group so there are multiple representations of the models when the groups present.

For the Extend Challenge part, the students have not been formally introduced to multiplication yet. Circulate the room and listen for problem solving strategies (possibly repeated addition). Some students may even extend the relationship between adding/subtracting whole numbers and decimals to multiplication, although this will be a more difficult relationship for the students to see. If they have trouble with this, remind them of the relationship between addition/subtraction between whole numbers and decimals. Tell the students they can, in fact, use this knowledge to multiply decimals. This could include using repeated addition to multiply. Encourage students to use their number sense if they have trouble with where to place the decimal by thinking about what would be a reasonable answer. Focus on student problem-solving here; it is a good introduction to multiplication of decimals.

Explore B Model Examples

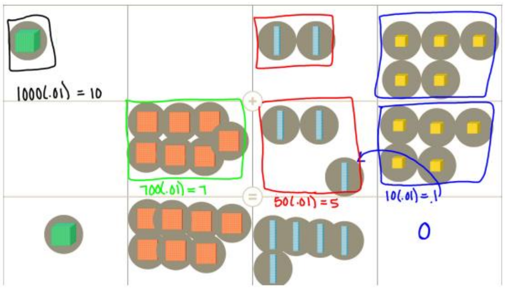
- Friday Dinner : $10.25, $7.25, $6.99, $4.99
- Part A - Base ten blocks letting one block equal .01



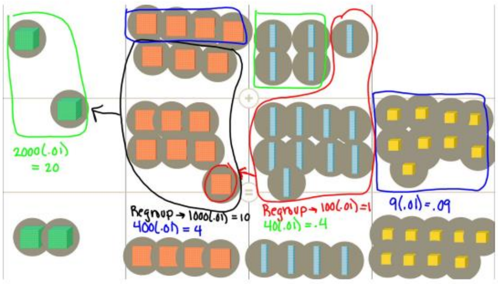
now add by regrouping using blocks

10.25 + 7.25 + 6.99 + 4.99

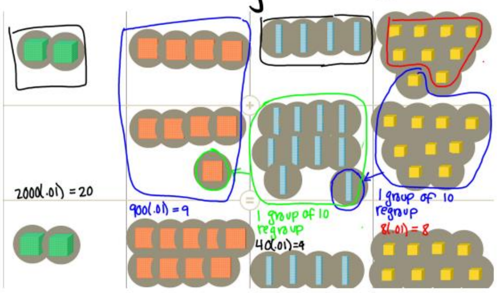
The first figure shows 10.25 + 7.25 = 17.50

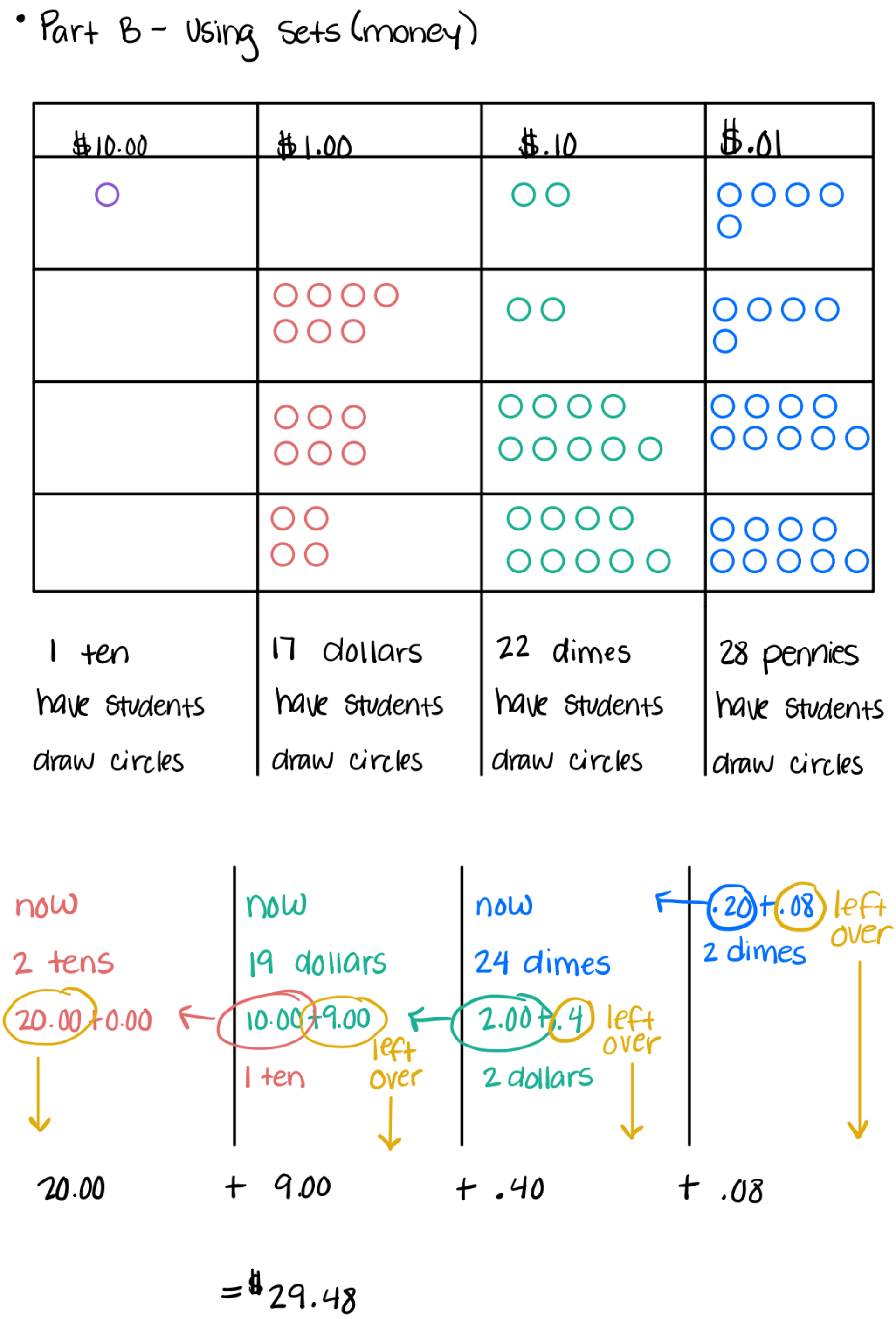


The second figure shows 17.50 + 6.99 = 24.49



The third figure shows 24.49 + 4.99 = 29.48





Citation

Tudor, M., Osborne, M., & Gundlach, M. (2023). Planning a Party. In A. Bicer (Ed.), *Creativity-Directed Mathematical Tasks for 5th Grade Common Core Classrooms* (pp. 179-186). ISTES Organization.

Task 14 - Candy Shop

Traci Jackson, Aylin S. Carey, Fay Quiroz

Mathematical Content Standard

CCSS.MATH.CONTENT.5.NBT.B.7

Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Mathematical Practice Standards

CCSS.MATH.PRACTICE.MP4

Model with mathematics.

CCSS.MATH.PRACTICE.MP5

Use appropriate tools strategically.

CCSS.MATH.PRACTICE.MP6

Attend to precision.

CCSS.MATH.PRACTICE.MP7

Look for and make use of structure.

Vocabulary

Decimal division, decimal modeling.

Materials

Printables (see Appendices A-G), base 10 blocks or virtual base 10 blocks:

https://mathigon.org/polypad

Lesson Objective

Students will investigate how to divide a decimal (hundredths) by a whole number in the context of a candy store. Although they initially begin with addition and multiplication, the focus is on making sense of a decimal divided by a whole number. Students use creativity to model a situation and represent their thinking through multiple representations. They begin the task by choosing the type and amount of candy to buy to share with their team with a weight constraint. Students explore division using multiple representations with area models, number lines, base ten blocks, and/or coins to find the cost per student to purchase the candy.

Engagement

Read the poem twice.

"Smart" by Shel Silverstein

My dad gave me a one-dollar bill
'Cause I'm his smartest son,
And I swapped it for two shiny quarters
'Cause two is more than one!

And then I took the quarters
And traded them to Lou
For three dimes - I guess he don't know
that three is more than two!

Just then, along came old blind Bates
And just 'cause he can't see
He gave me four nickels for my three dimes,
And four is more than three!

And I took the nickels to Hiram Coombs
Down at the seed-feed store,
and the fool gave me five pennies for them,
And five is more than four!

And then I went and showed my dad,
and he got red in the cheeks
And closed his eyes and shook his head-
Too proud of me to speak!

The first read is just for students to listen for what the son shows his father. Before reading the poem a second time, ask the students what the son understands about money. Use the following questions to help guide the discussion.

How much money did the student lose? What are different ways to calculate this?

Students may want to go progressively through the poem. For example: The son lost $.50 during the first transaction, then continues with trading 2 quarters for 3 dimes losing $.20. Alternatively, students may recognize that the son ends with 5 pennies and starts with $1.00, so altogether the son lost $.95. Stress that both ways of calculating are valid. There is additional information gained by going progressively through the poem because the loss for each transaction is known. Tell students that this poem was written by a famous author Shel Silverstein. He was born in 1930 and wrote over 400 children's poems and 800 songs for adults (New World Encyclopedia, 2019).

Number Talk: Write 741÷3 on the board. Ask students to solve in any way and as many ways as they can. During the number talk, be sure to highlight the area model, base 10 blocks, and the number line (Figure 1,2, and 3).

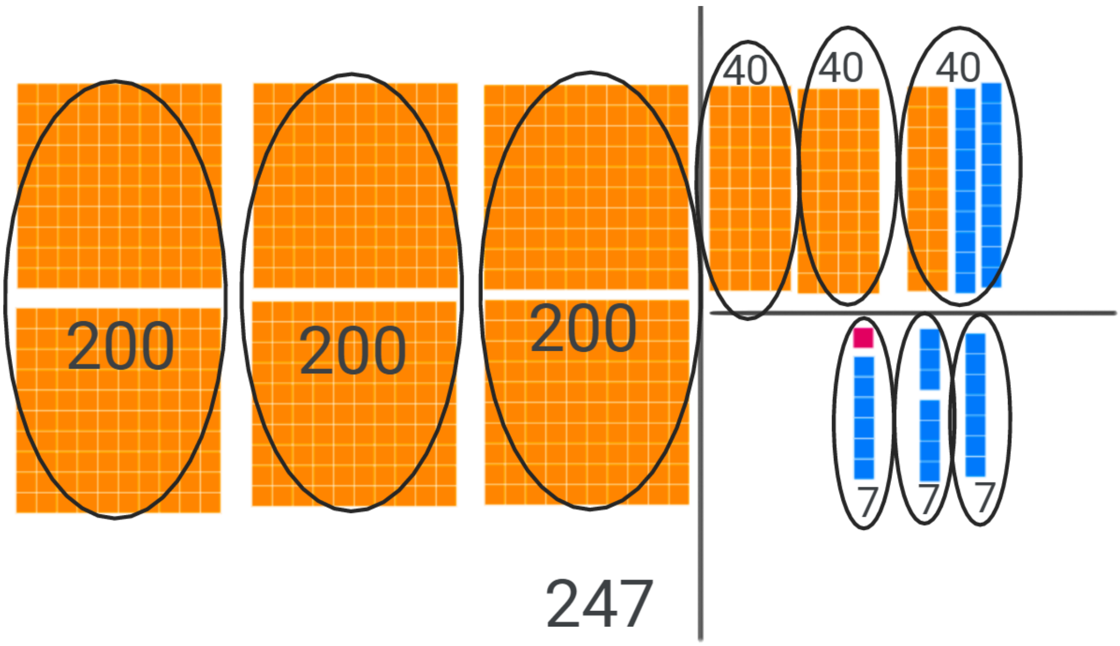


Figure 1. Example of using Base 10 Blocks for 741÷3

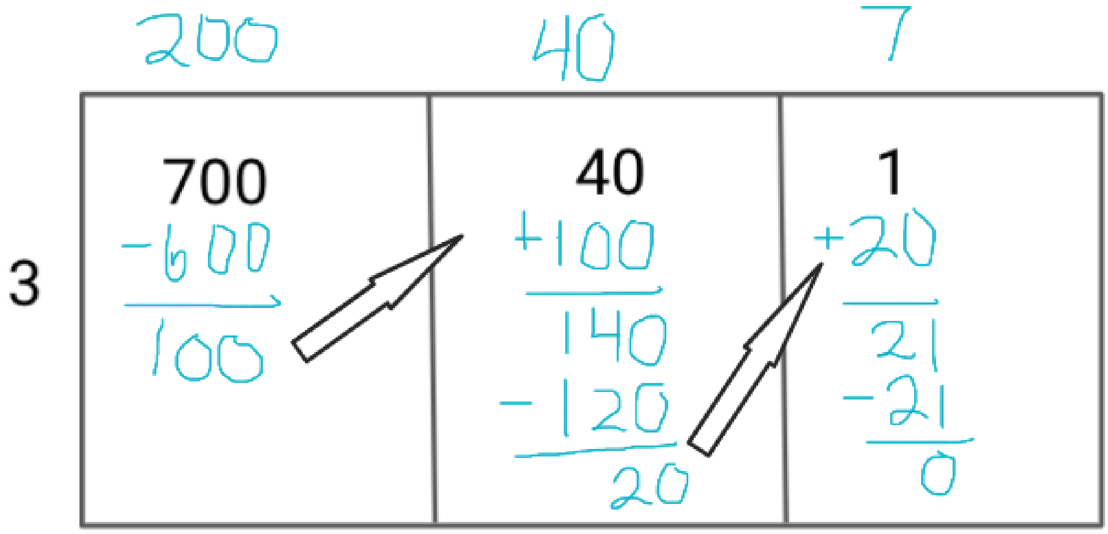


Figure 2. Example of using the Area Model for 741÷3

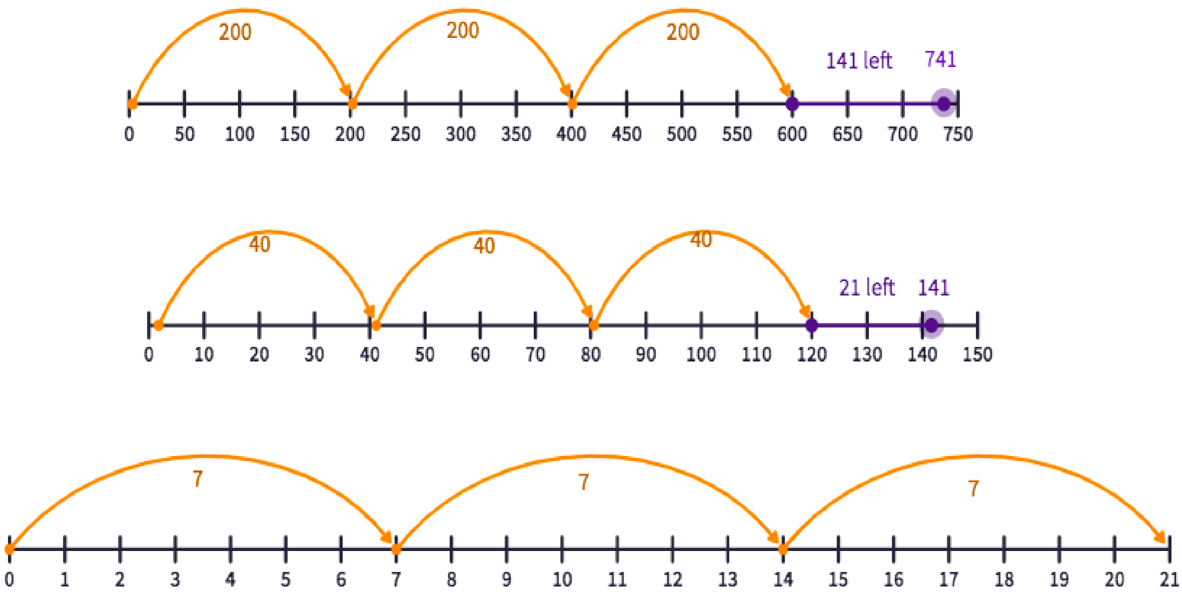


Figure 3. Example of using a Number Line for 741÷3

Explore

Tell students to pretend they are going on a field trip. Let students know that on their field trip, they will be stopping at a gift shop that offers bulk candy. Ask students if they have ever been to a bulk candy store where candy is in bins or barrels. Show an image or video of a bulk candy store (see Figure 4.)

Figure 4. Bulk Candy Shop Image

Tell students they are going to buy some candy for their team to share when they return from the field trip. They will need to plan ahead because there is limited time in the gift shop and the buses will be leaving quickly.

Provide them with a printout of the options and a chart (see Appendix A). Tell them they may choose which types of candy and how much of each (in pounds) they would like to purchase. Because teachers don't want students to get sick, each team may only buy a total of 2.5 pounds of candy.

Students work in groups of 4 to choose their candy and calculate the overall cost (and weight to be sure they haven't gone over). Provide students with access to base ten blocks, number lines, hundred grids, coins, and extra paper. Students then decide how to split the cost and candy among their team. Students may choose to do this in several different ways including dividing up different amounts of the certain candy choices, depending on the type of candy each student prefers. Students also may just divide the cost evenly by 4.

Encourage them to use models to show this division. It is important to note that students may have an amount of money that is not evenly divided by 4 (so they will have a thousandths digit). Mention that this happens at the store too and they just estimate a close penny value (usually round up).

Explain

Ask students to come back to a class discussion and share how much candy their group bought, how they decided to share and what representations they used to solve the problem. Acknowledge different strategies groups used to perform division. For example, if a group used an area model, look for breaking the cost up into dollars, tenths, and hundredths. If students had an extra amount left, ask how they chose to estimate the number by estimating to the closest penny or hundredths place.

Group 1 Task: Four students bought $11.12 worth of candy. If this group divides the cost evenly, explain how much money each member should pay.

After groups share their responses, tell students another class also went to the candy shop: There were four groups of students that are having a difficult time calculating what each person owes.

Tell students as a class they will work on the Group 1 together (see Appendix B) with their choice of a representation: base-ten blocks, area model or a number line to solve each problem. After completing the first group task, students will break off into pairs to work on the remaining groups.

Give each student a copy of Appendix B to represent the cost for each group member and record their representations.

Highlight each representation to the class (see Figure 5, 6, and 7) As students practice each representation, formatively assess those who may require more help with the content and those who are ready to be challenged.

After the class discussion, strategically pair off students to work on the following three group problems in Appendix B. As students work in their pairs, refer to their earlier observations to guide students in using a representation that may work best for them. While it is okay for students to use the same representation, encourage them to make connections to additional representations as well.

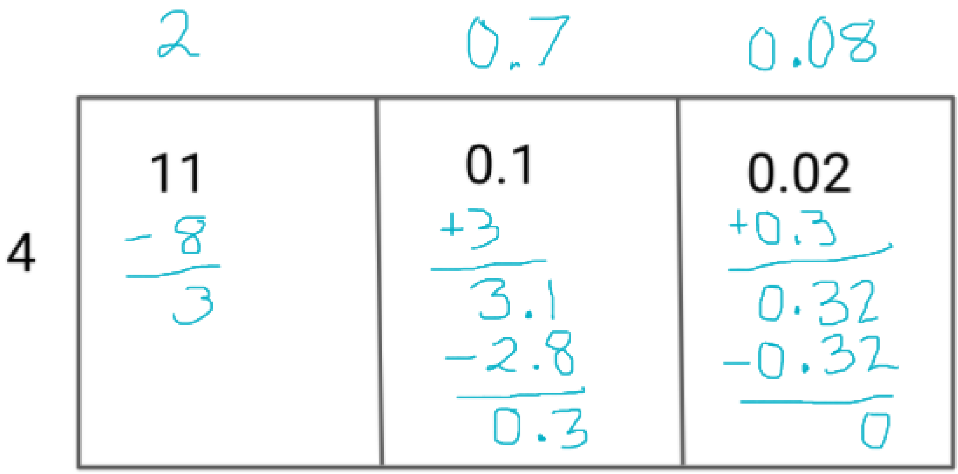


Figure 5. Example of using the Area Model to divide the Total Cost of $11.12 by Four

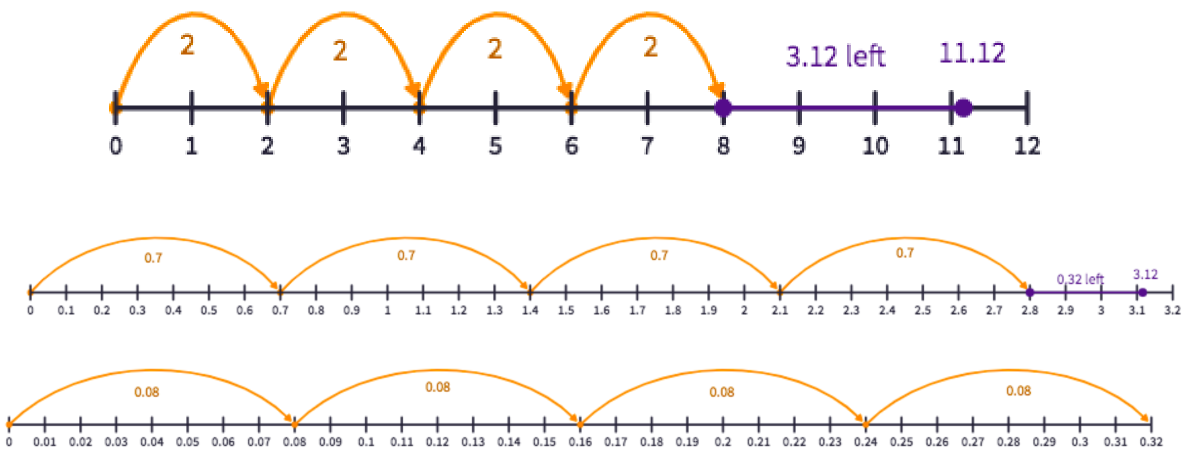


Figure 6. Example using a Number Line to divide the Total Cost $11.12 by Four

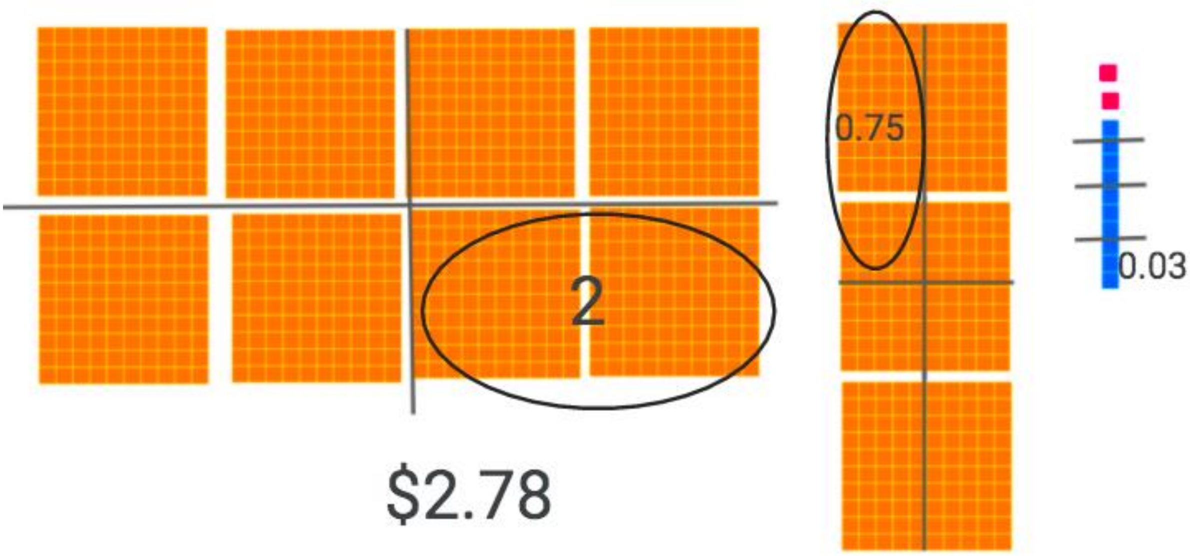


Figure 7. Example using Base-ten blocks of dividing the Total Cost $11.12 by Four

Extend

Another scenario to consider before introducing a decimal by decimal division would be figuring out the number of bags needed to split a pound of candies, if each bag should contain 0.25 pound of candies (see Figure 8). For this problem, encourage students to work as a group and discuss their findings.

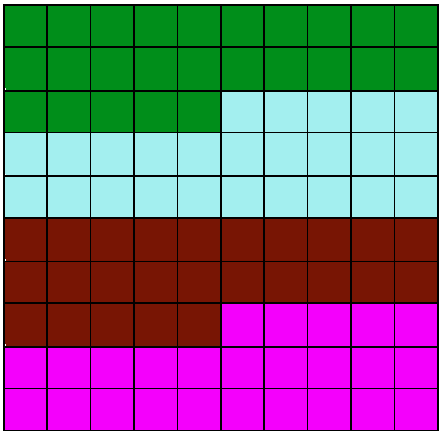


Figure 8. Possible Student Example of an Area Model for 1 ÷ 0.25

After students figure out the number of bags needed through multiple representations—area model, set model, and length model, bring the class together to share their ideas while writing the decimal division numerically, 1 ÷ 0.25 = 4, on the board. Here we can see the answer or use calculator to see the answer. Then, ask student to draw a conclusion that the quotient gets bigger as the divisor becomes smaller. For this realization, it would be ideal for students to create a number pattern (see Figure 9). Working with their group members, allow students some time for this number pattern activity. Connect this back to the activity to see this pattern in context. If the bags held .25lbs, how many bags would we need to make 1lb? What if the bags were 0.10 lb?

$$1 \div 0.25 =$$
$$1 \div 0.20 =$$
$$1 \div 0.15 =$$
$$1 \div 0.10 =$$
$$1 \div 0.05 =$$
$$1 \div 0.04 =$$
$$1 \div 0.03 =$$
$$1 \div 0.02 =$$
$$1 \div 0.01 =$$

Figure 9. A Number Pattern with Decimal Division

Ask students how fractions can also be expressed as decimals (e.g., $\frac{1}{4} = 0.25$). For the number pattern in figure 5, students can also divide by a fraction if they are not ready to divide by a decimal or use both fractions and decimals to make a comparison. Encourage students using both fractions and decimals simultaneously until they make a connection that fractions and decimals are similar in a way to express partial numbers and a representation of division.

After creating the number pattern, bring the class together and ask students to predict what would happen when they divide 1 by 0.00. Notice students' reactions and have a discussion on the concept of division by 0, if students are ready. More number patterns can be created to make a generalization. Allow students to discuss their findings. We expect students to see that 1/0 is undefined as there might be 0 zero 1 zero, 2 zero, etc. In one explanation teachers may use to help students recognize the answer times 0 will not equal 1 because anything times zero equals zero.

Record students' understanding of division by a decimal and then introduce a decimal by decimal division with this example:

What if the jar had a total of 2.75 pounds of candies and a single bag could only hold 0.25 pounds ($3/bag). Now the students need to figure out how they could split 2.75 evenly so that each bag would contain 0.25 pounds of candies (see Figure 10).

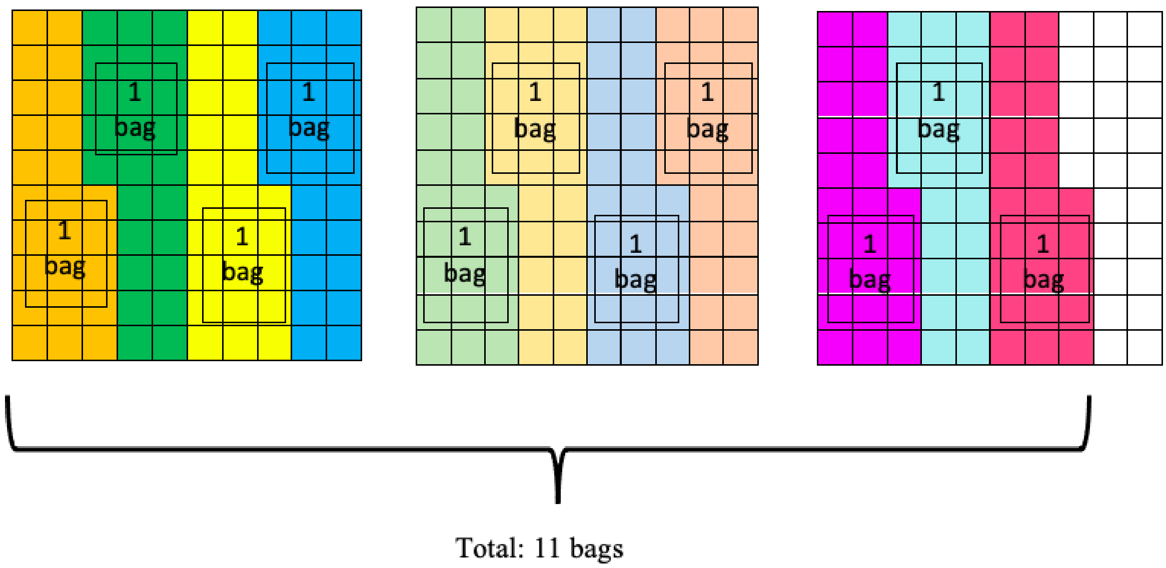


Figure 10. Possible Student Example of an Area Model for 2.75 ÷ 0.25

Again, encourage students to solve the problem through multiple representations–area model, set model, and length model. Look for students' understanding of the connection between decimals and fractions and how they apply their understanding to correctly perform division. There are many options for students to practice dividing a decimal by a decimal. Students can create their own decimal division problems, or students may write problems to let other groups solve. Examples of possible created student problems are:

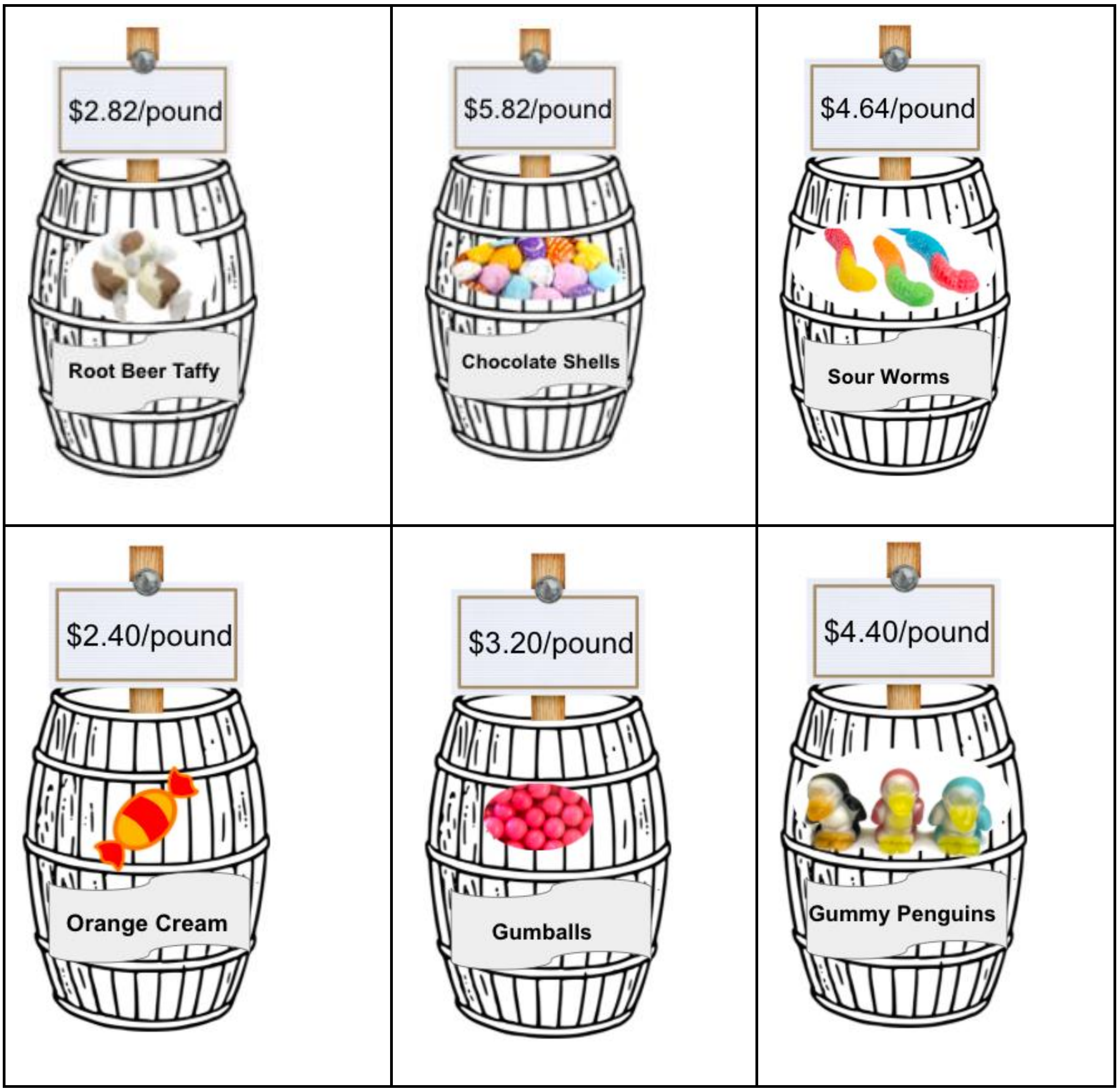
> If you have $15.82, How many pounds of sour worms ($4.64 per pound) could you buy?
>
> How many 0.27 pounds of bags could you buy of gumballs if you have $13.00. This will allow students the chance to interpret the remainder (15 bags) or discuss the idea of a partial 0.27-pound bag.

Evaluate

Look first for whether students have a strong understanding of the connection between decimals and fractions and they can correctly perform division by a fraction. While it is not the aim of this lesson, another goal is to have students interchangeably use both fractions and decimals to perform division by a decimal. For students who have difficulties with converting fractions to decimals or decimals to fractions, more activities can be provided with a commonly used fraction, such as 1/5, to convert a decimal by using base ten materials. Since students previously have worked with concrete models, drawings, and strategies when learning addition, subtraction, and multiplication, a short review on these concepts can be helpful for students who have difficulties with conversions.

Throughout the lesson, look for flexibility in using any models (e.g., area, length, and set) to represent decimal division. Note that the area model can be difficult for students with decimals at first compared to whole numbers. Look for students' understanding of how dividing a decimal number generates a smaller dividend and the connection between multiplication and division by a decimal when the area model is used. Continuously challenge students in creating additional representations of a mathematical model.

Appendix A



Type of Candy	Cost per Pound	Pounds	Cost of Candy
	Total Pounds	Total Cost	

Appendix B

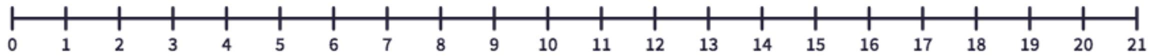
Group 1 bought $11.12 worth of candy and they have four people in their group. How much should each group member pay? Use a representation to show your answer.
Group 2 bought $15.22 worth of candy and they have three people in their group. How much should each group member pay? Use a representation to show your answer.

Group 3 bought $21.45 worth of candy and they have five people in their group. How much should each group member pay? Use a representation to show your answer.
Group 4 bought $49.98 worth of candy and they have six people in their group. How much should each group member pay? Use a representation to show your answer.

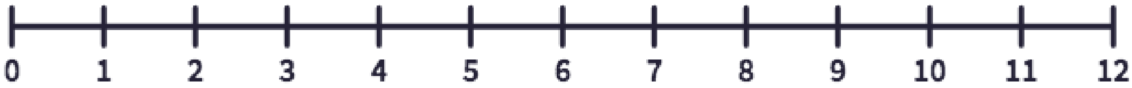
Appendix C


0 50 100 150 200 250 300 350 400 450 500 550 600 650 700 750

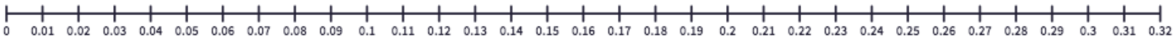

0 10 20 30 40 50 60 70 80 90 100 110 120 130 140 150


0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21

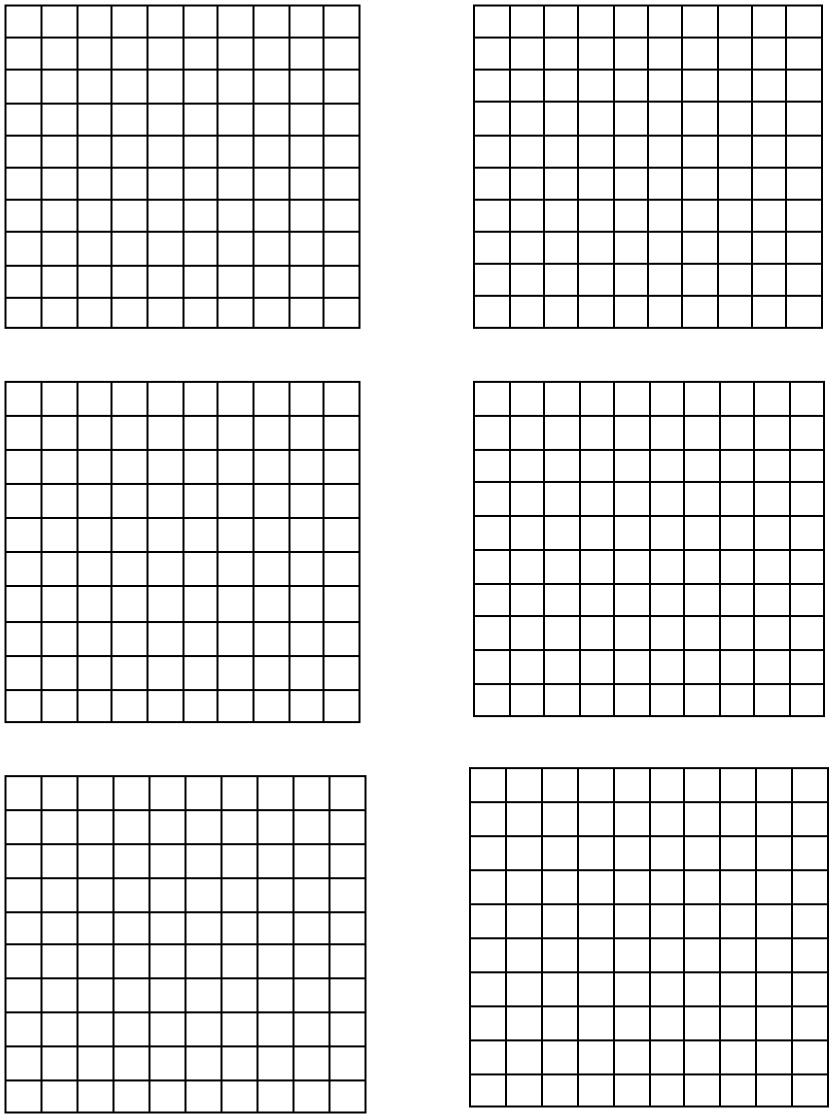
Appendix D


0 1 2 3 4 5 6 7 8 9 10 11 12


0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 1.9 2 2.1 2.2 2.3 2.4 2.5 2.6 2.7 2.8 2.9 3 3.1 3.2


0 0.01 0.02 0.03 0.04 0.05 0.06 0.07 0.08 0.09 0.1 0.11 0.12 0.13 0.14 0.15 0.16 0.17 0.18 0.19 0.2 0.21 0.22 0.23 0.24 0.25 0.26 0.27 0.28 0.29 0.3 0.31 0.32

Appendix E



Appendix F

Candy store Image
https://www.candyconceptsinc.com/Barrels_c_887.html

Candy barrels built from
https://www.nicepng.com/

Number line and base 10 blocks created through
https://mathigon.org/polypad

Shel Silverstein. (2019). *New World Encyclopedia*. https://www.newworldencyclopedia.org/p/index.php?title=Shel_Silverstein&oldid=1026681.

Citation

Jackson, T., Carey, A. S., & Quiroz, F. (2023). Candy Shop. In A. Bicer (Ed.), *Creativity-Directed Mathematical Tasks for 5th Grade Common Core Classrooms* (pp. 187-204). ISTES Organization.

Task 15 - Sun Catchers

Chuck Butler, Jennifer Kellner, Amy Kassel

Mathematical Content Standards

CCSS.MATH.CONTENT.5.NBT.B.7

Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Supporting Standards

CCSS.MATH.CONTENT.5.NF.B.5

Interpret multiplication as scaling (resizing), by:

CCSS.MATH.CONTENT.5.NF.B.5.A

Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.

CCSS.MATH.CONTENT.5.NF.B.5.B

Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1.

Mathematical Practice Standards

CCSS.MATH.PRACTICE.MP2

Reason abstractly and quantitatively.

CCSS.MATH.PRACTICE.MP4

Model with mathematics.

CCSS.MATH.PRACTICE.MP7

Look for and make use of structure.

Vocabulary

Decimal, multiplication, decimal multiplication, inches

Materials

Semi-transparent origami paper sheets of various sizes, glue sticks, base 10 blocks, grid paper, number lines, rulers (inches)

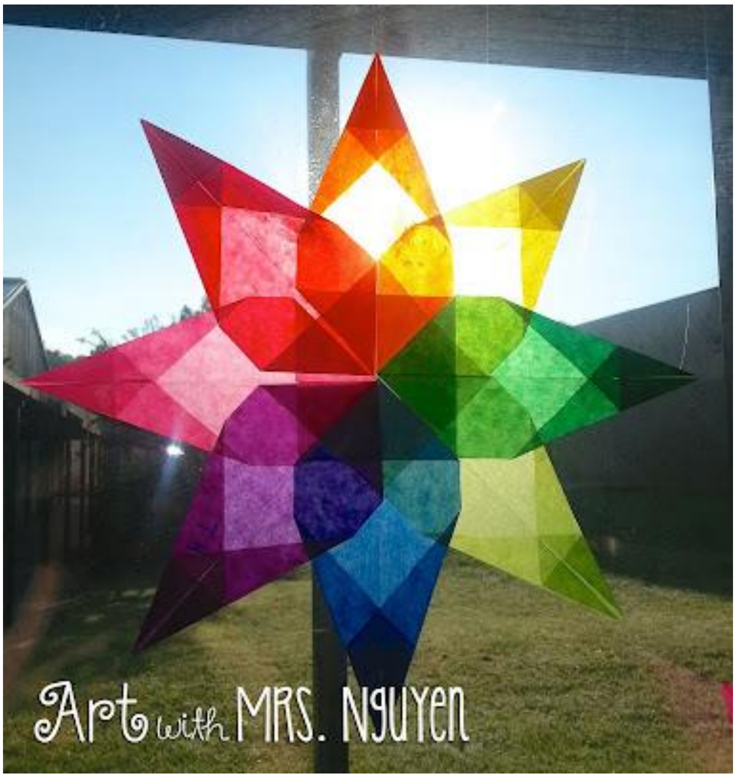
Lesson Objective

Students will be able to multiply decimals to the hundredth using models or strategies based on place value and explain their reasoning. The task's aim will be to develop students' creative thinking skills by challenging them to find multiple representations of decimal multiplication while creating sun catchers of various sizes.

Engagement

(20-30 minutes) To engage students, show them a picture of the radial origami sun catchers (Appendix A) from the website Art with Mrs. Nguyen (Nguyen, 2022).

Have students explore the historical context about Origami by watching the video, What is Origami? (origami 4 kids, 2020). Engage students in a brief discussion about the history of origami. Then teach students to create their own Radial Origami Suncatcher (Nguyen, 2022) (Appendix B). Practice folding one of each of the different folds with the students. Then have students choose which fold they wanted to do and select 8 different colors of origami paper. Students will create their original radial origami suncatcher.



Basic Fold Suncatcher (Nguyen, 2022)

Checkerboard Fold Suncatcher (Nguyen, 2022)

Figure 1

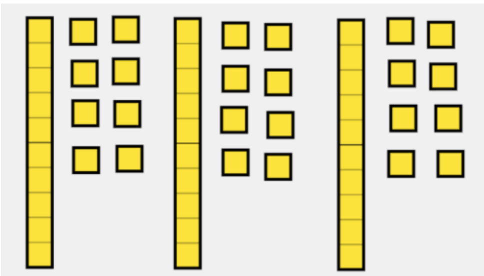
Explore

(30 minutes) The students will use either a $3'' \times 6''$ rectangle to make the Basic Fold Suncatcher or a $6'' \times 6''$ square to make the Checkerboard Fold Suncatcher. Then students will be creating different sizes of suncatchers to decorate the classroom. Note: If the teacher would like to make the sizes of rectangles more challenging or different to provide variety for students, the teacher can change the sizes to accommodate the needs of the students.

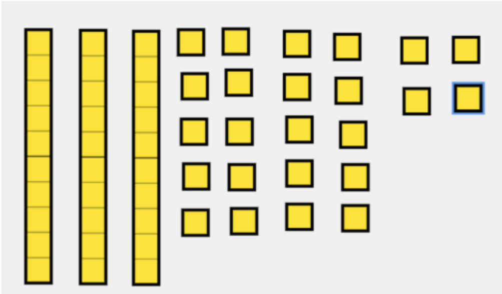
For the larger suncatchers, the teacher should facilitate a class discussion asking students what kinds of numbers would make the dimensions of the initial rectangle/square larger. The teacher should question students until they decide multipliers greater than 1 will make the dimensions larger. The teacher should set the rule that the multiplier must be a decimal number greater than 1 and less than 3 (The teacher can set this value to ensure the paper pieces will accommodate the potential maximum size).

Students may ask if they can switch designs. They may choose to do one of each. Tell students they need to decide how much bigger they wish to make their suncatcher by choosing a multiplier for the dimensions of their initial piece. The teacher may need to help students recognize that they need to use the same multiplier for each dimension.

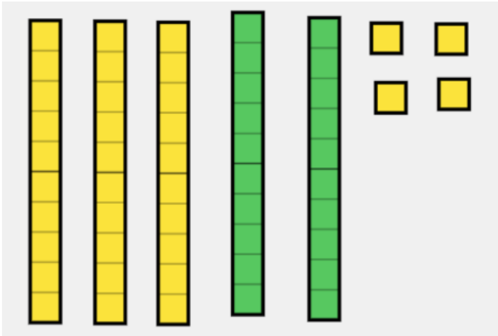
Once the students have each decided on their own multiplier, the teacher should hand out base 10 blocks, grid paper (Appendix C), number lines (Appendix D) and ask the students to represent how they calculated their new dimensions in multiple ways (The Math Learning Center, 2022, University of Colorado Boulder, 2022). For example, if a student selected a multiplier of 1.8, the student could show the following models to represent 3×1.8:



Note. Base 10 Model of 3 groups of 1.8 or 3×1.8 (the long represents 1 and the small square represents 0.1. Teachers may need to introduce students to the change in base-10 block values, long represents a whole and the small square represents one-tenth.)

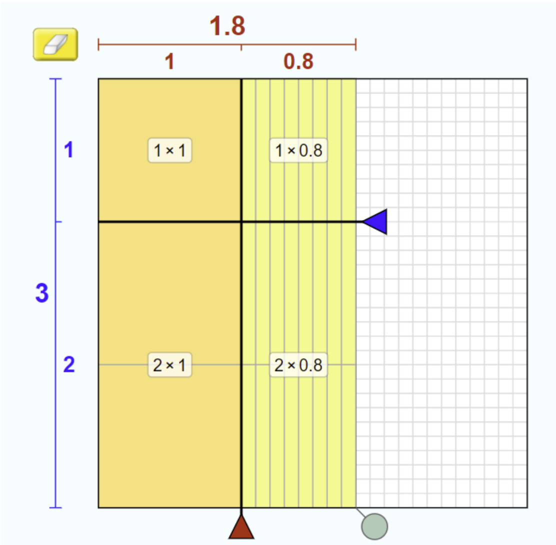


Note. Students may place the longs together to create 3 and the small squares together to create 2.4 or a student may explain it as $3 \times 1 + 3 \times 0.8 = 5.4$



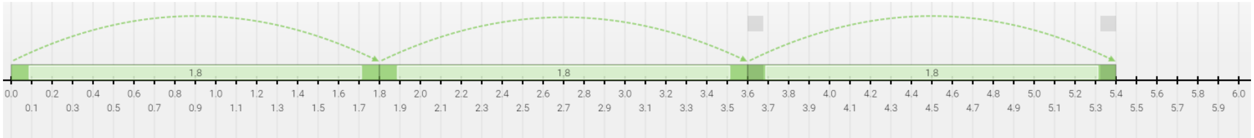
Note. Students may regroup ten small squares as one long. This can be done two times with 4 small squares left over. $5 \times 1 + 4 \times 0.1 = 5.4$.

Figure 2



Note. Area Model. Students may explain the area model as splitting the 3 into a group of $1 + 2$ and the 1.8 into $1 + 0.8$. Then the areas of the smaller rectangles can be calculated and summed as $1 \times 1 + 1 \times 0.8 + 2 \times 1 + 2 \times 0.8 = 5.4$.

Figure 3

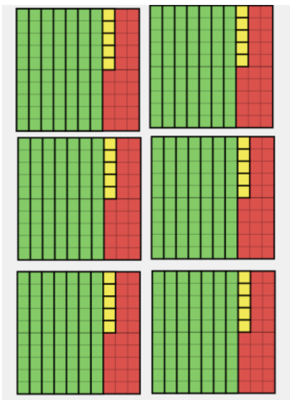


Note. Number Line Model. Students may explain this model as 3 hops of 1.8 or

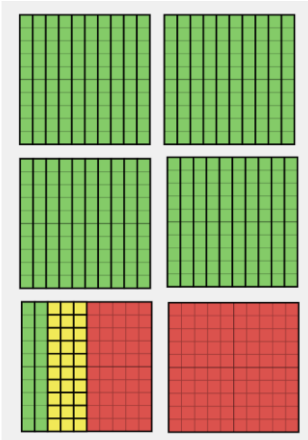
$3 \times 1.8 = 5.4$.

Figure 4

The students should record the dimensions for their new rectangle/square on the recording sheet (see Appendix E). Teachers may ask students to also record their models. For the smaller suncatchers, the teacher should facilitate a class discussion asking students what kinds of numbers would make the dimensions of the initial rectangle/square smaller. The teacher should question students until they decide multipliers less than 1 will make the dimensions smaller. The teacher should set the rule that the multiplier must be a decimal number greater than 0.5 and less than 1 (The teacher can set this value to ensure the paper pieces will still be large enough for students to fold for the potential minimum size). Students may ask if they can switch designs. They may choose to do one of each. Tell students they need to decide how much smaller they wish to make their suncatcher by choosing a multiplier for the dimensions of their initial piece. The teacher may need to help students recognize that they need to use the same multiplier for each dimension. Once the students have each decided on their own multiplier, the students use the base 10 blocks, grid paper (see Appendix C) and number lines (see Appendix D) to represent how they calculated their new dimensions in multiple ways (The Math Learning Center, 2022,). For example, if a student selected a multiplier of 0.75, the student could show the following models to represent 6×0.75:



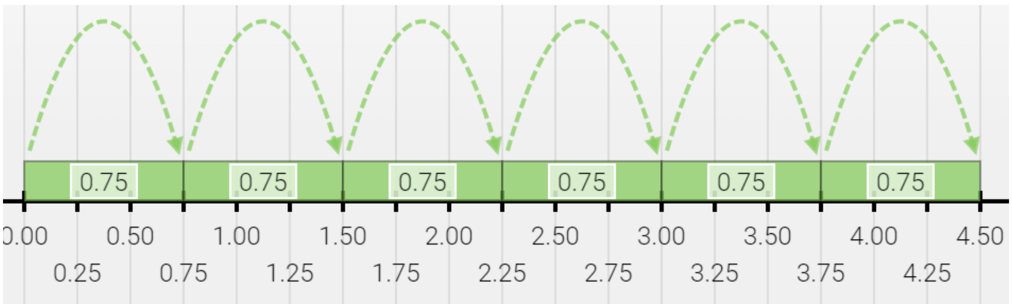
Note. Base 10 Model with the red squares representing 1 unit and the green longs representing 0.1 and the yellow squares representing 0.01. Students may explain this model as $6 \times (7 \times 0.1 + 5 \times 0.01) = 4.5$



Note. Students may regroup the thirty 0.01 yellow squares and regroup twelve of the 0.1 green longs to complete the red 1 units. Students may show this $4 + 2 \times 0.1 + 30 \times 0.01$

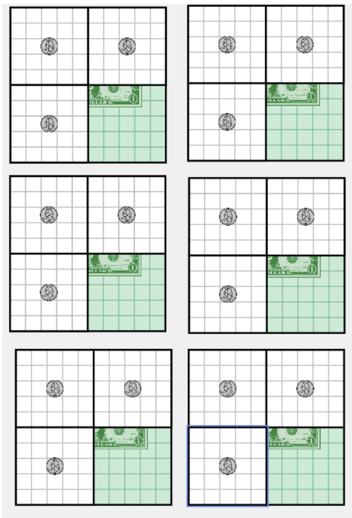
Note. Students may regroup the thirty 0.01 yellow squares into three 0.1 longs. Students may show this as $4 + 5 \times 0.1 = 4.5$.

Figure 5

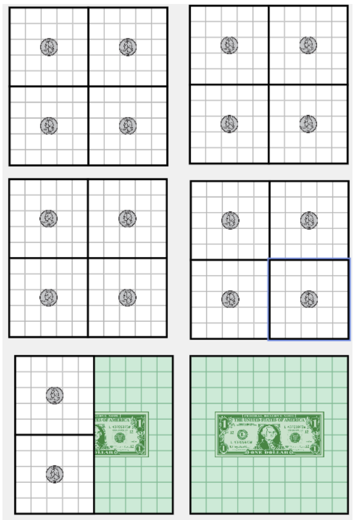


Note. Number Line Model. Students may explain this model as representing six hops of 0.75 or $6 \times 0.75 = 4.5$

Figure 6



Note. Money Model. Students may explain this model as representing $0.75 of 6 dollars or 6×0.75.



Note. Students may explain this as regrouping 3 quarters ($0.75) + 1 quarter ($0.25) to complete the dollars yielding the result of $4.50 or $4 \times 1 + 2 \times 0.25 = 4.50$

Figure 7

The students should record the dimensions for their new rectangle/square on the recording sheet (see Appendix E).

At the conclusion of the explanation, the students can measure and cut the new rectangles or squares and make their new suncatchers. Teachers may wish to check students' measurements before having them cut.

Explain

(30 minutes) During the explain phase, teachers can facilitate students' connections between multiple representations such as the base 10 block model, grid model, area model, and number line model by asking students to explain their models to one another, offering explanations to support students' connections between representations, writing the equation for the multiplication, and performing the calculations.

For example, as students are working on creating their models of the decimal multiplications, the teacher should check in with students to ensure that they are making sense of multiplication as equal groups and making sense of the decimal in their representations. For example, if students are struggling with representing multiplication as equal groups, the teacher may ask the student to represent 1 group of 1.8 with base 10 blocks supporting students in identifying the place value for each block. Then the teacher can further support the student by having them create a total of 3 groups of 1.8 and helping the student regroup, creating five groups of 1 long and four groups of 0.1. The teacher should then ask students to write the equation representing their model $(5 \times 1 + 4 \times 0.1 = 5.4)$.

As the teacher is observing the students making their models, the teacher should determine which models students will present to the class, paying particular attention to the scaffolding of the models, connecting the representations, and by selecting increasingly complex multiplications and models for the presentations. For example, the teacher may ask a student to represent the multiplication using the base 10 blocks, followed by the number line model, followed by the area model, each time supporting students' connections of the model to the equation. If students are struggling with this, the teacher can support students by having them write their numbers in expanded form to make the connection to place value. Place value can be illuminated through the base 10 model by representing

$3 \times 1.8 =$

$3 \times 1 + 3 \times 8 \times 0.1 =$

$3 \times 1 + 24 \times 0.1 =$

$3 \times 1 + 2 \times 1 + 4 \times 0.1 =$

$5 \times 1 + 4 \times 0.1 = 5.4$

through the regrouping process.

The teacher may also support struggling students by representing multiplication as repeated addition. This is most evident in the number line model, i.e.,

3 hops of $1.8 =$

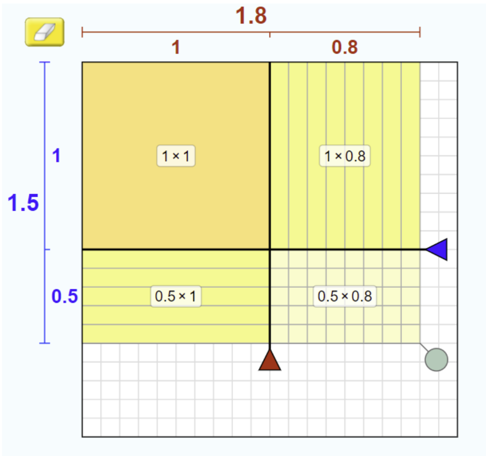
$3 \times 1.8 =$

$1.8 + 1.8 + 1.8$

$= 5.4.$

Extend

The teacher may extend students' creative thinking by asking them to observe patterns of measurement and area created within the shapes of the suncatchers. For example, the teacher may ask students what patterns they think they might find by completing an "I notice…" and "I wonder…". Students can then test their conjectures by finding a square within each of their original suncatchers, measure the dimensions with a ruler and calculate the area (see Figure 9) for an example). Then students can measure the dimensions and calculate the area of the square in the same position in each of the larger and smaller suncatchers. The graphic organizer may help students keep track of their measurements (see Appendix F). The teacher may find it helpful to collect all the student data in one location such as on the classroom display so that students have more data with which to generalize. The teachers should prepare questions to help students recognize that linear dimensions will change by the same scale factor used to scale the dimensions of the original shape. Teachers may ask questions such as, "What relationship do you see between the length of the original square and the length of the square in the new suncatcher?". If students continue to struggle, the teacher may ask, "What do you think you could multiply the length of the original square by to get the length of the square in the new suncatcher?". Having all the class data collated may help students initially recognize patterns from easier numbers and then they can test their conjecture on other numbers. As students move to exploring the relationship between the areas of the squares and the scale factor, the teacher should be prepared to support their learning by asking questions that will direct them to multiply by (scale factor) $\times$ (scale factor) or the (scale factor)^2. The teacher may ask students to explore what happens when you multiply the area of the square in the original suncatcher by the scale factor twice. For example, if the area of the original square is 1.5 in^2 and the scale factor was 1.8, the teacher may then ask students to calculate $1.5 \times 1.8 = 2.7$ and then calculate $2.7 \times 1.8 = 4.86$ (See Figure 8 for a sample area model of the multiplication). Students could then compare 4.86 to the area of the square in the larger suncatcher. Students may find it easier to initially see this pattern with easier numbers so collecting class data would allow the teacher to point out specific values to help students get started.



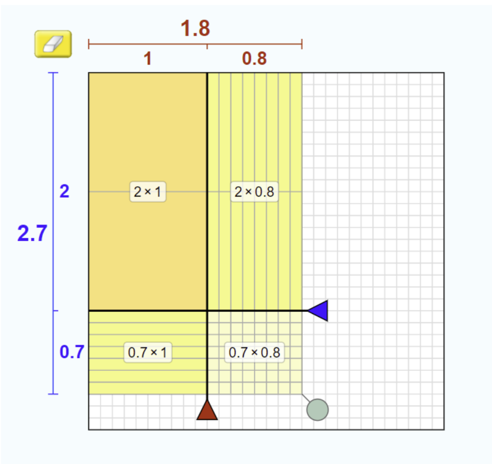
Note. Students may model the calculation for the area of the square using an area model as represented here showing the equation

$$1.5 \times 1.8 =$$

$$(1 + 0.5) \times (1 + 0.8) =$$

$$1 \times 1 + 1 \times 0.5 + 1 \times 0.8 + 0.5 \times 0.8 =$$

$$1 + 0.5 + 0.8 + .4 = 2.7.$$



Note. Students will need to multiply by 1.8 again to represent how the area of the square is scaled from the original square. The multiplication of 2.7×1.8 can also be represented using the area model. Students may write the equation as

$2.7 \times 1.8 = (2 + 0.7) \times (1 + 0.8) = 2 \times 1 + 2 \times 0.8 + 1 \times 0.7 + 0.7 \times 0.8 = 2 + 1.6 + 1.4 + 0.56 = 4.86.$

Figure 8

There are several real-world considerations the teacher should be prepared to address with students in this extension, including how to measure using a ruler, how to determine units and parts of a unit on a ruler, and error in measurement.

Note. Images with squares marked for measurement.

Figure 9

Evaluate

Formative assessment of the standard will occur when the students explain the models and their connections between multiple representations such as the base 10 block model, grid model, number line model, and area model to decimal multiplication.

- If students are creating base 10 block, grid, or number line models, the teacher should look to see that the students understand that multiplication is creating equal groups in their representation, followed by appropriate regrouping.
- As the students create the equations that model their decimal multiplication, the teacher should ask students to explain where each value is represented in their model, i.e., 3 hops of 1.8.
- If students are creating area models, the teacher should look and listen that the students are using "friendly" numbers to decompose the original number to create mental math problems.
- As students are creating the equations for the area model, the teacher should look to see that the decomposition and corresponding products are represented accurately in

the equation and that students can clearly identify where each value in the equations are represented in the model, i.e., in Figure 3, the students may state that 3 can be decomposed into 1 + 2 and 1.8 can be decomposed into 1 + 0.8. The teacher should see that the students understand that the areas of the rectangles are calculated by their corresponding length x width. Finally, the teacher should ensure students can articulate that the product is represented by the area of the entire rectangle, so students should sum the smaller rectangles to obtain the final product.

Teachers will continue to formatively assess students' understanding of decimal multiplication as students explain their recording sheets and how multiplying by decimals can resize an image. Teachers should listen for students' statements that indicate they recognize multiplying the length of the original rectangle by a value larger than 1 will make the length of the new rectangle larger by that scale factor and similarly, multiplying by a value between 0 and 1 will make the length of the new rectangle smaller.

Teachers can use formative assessments for the extend portion by having students explain the changes they find for the dimensions of the squares found in the different suncatchers. The teacher should look for the students to recognize that the lengths of the squares in the resized suncatchers change by the same multiplier by which the student originally multiplied the rectangle length.

As students explore the relationship comparing the area of the square in the original suncatcher to the area of the square in the new suncatcher, the teacher should listen for students to explain that the area of the new square is the same as multiplying the original area by the scale factor by the scale factor. Teacher should also listen for students to explain how a scale factor equal to, less than, or greater than 1 impacts the size of the suncatcher. Multiplying by a scale factor greater than 1 would increase the size of the suncatcher and multiplying by a scale factor less than one would decrease the size of the suncatcher.

References

Nguyen, M. (2022). *Art with Mrs. Nguyen*. Retrieved February 10, 2022, from Raidal Origami Suncatchers: https://www.artwithmrsnguyen.com/2014/08/radial-origami-suncatchers-5th.html origami 4 kids. (2020, May 2). *What is Origami*. Retrieved February 10, 2020, from https://www.youtube.com/watch?v=3ZsOOzFQkIw

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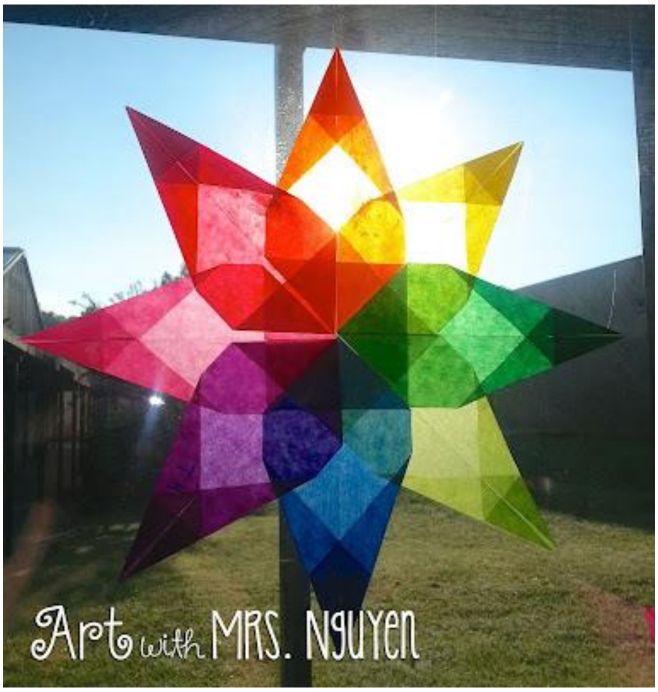
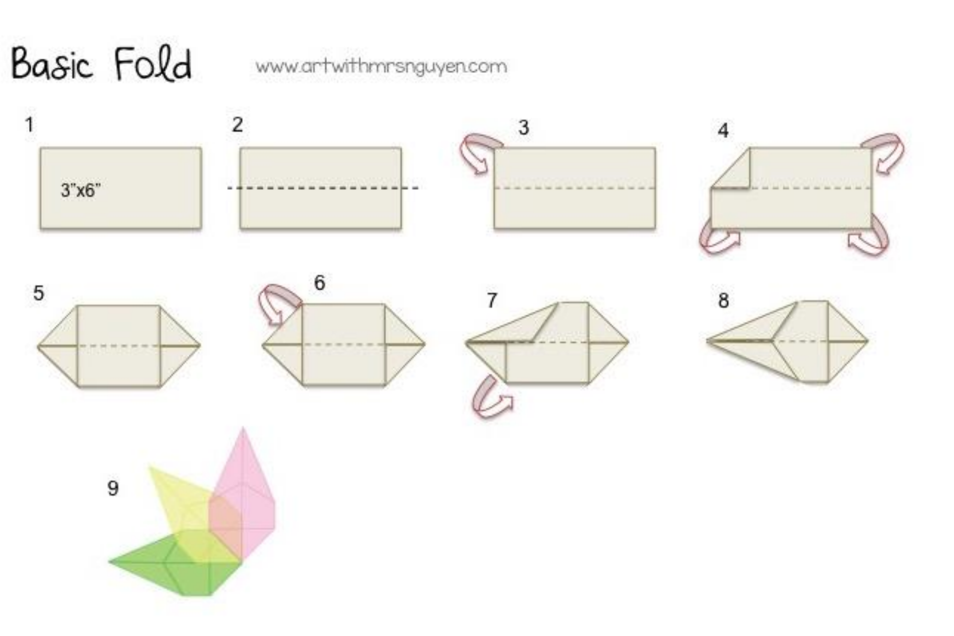
Appendix A



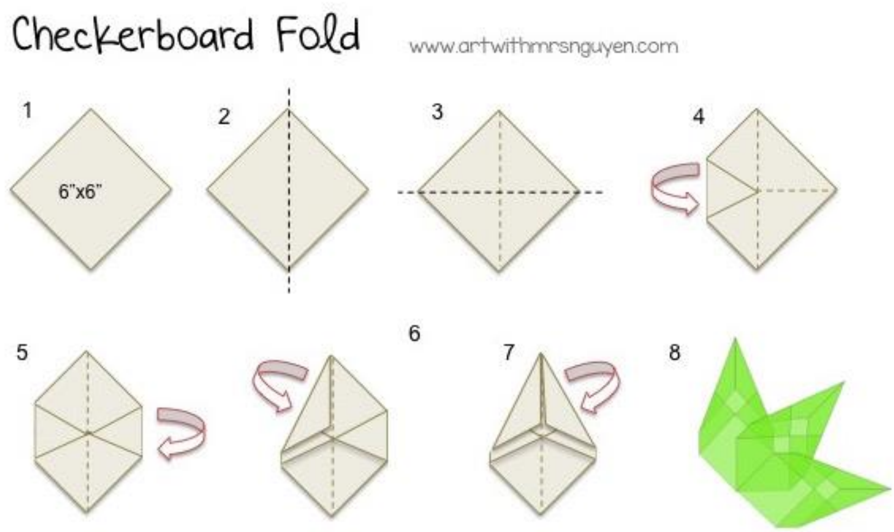
Basic Fold Suncatcher (Nguyen, 2022)

Checkerboard Fold Suncatcher (Nguyen, 2022)

Appendix B

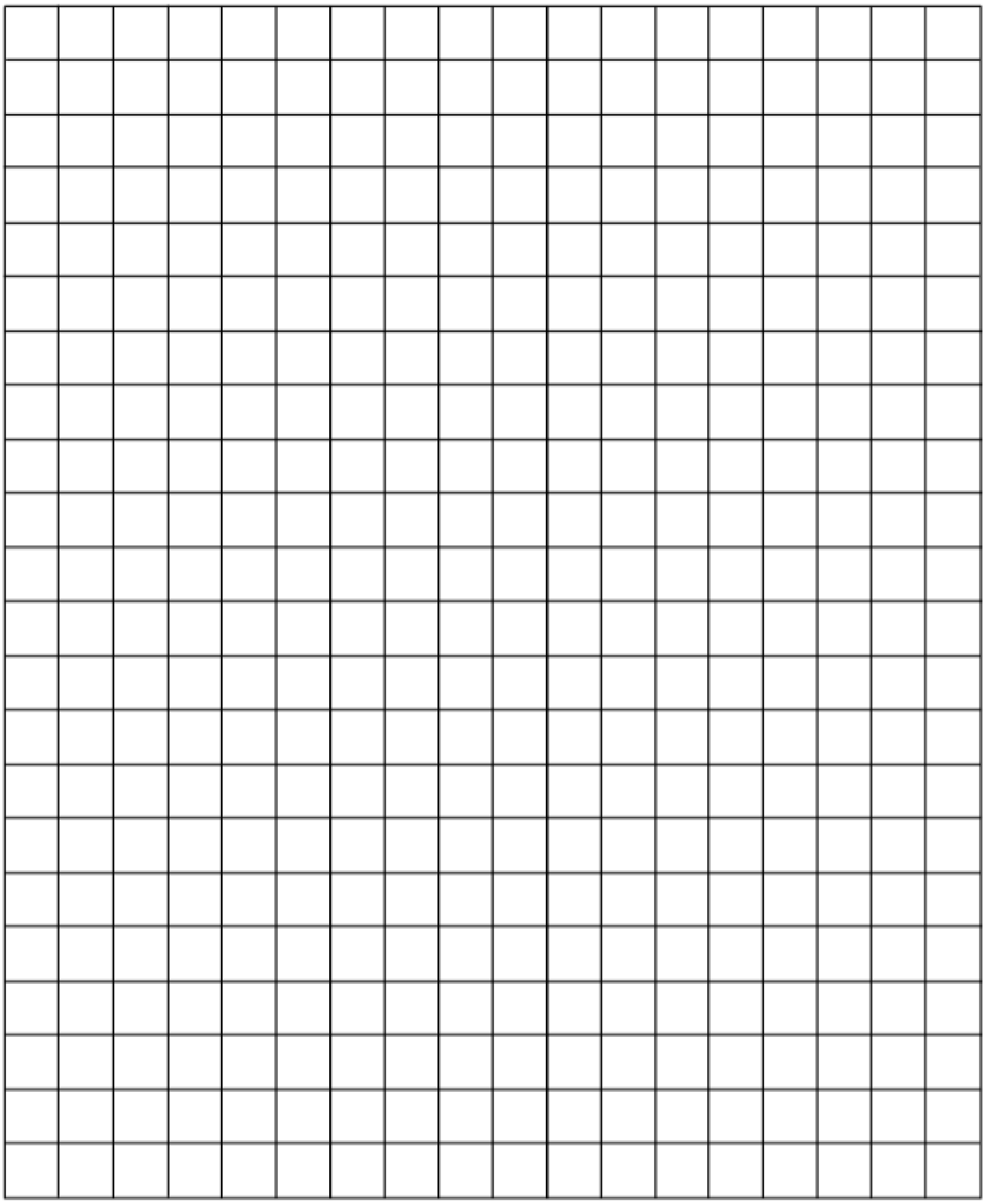


Basic Fold Suncatcher (Nguyen, 2022)



Checkerboard Fold Suncatcher (Nguyen, 2022)

Appendix C



Appendix D

Appendix E

		Scaled Measurements	
Suncatcher Name	**Original Measurement**	**______ Times as Large**	**_______ Times as Small**
Basic Fold	**3"**		
Basic Fold	**6"**		
Checkerboard Fold	**6"**		

Appendix F

Suncatcher Name	Measurement of an original square	Area of the original square in the suncatcher	Scale Factor	Measurement of the same square in the new suncatcher	Area of the same square in the new suncatcher

Citation

Butler, C., Kellner, J., & Kassel, A. (2023). Sun Catchers. In A. Bicer (Ed.), *Creativity-Directed Mathematical Tasks for 5th Grade Common Core Classrooms* (pp. 205-226). ISTES Organization.

Task 16 - Designing Mosaic Tiles

Geoff Krall, Helen Aleksani

Mathematical Content Standards

Focus Content Standard

CCSS.MATH.CONTENT.5.NBT.B.7

Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Mathematical Practice Standards

CCSS.MATH.PRACTICE.MP2

Reason abstractly and quantitatively.

CCSS.MATH.PRACTICE.MP5

Use appropriate tools strategically.

CCSS.MATH.PRACTICE.MP7

Look for and make use of structure.

Vocabulary

Mosaic, percent, tenths, hundredths, grid

Materials

Grid paper or a digital tool such as mathigon.org/polypad, Mosaic tile recording sheet (see Appendix B)

Lesson Objective

Students create a mosaic tile design on a 10x10 grid and then use decimal multiplication to determine the cost of their mosaic based on a pricing sheet. Students will demonstrate their understanding of multiplication models with decimals. Students will demonstrate and experience mathematical creativity ny crafting mosaic designs using physical or digital tools.

Engagement

(15 minutes) Play the following video and ask if students recognize what it is. https://www.youtube.com/watch?v=BOvAqlYWiI4

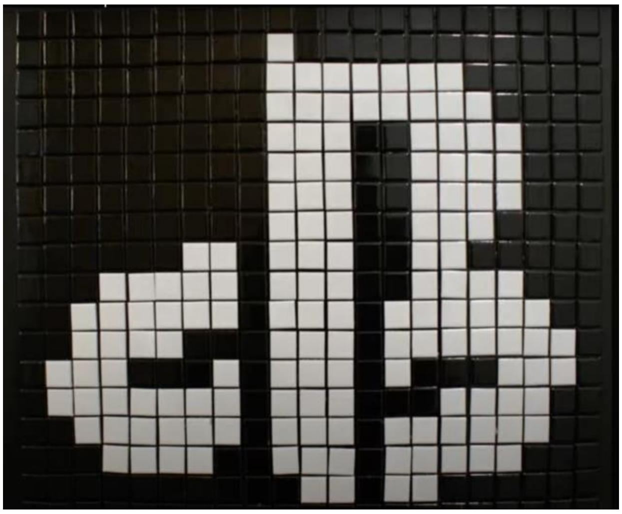


Figure 1. A Screenshot of the Youtube Video showing the Design of the Mosaic

Students that play video games might recognize it as the logo for the Sony Playstation video game console. Ask students what they notice and wonder about the design. Students may notice things such as the following:

- I notice there are black tiles and white tiles.
- I notice there are more black tiles than white tiles.
- I notice there are probably over a hundred tiles.

Students may wonder the following:

- I wonder how many black/white tiles there are?
- I wonder what percentage of tiles are black/white?

You may wish to have students find the number of total tiles by counting the number of tiles along the length and width of the design to review area concepts. You also may wish to have students calculate the ratio of black tiles to white tiles or total tiles.

Now show the following pictures (Figure 2) and ask students where they have seen this kind of design.

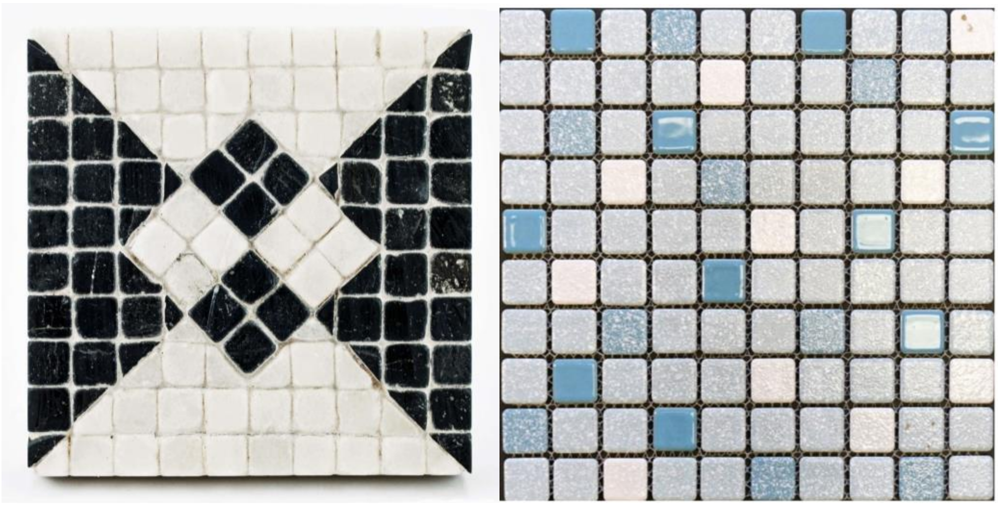


Figure 2. Two Examples of Mosaic Tiling

Students may notice that they have seen these types of tiles in bathrooms or lobbies of buildings. Tell students that mosaics are a very famous type of art design and that mosaics are used in many cultures throughout the world.

Ask students, “about how much of the mosaic on the left is black and about how much is white?” Students will conjecture about half and half. Ask, “how can we be sure?” Students may count each color of tiles or share how they see four triangles with their tips slightly rotated. Ask students, “what fraction of the mosaic is black? What fraction is white?” Students will respond ½ and ½. Note that there are exactly 100 small tiles in this mosaic, 50 of which are black, 50 of which are white. Now ask students what decimal of the mosaic is black and white (0.5 and 0.5).

Explore

(20 minutes) *Create a mosaic tile.*

Using physical or digital manipulatives or grid paper, have students create a mosaic tile on a 10x10 grid (see Appendix A). Tell students, "Now that you have seen examples of mosaics, we are going to create our own. And, more than that, we will design them and figure out how much it would cost to create the tile in real life."

Have students create mosaics using the following colors: Black, Green, Purple, Orange, Red, Blue, White. Yellow

Instruct students that they must use at least three different colors in their design. Students may choose to create designs of abstract patterns or actual objects or animals. For examples, see Figure 3a and Figure 3b.

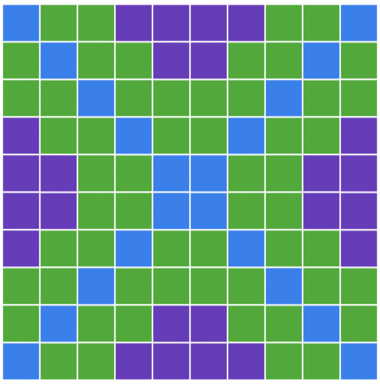


Figure 3a. An Example of an Abstract Design on a 10x10 Mosaic Tile

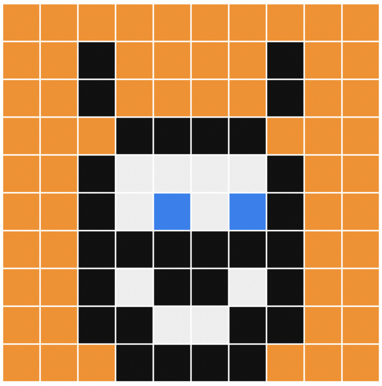


Figure 3b. An Example of an Animal on a 10x10 Mosaic Tile

While students are creating their design, float around to different students to identify one or two examples to showcase to the whole class. Ideally, choose a student who you would like to elevate their academic status. Once students have completed their designs, showcase one example for the class. For this lesson plan, we will use Figure 3a as the example.

Ask the class the following questions and record the answers on the board:

- How many tiles are on the mosaic in total? (10x10 =100 tiles)
- How many blue tiles does it have? (20 tiles)
- How many purple tiles does it have? (24 tiles)
- How many green tiles does it have? (56 tiles)

Use the "how many" questions as an opportunity to engage students in an image-based number talk. You can even ask students "how many do you see and how do you see them?"

Once you have recorded the number of tiles, make the connection to fractions:

- What fraction of this mosaic is blue? (20/100 or ⅕)
- What fraction of this mosaic is purple (24/100 or 12/50 or 6/25)
- What fraction of this mosaic is green? (56/100 or 28/50 or 14/25)

Then make the connection to decimals.

- What is the decimal that represents blue in this mosaic? (20/100 = 0.2)
- What fraction of this mosaic is purple (24/100 =0.24)
- What fraction of this mosaic is green? (56/100 0.56)

Note that students might need some additional explanation or review of why 20/100 = 0.2. Explain to students that 20/100 is equivalent to 0.20 and that 20 hundredths is the same as 2 tenths. It might help to showcase this in a separate grid (see Evaluate section).

Once the class has identified all decimals in the mosaic, ask students to add the three decimals together to ensure that they add up to 1.00. In this case, the entire mosaic is one whole and each tile is one hundredth, or 0.01:

0.2 + 0.24 + 0.56 = 1.00

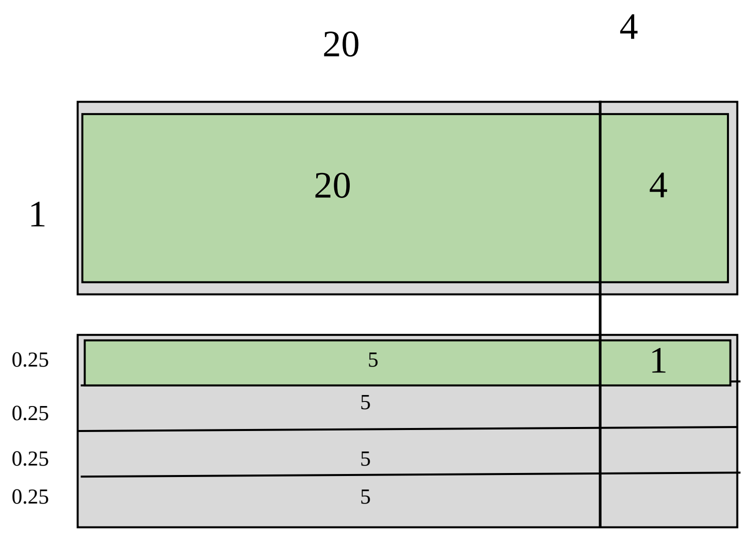
Once again, students may need to be reminded that 0.2 and 0.20 are equivalent. Students may require a demonstration of the above addition problem:

0.20 + 0.24 = 0.44

0.44 + 0.56 = 1.00 = 1

Explain

(20 minutes) Now tell students we are going to find the cost of our mosaics. Each tile color will have a different unit price. So we will have to multiply the number of each color tile by the price in order to find the total. Using your showcase example, walk students through multiplying decimals with a model. Once again, we will use Figure 3a as our example. Tell students, "Purple tiles cost $1.25 for each tile. And our example has 24 purple tiles. So we must multiply 24 by $1.25." Walk students through a multiplication model of decimals.



$$1.25 \times 24 = (1 + 0.25) \times (20 + 4) = 20 + 4 + 5 + 1 = 30$$

Figure 4. One Potential Model of Decimal Multiplication

Using an area model, we notice that we are multiplying 24 by 1 and 24 by 0.25 (or one fourth of one). Therefore, the total cost of all of our green tiles is $24 + $6 = $30.

Give students the pricing sheet (see Appendix A) and recording sheet (Appendix B). Have students mark their recording sheet with the number of each type of tile, the fraction representation of the tiles, the decimal representation of the tiles and the cost of the mosaic. Be sure to have students include a multiplication model for their decimal multiplication.

Extend

(30 minutes) In this section, students will be working in groups of 4 to combine their designs

and create a new mosaic logo. In their groups, students are going to pick a 5x5 portion of their created design. It could be any part of their design. There are many possibilities. For example, students can do the following**:**

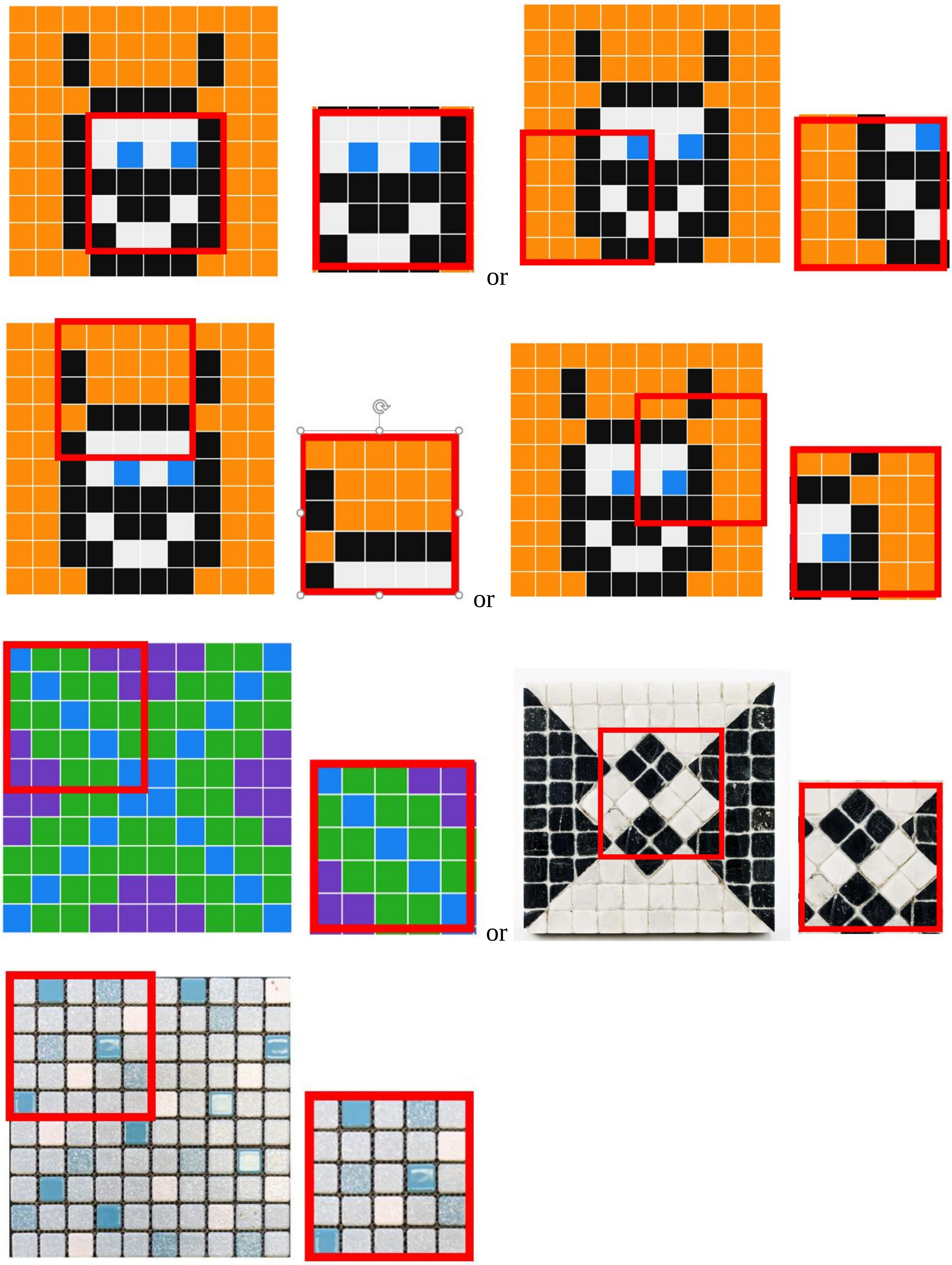


Figure 5. Creating a Logo in Groups based on their Designs

Explain to students that every group will be presenting their project at the end to the class. Their design needs to represent a story or an idea. Once the students finish choosing their 5x5, they are required to combine their selections onto a 10x10 grid to build a new logo that is a combination of all four students in the group. Students will use the pricing sheet and the mosaic title recording sheet given to them earlier to calculate the cost of their new design (Appendix A and Appendix B). Then, the group will describe the representation of their logo design and the story behind their selection to their class. They will also discuss the cost of their design as well.

The following is an example of a group logo.

Figure 6. A Finished Group Logo

Students may even wish to alter their design to make it mesh better together. If there is time in class, and students wish to do so, allow students to redesign their combined mosaics to make it more aesthetically pleasing.

Evaluate

In this task, teachers need to pay attention to students' reasoning of decimals and fractions. Students need to demonstrate their understanding of the connection between decimals and fractions. They also need to demonstrate understanding of multiplying and adding fractions. Teachers can revisit the area model to help build student understanding of the concept. When revisiting the area model, provide students with color pencils to help understand the concept of area and the representation of each decimal. You may wish to have students use additional grids to represent equivalent fractions (Figure 7).

Students may not recall some fraction equivalency facts, such as 20/100 = 2/10 = 0.2 = 0.20. In order to help students understand the connections, consider showcasing the following diagrams (Figure 7). Ask students, “what is the same about these pictures? What is different?” Highlight the fact that the amount of orange is the same in each picture, even though it is partitioned differently. The figure on the right represents 2 out of 10 (or two tenths or 0.2), the figure on the left represents 20 out of 100 (or 20 hundredths or 0.02).

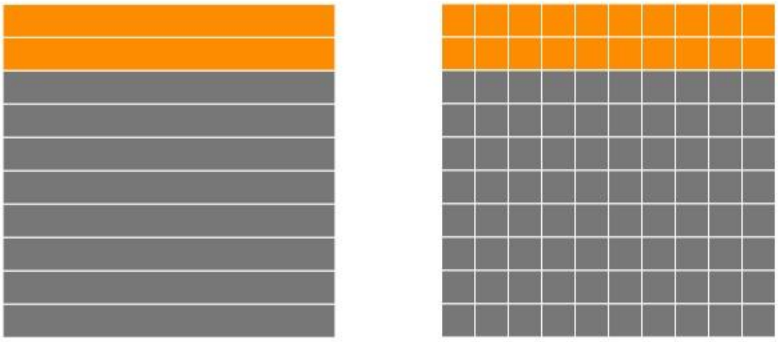
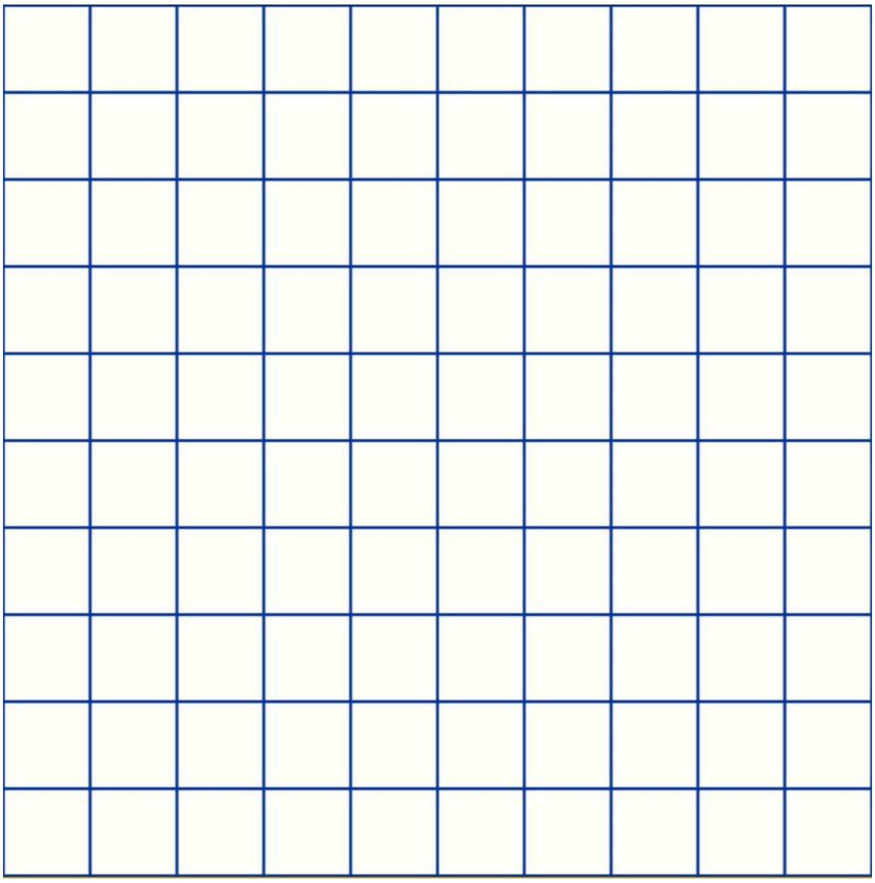


Figure 7. Diagrams showcasing the Equivalency of Certain Fractions and Decimals

Appendix A

A 10 x 10 Grid



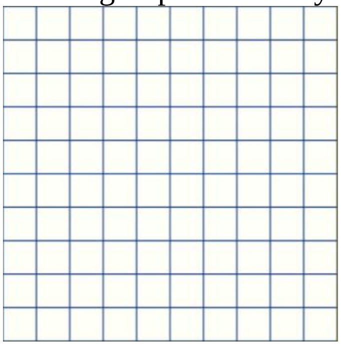
Pricing sheet for the tiles

Black	$1 per tile
Purple	$1.25 per tile
Green	$1.50 per tile
Orange	$1.75 per tile
Red	$2 per tile
Blue	$2.25 per tile
White	$2.50 per tile
Yellow	$2.75 per tile

Appendix B

Mosaic Tile Recording Sheet

Mosaic Tile Recording Sheet
Name:
Drawing or picture of my mosaic:



Fill out this chart with the color of your tiles and the numerical representation of each color compared to the whole mosaic. Use the space below to demonstrate your model of decimal multiplication.

Color of tiles	Number of tiles	Cost of the color tile ($/tile)	Cost of all color tiles ($)	Fraction of mosaic	Decimal of mosaic
Total Tiles					

Decimal Multiplication area models

Citation

Krall, G. & Aleksani, H. (2023). Designing Mosaic Tiles. In A. Bicer (Ed.), *Creativity-Directed Mathematical Tasks for 5th Grade Common Core Classrooms* (pp. 227-238). ISTES Organization.

SECTION 5 - USE EQUIVALENT FRACTIONS AS A STRATEGY TO ADD AND SUBTRACT FRACTIONS

Task 17 - What's Your Favorite Recipe?

Michelle Tudor, Michael Gundlach, Melena Osborne

Mathematical Content Standards

CCSS.MATH.CONTENT.5.NF.A.1

Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)*

CCSS.MATH.CONTENT.5.NF.A.2

Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.*

Mathematical Practice Standards

1. MP 2 Reason abstractly and quantitatively.
2. MP 5 Use appropriate tools strategically.
3. MP 7 Look for and make use of structure.
4. MP 8 Look for and express regularity in repeated reasoning.

Lesson Objective

Students will use their understanding of equivalent fractions to add and subtract unlike fractions. To activate students' mathematical creativity, they will create multiple representations of commonly applied fractions, such as ½, ¼, and ⅛. Students will be given the opportunity to develop reasoning for understanding the standard algorithms for adding and subtracting fractions by using area and length models. Students will be challenged to modify a recipe by only using certain measuring cups enabling them to apply what they have learned. Throughout this lesson, students will estimate answers by using ½ or 1. This lesson is intended to take 2-3 days.

Materials

Square paper (paper strips can be used as an alternative here), fraction bars or tower (can use https://mathigon.org/polypad#fraction-bars for fraction bars), grid paper (area models), colored pencils, family recipes (students will bring these, but bring extra just in case)

Vocabulary

Equivalent fractions, numerator, denominator, common denominators

Engagement

A couple of days before this lesson, ask students to find a favorite family recipe and get a copy of it. They will then bring this in to share with the class. Have a couple of recipes ready in case students forget or do not have a favorite recipe. Make a batch of sugar cookies (or ask a parent volunteer) and bring the cookies with the recipe to the students. Show students the recipe you used while they eat the cookies.

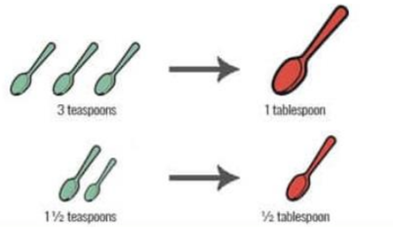
Allow students to share the recipes they have brought to class and discuss why they are a favorite. Promote discussion by asking students if they have ever helped a family member to make the recipe. Ask what types of measuring utensils they used. What did they notice about the units on the utensils? What would you have to do with the recipe if you wanted to make it for more or fewer people? This can lead to the explanation of equivalent fractions.

During this discussion, you should review the abbreviations used in recipes, like tsp and tbsp and as a helpful hint, tell students that 3 tsp = 1 tbsp. A different approach to this would be to give students teaspoons and tablespoons, along with a small amount of flour, and have them explore how they can find this equivalency (without first telling them). They could do this with several different measurements.

For example, you could also take in some different measuring cups and have them find how many 1/4's make 1/2. There are several ways you can make this more engaging and fun for students, only a couple of ways are listed above.

Sugar Cookie Recipe

- 3 cups all-purpose flour
- 3/4 teaspoon baking powder
- 1/4 teaspoon salt
- 1 cup unsalted butter, softened
- 1 cup sugar
- 1 egg, beaten
- 1 tablespoon milk
- Powdered sugar, for rolling out dough



Explain A

It is important to remind students of what equivalent fractions are before you introduce addition and subtraction of unlike fractions. Here, use the fractions bars/towers or https://mathigon.org/polypad#fraction-bars and let students explore equivalent fractions. For example, have students show equivalency between 1, two 1/2's, three 1/3's, four 1/4's, five 1/5's, and so on all the way through twelfths.

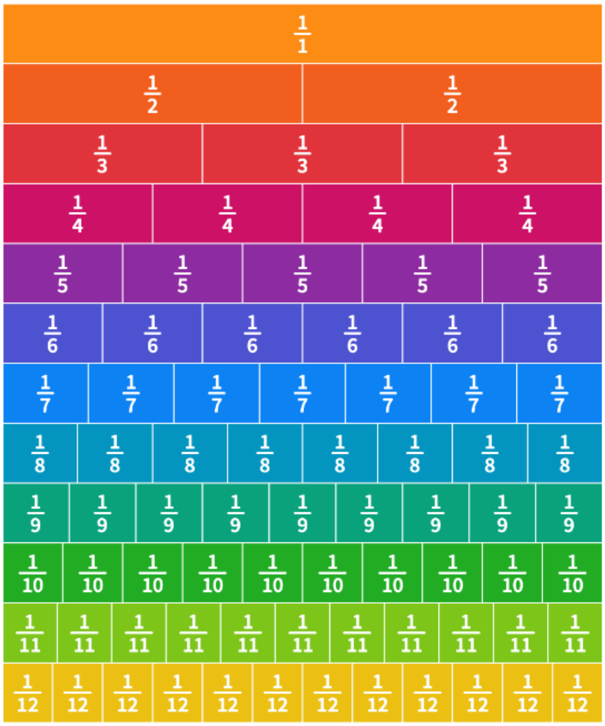


Figure 1. Fraction Bars

Then, have the students break this down more. Using the fraction bars/towers, have students create equivalent fractions for ½ using halves through twelfths. While manipulating the fraction bars, most students will notice that this cannot be done with thirds, fifths, sevenths, and ninths.

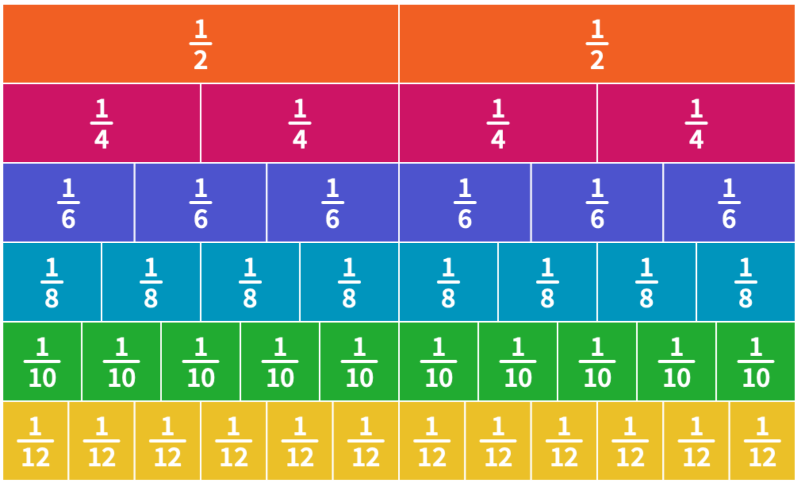
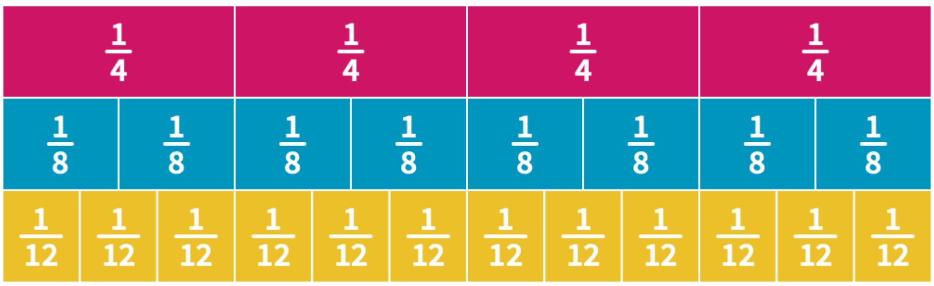


Figure 2. Equivalent Fractions

Have students write out the equivalent fractions after creating them with the fraction bars:
1/2 = 2/4 = 3/6 = 4/8 = 5/10 = 6/12

Ask them if they see a pattern, and to list the next three fractions that are equivalent to 1/2 (7/14, 8/16, 9/18).

Now, extend this to 1/4 and 3/4. Students can use the fraction bars again to help them with finding equivalent fractions.



Once again, have students list the next three equivalent fractions after they see 1/4, 2/8, 3/12 (4/16, 5/20, 6/24) and 3/4, 6/8, 9/12 (12/16, 15/20, 18, 24) and ask them to identify the pattern again.

Remind students that they can find equivalent fractions using multiplication. For example,

$$\frac{3}{4} = \frac{3 \times 2}{4 \times 2} = \frac{6}{8}$$

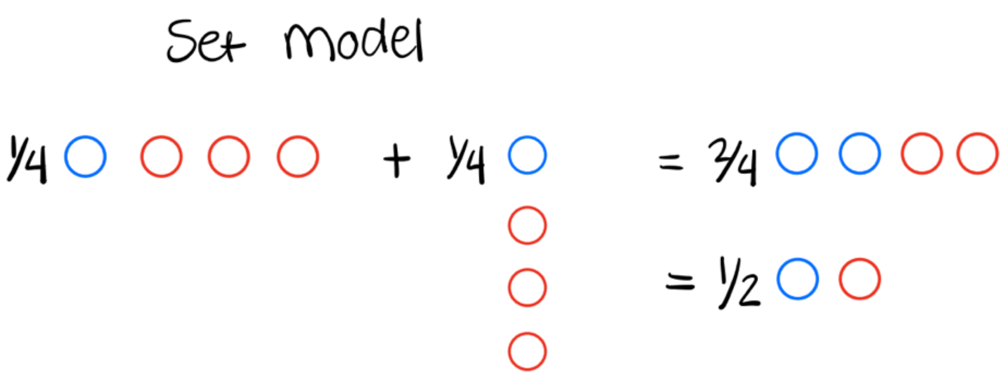
$$\frac{3}{4} = \frac{3 \times 3}{4 \times 3} = \frac{9}{12}$$

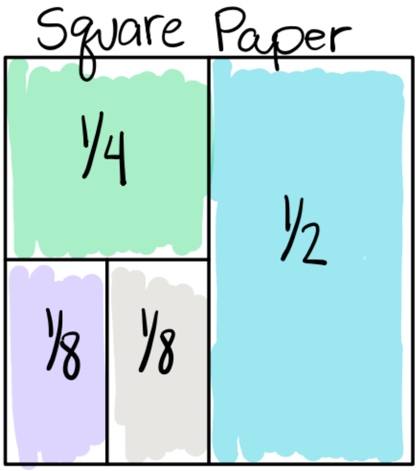
Make sure to point out that to keep the fractions equivalent, the numerator and denominator are being multiplied by the same number (refer to the previous patterns and fraction bars if students are confused about this). Have students use this fact to reduce a few fractions completely (8/10, 10/15, 12/24). You can give more examples here if students are struggling.

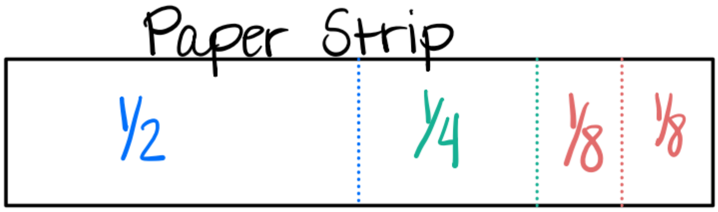
Explore A

For this part of the lesson, students can use square paper (Appendix A), paper strips (or fraction bars sets, if you have them) (Appendix B), or manipulatives to represent set models or all three. Have students use the recipe they have and find equivalent fractions for the amount of ingredients needed. For example, if 1/2 cup is needed, they could also use 1/4 cup 2 times or 1/8 cup 4 times. Ask them to find two equivalents for each of the ingredient amounts and write the corresponding expressions. Students should justify their work using either square paper (have students shade 1/2, 1/4, and 1/8 using different colors), paper strips (they can show the 1/2, 1/4, and 1/8 by folding the paper strips), fraction bars, or set models. Circulate and check on these conversions as students are working.

Examples of Models







Equivalent Fractions for 3 models:

$$\frac{1}{4}+\frac{1}{4}=\frac{1}{2}$$
$$\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}=\frac{1}{2}$$
$$\frac{1}{4}+\frac{1}{2}=\frac{3}{4}$$

Explain B

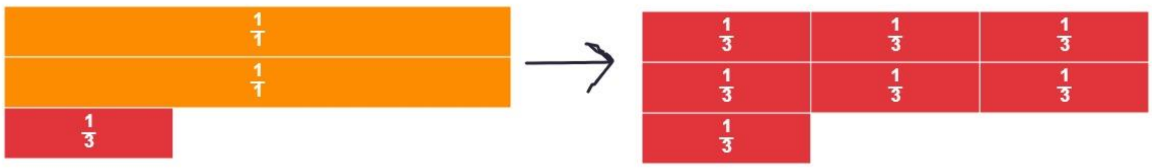
Once students have finished Explore A, they are ready to explore the addition and subtraction of fractions. Explain to students that they will add and subtraction fractions with unlike denominators by creating equivalent fractions so that the fractions have common denominators. Show students fraction bars that represent 1/2 and 1/2 and tell students they are going to add the fractions and subtract the fractions. Students should notice that it is impossible to find a total sum of difference with the unlike parts. Use additional fraction bars to show equivalent fractions for 1/2 (e.g., 3/6) and 1/3 (e.g., 2/6) and then ask students to add and subtract. Do this with several different examples; be sure to show the model(s) for each fraction so students will see the connection and reason for needing common denominators.

Explore B

At this point, have the students go back to their recipes. Have students add all dry ingredients (for total dry ingredients) and add all of the liquid ingredients (for total liquid ingredients).

There will likely be some whole numbers in the recipes, have students discuss together how they can represent a whole number as a fraction. Tell them to think about what a fraction represents and ask them how they can create an equivalent fraction for the whole number.

For example, the students may see 2 cups of flour and 1/3 cup of sugar in their recipe. To add this, the students will need to change 2 to 2/1 so that they can visually see that 2 has a denominator of 1. Once the students see this, they will know to find a common denominator between 1 and 3. Try to let the students come up with 2/1 on their own; give them some time to think and problem solve before you tell them what it is. Additionally, the mathigon tiles mentioned previously have a "rename" option that can be used to split a "1" tile into smaller pieces. This can be used to help students see that 2/1 = 6/3. An example is shown below.



Extend

After students have created equivalent fractions for their recipe, place them in groups of 2-3 students who have similar recipes. Students will be given a recipe and will be asked to determine the least number of measuring utensils that could be used to make the whole recipe with the stipulation that any cup or spoon used must be full (They cannot use a 1-cup measuring cup and say they will fill it halfway). Students can use square paper, fraction bars or strips or set models to help them. The students will then write the recipe and designate the measuring utensil to use with the ingredient amount. A half cup of flour (use 2- ¼ cups) Students will copy the recipe with measuring utensil units onto a poster paper. Posters will be placed around the room and students will do a gallery walk to compare the results.

Recipe for Extend

The Best Eggless Chocolate Chip Cookies

1tsp baking soda

½ cup brown sugar

1 cup chocolate chips

1 ½ flour

¼ cup sugar

1 tsp vanilla

2 tbsp vegetable oil

½ cup butter

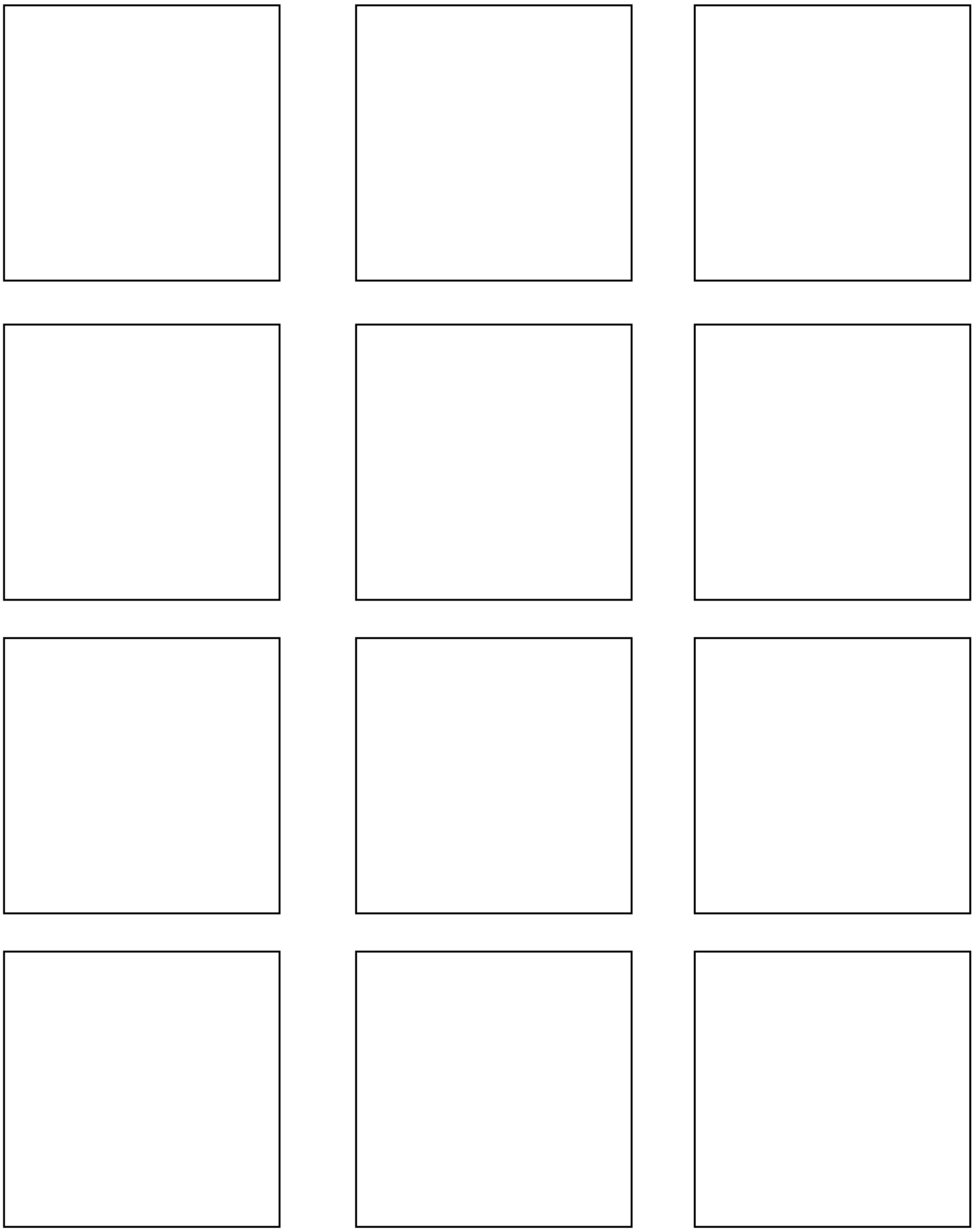
1 tbsp water

Evaluate

During the explore and extend portions of the lessons, ensure students are using equivalent fractions correctly. These are good times to formatively assess whether students have mastered the idea that equivalent fractions are created by dividing a quantity into equal pieces. Look for students who are struggling to add fractions with unlike denominators when they are working on creating equivalent fractions for the recipe and when they are working on adding the total amount of dry and wet ingredients. Pull these students aside and guide them through creating equivalent fractions with the fraction strips. Ask students to use the bars to show you equivalent fractions. For example, students will line up the bars for halves, thirds and sixths to show how the sixths can be used as a common denominator to add halves and thirds. Make sure they see the connection of the strips to the numbers. Ask students: How many sixths are in one half? How many sixths are in one third? Can we add these together using the sixths? What would the fraction in sixths be? Students without a solid foundation in fraction number sense may need extra time on this concept.

As a summative assessment, you can collect students' modifications to family recipes in the "explore" portion of the lesson to determine if they have produced proper equivalent fractions. Also, the posters from the "extend" portion of the lesson can be graded for accuracy. You should also determine if there was a way to use a smaller number of measuring utensils than stated by the student, but we recommend not assigning a grade for this. Students should be assessed on their ability to produce correct equivalent fractions and fraction sums.

Appendix A (Square Paper)



Appendix B (Strip Paper)

Citation

Tudor, M., Gundlach, M., & Osborne, M. (2023). What's Your Favorite Recipe? In A. Bicer (Ed.), *Creativity-Directed Mathematical Tasks for 5th Grade Common Core Classrooms* (pp. 239-250). ISTES Organization.

Task 18 - Birthday Celebration

Aylin S. Carey, Fay Quiroz, Traci Jackson

Mathematical Content Standards

CCSS.Math.Content.5.NF.A.1

Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)*

CCSS.Math.Content.5.NF.A.2

Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.*

Mathematical Practice Standards

CCSS.MATH.PRACTICE.MP2

Reason abstractly and quantitatively.

CCSS.MATH.PRACTICE.MP4

Model with mathematics.

CCSS.MATH.PRACTICE.MP6

Attend to precision.

CCSS.MATH.PRACTICE.MP7

Look for and make use of structure.

Vocabulary

Equivalent fractions, mixed number, numerator, denominator, sum, and difference.

Materials

Fraction bars
Hundred grids
Online manipulatives (see References)

Lesson Objective

Students will be able to add and subtract fractions with like and unlike denominators (including mixed numbers) by using visual fraction models to assist with finding the least common denominator to solve and make sense of their findings.

Combined with students' understanding of equivalent fractions to add and subtract, the effective use of visuals and manipulatives in mathematical tasks will help promote the mathematical creativity of students.

This lesson will be completed in two-class time.

Engagement

(15 minutes) Start the lesson by telling students that one of their classmates' birthday is coming up and asking them how they would like to celebrate. Have them share ideas and mention how helpful it would be to create a list of items (see Figure 1).

Tell students that you already have some cake designs and need help with figuring out how many cakes to order, cutting arrangements, and also with birthday decorations. Let students know that you created a lesson that involves addition and subtraction of fractions to help better calculate with the measurement of birthday items while learning fraction equivalence.

Cake
Popcorn
Fruit Punch
Paper Plates
Popcorn Boxes
Plastic Utensils
Napkins
Birthday Banner
Balloons
Ribbons & Streamers

Figure 1. Possible Birthday List

Explore

(40 minutes) Before adding and/or subtracting fractions, have students' minds refresh about fraction equivalence through fraction circles and bars (see Figure 2).

To help choose a cake design and explore cutting arrangements, introduce both circular and rectangular shaped cakes that are divided into parts (with numbers). This way students get to practice the part-whole concept of fractions and partitioning. Have students think about fair (equal) shares.

Some questions to pose: "How many pieces of cake will each student get if four students are sharing one piece of cake?", "Suppose the whole cake is to be shared fairly among 12 students. How much will each student get?", and "Can a half cake be shared fairly with 8 people?". Students can also practice equivalence through manipulatives such as fraction towers: https://mathigon.org/polypad#fraction-bars.

After refreshing their understanding of equivalent fractions, introduce an interactive game (https://www.abcya.com/games/equivalent-fractions-bingo) and remind students that they will play the Equivalent Fractions Bingo game at the birthday party. This interactive game also would help students refresh their understanding of equivalent fractions in a fun way while helping them develop fluency. Teachers can choose to add numbers and charts or only numbers in the display and decide the grid size: 3x3, 4x4, or 5x5 (see Figure 3).

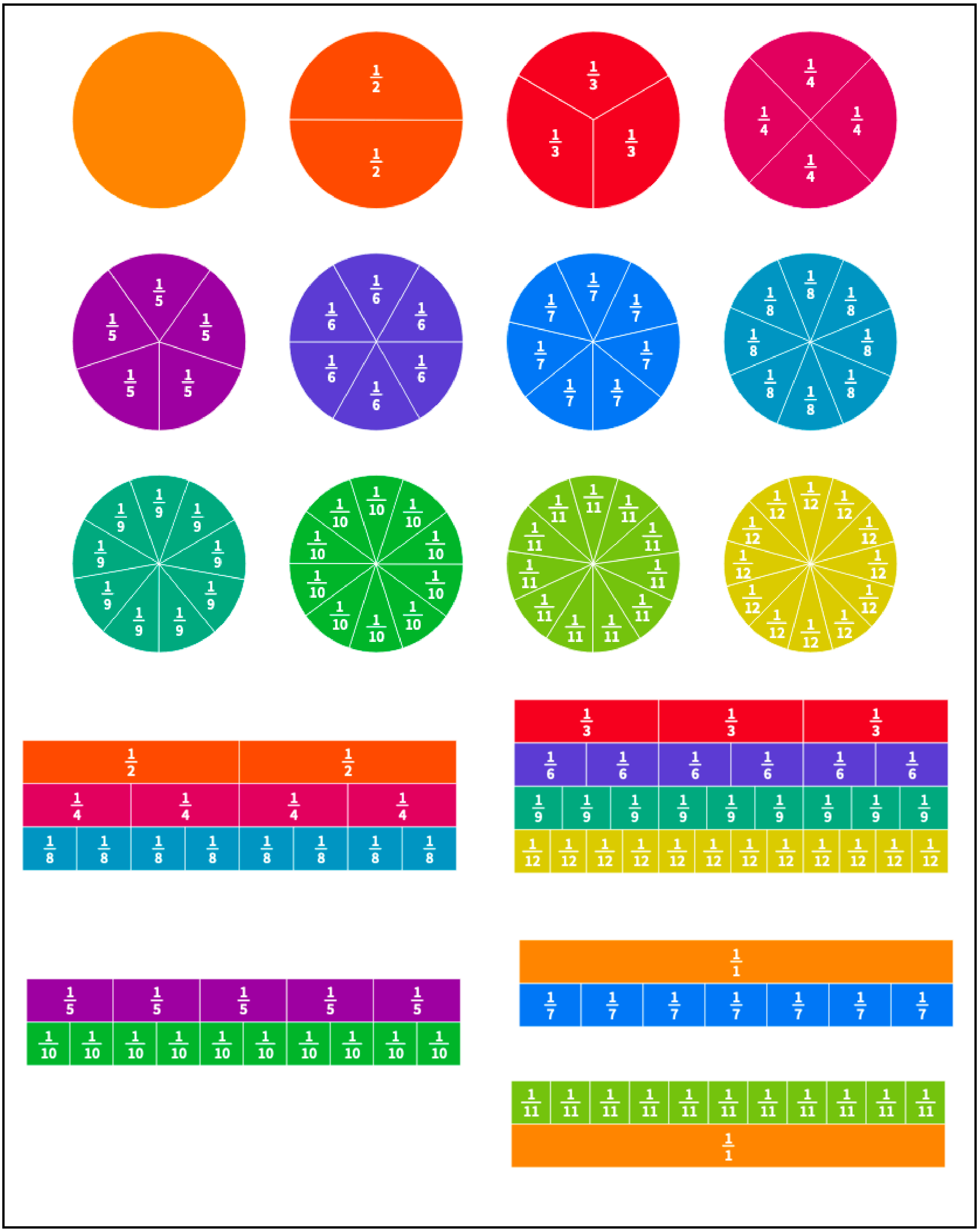


Figure 2. Examples of Cake Designs and Cutting Arrangements

Figure 3. Equivalent Fractions Bingo Game

After having students explore the game for a short period of time, bring the class back together and explain they are going to take all of the ideas they discussed in the engagement part to work with their groups of 3-4 students to plan the party. Tell students the first item they will need to decide are the flavors of the cakes. Each group will be responsible for putting an order together with 4 different options for cakes (see Appendix A). Teachers can tell students the bakery has these options for flavors: Chocolate, vanilla, strawberry, cookies and cream, and triple berry. Teachers can remind students that they are in a friendly competition with the other groups, so each cake needs to be designed creatively using all five flavors. For example, students can choose to have half of their cake with one flavor, the other half needs to have the other four flavors. Students need to write an equation under each cake so the baker understands which fraction of the cake each flavor will be (see Figure 4). If time allows, students can be challenged to write two equations to represent each of their cake designs. Teachers can look for students' understanding of the relationship between the part and the whole. Also, teachers should pay close attention to students' understanding about dividing the parts into smaller pieces, but the fraction would be getting larger. Teachers want to see students' understanding of the part-whole relationship between numerator and denominator in relation to the whole cake. Teachers should monitor students and have discussions with group members to address any of these misconceptions.

$$\frac{1}{2}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}=1$$

$$\frac{1}{2}+\frac{4}{8}=1$$

Figure 4. A Student Example of a Cake Design

Once students plan the cake design, their second task is to ensure that they have enough streamers and ribbons to decorate the edges of the tables. Each table will have a color theme that represents the birthday kid's favorite colors: Green, blue and yellow. The food table will have all of the colors by using the leftover streamers and ribbon (see Appendix A). Teachers can look for students creating equivalent fractions using their fraction bars and their equations. With fraction bars, students can adjust how a fraction looks to use ideas that makes sense to them. For example, assuming that there was $\frac{2}{3}$ of green ribbon left on the roll, and the table needed one half of the roll. Using the fraction bars, students may notice that $\frac{2}{3}$ cannot be divided in one half and be able to connect their previous knowledge to the task. The student may change the $\frac{2}{3}$ to $\frac{4}{6}$ so that both fractions would be two parts away from the whole. Another student may change both fractions to a common numerator of 6. Similarly, students may divide ½ into 3 sections to also make sixths or an equivalent fraction of $\frac{3}{6}$. Finally, the goal is to have students write the fractions numerically to represent the equivalent fractions that they create with their fraction bars and subtract the equivalent fractions ($\frac{4}{6} - \frac{3}{6} = \frac{1}{6}$) (see Figure 5).

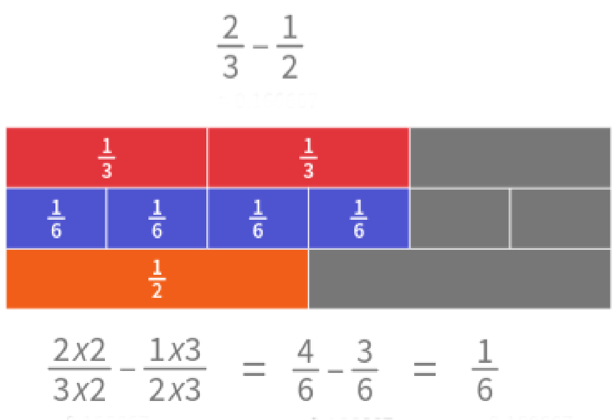


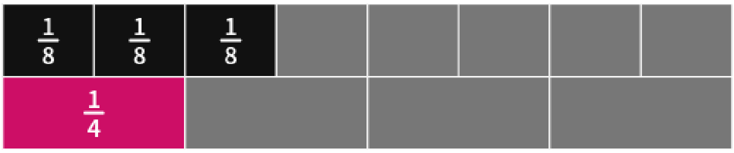
Figure 5. A Student Example for the Green Table, and How They May Use Fraction Bars and Equations to Figure Out the Leftover Streamer

Explain

(30 minutes) Discuss the different ways of how students used common denominators to add and subtract fractions with cakes and ribbons from the explore section. Teachers can explain how the fraction bar (refer to the streamer example above) can be sectioned into sixths before "cutting" $\frac{1}{2}$ of the green streamer. Teachers should have a conversation with students about how to create common denominators by looking for the equivalent fraction. For example, teachers can help students notice a common multiple between the two denominators, like denominators that are 2 and 5 have a least common multiple of 10. Visually the teacher could draw halves and fifths on a fraction bar directly above one another and model how each of the halves bars can be split into tenths by multiplying each half by a fifth. Teachers can model the multiplication numerically that matches the representation. Similarly, the fifths bar could also be split into tenths, but this time by multiplying each fifth by a half. Again, teachers can model the multiplication numerically to match the representation.

Now introduce a scenario to help students calculate how much cake was eaten during the party. For example, assuming that $\frac{3}{8}$ of the Chocolate Cake and $\frac{1}{4}$ of the Strawberry Cake are left. The task is to calculate how much cake is leftover. Have students pair up and hand out the story problems (see appendix B). Teachers are looking for students who are creating models and equations with common denominators before adding or subtracting the fractions. Some students may visualize $\frac{1}{4}$ as $\frac{2}{8}$ since both fractions would be two parts away from the whole or change $\frac{1}{4}$ to a common denominator of 8 (see Figure 6).

$$\frac{3}{8}+\frac{1}{4} =$$



$$\frac{3}{8}+\frac{1}{4} = \frac{3}{8}+\frac{1x2}{4x2} = \frac{3}{8}+\frac{2}{8} = \frac{5}{8}$$

Figure 6. A Student Example of How Much Chocolate Cake and Strawberry Cake are Leftover

Extend

To observe different ways of finding equivalent fractions to add or subtract fractions and have students use their fractions and number sense of fractions to estimate, it is time to have students make some punch for the birthday party. They have a pitcher that holds 7 cups, however they need to leave room at the top ($\frac{1}{3}$ of a cup) so they can carry the pitcher. Ask students how many cups of juice they can have in the pitcher ($6\frac{2}{3}$).

Give students the possible juice choices (see Figure 7). Ask students to estimate a combination of juices that would fit in the pitcher. A possible response may be Lemon Twist and Perfect Peach because a student may add up the whole numbers first ($3+2=5$) and calculate the difference between the amount of pitcher can hold and five ($6\frac{2}{3} - 5 = 1\frac{2}{3}$) and estimate whether $\frac{3}{4}+\frac{3}{8}$ will be less than the $1\frac{2}{3}$. A follow up question can be what juice to add to the combo.

Juice Options

Lemon Twist	3 ¾ cups
Wild Strawberry	1 ⅙ cups
Great Grape	1 ½ cups
Perfect Peach	2 ⅜ cups
Classic Apple	4 ⅕ cups
Cherry Jubilee	1 ⅖ cup

Figure 7. Possible Juice Choices

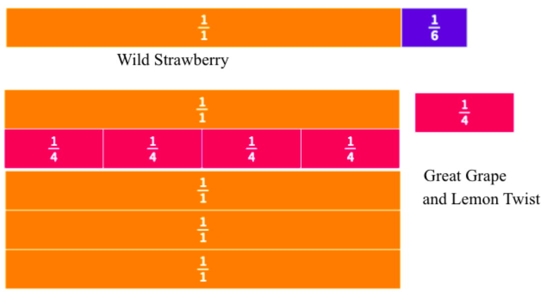
The next task is to combine Wild Strawberry, Great Grape, and Lemon Twist (see Figure 8). Some questions to pose: "How much juice would Wild Strawberry, Great Grape, and Lemon Twist make and will this mix fit in the pitcher?", "If yes, how much space will be left?", "If not, how many cups would overflow?", and "What other juice choices would make a juice blend?". Provide some time to students to discuss and calculate, encouraging students to use various methods for finding a common denominator.

Figure 8. Problem Set-Up Using Fraction Bars

After students complete the task, have them share their representation of common denominators to add the juices together (see Figure 9). Ask students to then create their own juice blends, stating the total amount of juice, the space left in the pitcher, or the amount of juice that overflowed.

(Step 1) Set up the problem.

(Step 2) Break 1 ½ into 1 $^{2}/_{4}$.



(Step 3) Add Great Grape (1 ½ cups) and Lemon Twist (1 ¾ cups).

(Step 4) Find a common denominator and add twelfths together.

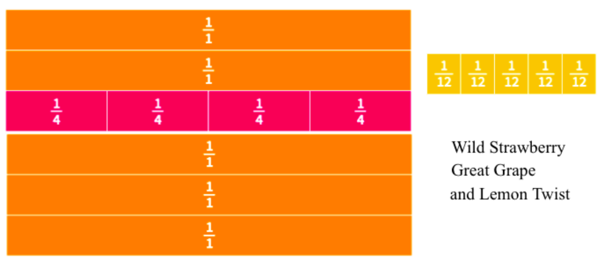


Figure 9. Possible Student Solution

Evaluate

One assessment to consider before beginning the lesson is looking for students' counting fractional parts correctly and understanding the relationship between the parts (the numerator) and the whole (the denominator). Students should understand the iterative concept that the top number (numerator) counts and the bottom number (denominator) tells what is being counted.

As students work through planning the birthday cake designs, decorating the tables and eating the cake, teachers should look for students' flexibility in using models like area model, fraction bar or number line to represent fractions. Students' flexibility of using contexts and models help understanding equivalence. For example, in the green table streamer example, students manipulate the fraction bar to create equivalent fractions, and use multiplication to match their representation. Teachers can ask students "why" they choose to make specific decisions to gain understanding into students' fraction number sense. If students struggle, provide students with more opportunities to play with manipulatives and ask them to represent commonly applied fractions such as $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{3}{4}$, and $\frac{5}{10}$.

When students are working on the juice problem, look for students using benchmark fractions such as $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$, and $\frac{1}{8}$ when estimating. For example, they may estimate $2\frac{3}{8}$ as a little more than $2\frac{1}{4}$ since $2\frac{2}{8} = 2\frac{1}{4}$. When working the juice problem, look for different ways of students' adding the fractions. For example, they may add $1\frac{1}{6} + 1\frac{1}{2}$ and simplify the sum to $2\frac{2}{3}$ before adding the $3\frac{3}{4}$ value. An improper fraction will occur sometime in the process. Listen for students' understanding of an improper fraction containing a whole and part and whether students can calculate the amount correctly. For example, if a student adds Great Grape and Lemon Twist first, observe how this student calculates ($\frac{1}{4}$ more than one whole) and whether the student uses fraction bars to model. Once the total amount of juice is calculated ($6\frac{5}{12}$), listen for a reasoning that would fit into the pitcher since $\frac{5}{12}$ is less than $\frac{1}{2}$, $\frac{1}{2}$ is less than $\frac{2}{3}$ and the pitcher holds $6\frac{2}{3}$ cups. Ask how much more juice would fit into the pitcher exactly. Students may subtract by creating common denominators and adding up $6\frac{5}{12}$

juices to fit into 6 $\frac{2}{3}$, that is equivalent to 6 $\frac{8}{12}$, so they need 3 more 12th, or $\frac{1}{4}$ cup. They may also use fraction bars and create the amount of the pitcher (6 $\frac{2}{3}$) and fill the pitcher with "juice fraction bars" until they have $\frac{1}{4}$ left.

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Appendix A

Cake Order (Part I)

As a group create four cakes with different amounts of flavors in each cake. Write the equation representing the amount of each flavor. The flavors are: chocolate, vanilla, strawberry, cookies and cream, and triple berry.

Equation:	Equation:
Equation:	Equation:

Decorate the Tables (Part II)

Students need to use the information below to figure out how much ribbon or streamers are needed for each table. Draw a model and an equation to show your thinking.

Tables for the Birthday Party	**Models and Equations**
1. ***Green Table:*** *There is* $\frac{2}{3}$ *green ribbon left and you will need to use ½ of that for the green table.*	
2. ***Blue Table:*** *There is* $\frac{7}{12}$ *of the blue streamer left on the roll. The blue table will need* $\frac{1}{4}$ *of the streamer left on the roll.*	
3. ***Yellow Table:*** *There is* $\frac{5}{6}$ *of the yellow ribbon left on the roll. The yellow table will need* $\frac{2}{3}$ *of the ribbon left on the roll.*	
4. ***The Food Table:*** *The food table will use all of the leftover ribbon and streamers from the green, blue and yellow tables.*	

Appendix B

Story Problems

It is time to eat cake!! WOOHOO!! The baker wanted to know which cakes were the favorite amongst the guests. Can you help the baker figure this out? Draw representations and write equations to show the baker what was eaten during the party.

One of the cakes was cut into 12 ***pieces and*** $\frac{4}{6}$ ***of that cake was eaten. How much was left?***	***On one table there was*** $\frac{2}{5}$ ***of the Triple Berry and*** $\frac{2}{3}$ ***of the Cookies and Cream left. How much cake was leftover?***
On another table there was $\frac{3}{8}$ ***of the Chocolate Cake and*** $\frac{1}{4}$ ***of the Strawberry Cake left. How much cake was leftover?***	***Another cake was cut into*** 24 ***pieces and*** $\frac{5}{8}$ ***of that cake was eaten. How much was left?***

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Citation

Carey, A. S., Quiroz, F., & Jackson, T. (2023). Birthday Celebration. In A. Bicer (Ed.), *Creativity-Directed Mathematical Tasks for 5th Grade Common Core Classrooms* (pp. 251-266). ISTES Organization.

Task 19 - Tile Patterns

Jennifer Kellner, Amy Kassel, Chuck Butler

Mathematical Content Standards

CCSS.MATH.CONTENT.5.NF.A.1

Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators.

For example, $\frac{2}{3}+\frac{5}{4}=\frac{8}{12}+\frac{15}{12}=\frac{23}{12}$ (In general, $\frac{a}{b}+\frac{c}{d}=\frac{(ad+bc)}{bd}$.)

Mathematical Practice Standards

CCSS.MATH.PRACTICE.MP4

Model with mathematics.

CCSS.MATH.PRACTICE.MP5

Use appropriate tools strategically.

CCSS.MATH.PRACTICE.MP7

Look for and make use of structure.

Vocabulary

Equivalent fractions, denominator, numerator, common denominator, unlike denominator, adding fractions, subtracting fractions, mixed number, trapezoid, rhombus, hexagon

Materials

Pattern Blocks (hexagon, trapezoid, rhombus, and triangle), hexagon grid paper, colored pencils

Lesson Objective

Students will use equivalent fractions to add and subtract fractions with unlike denominators. This lesson will promote creative thinking by asking students to write addition and subtraction equations using pattern blocks. They will create designs using the pattern blocks and find the area using fractions with unlike denominators with multiple representations.

Engagement

(20-30 minutes) Teachers may begin this lesson by having students explore the history of Ceramic Tiling. A brief history can be found at whytile.com (Tile History - Why Tile). Ceramic tile can be found in nearly every culture in the world. Social, political, and economic influences have impacted ceramic tile designs over the course of centuries. Ceramic tile is very durable and hence, has greatly contributed to understanding history through ceramic tile designs (Heritage, 2022). Students may be asked to project their favorite example of ceramic tiles and tell which time period it came from the website. Figures 1, 2, and 3 are examples from Moorish Tile, Islamic Tile, and Modern European Tile.

Figure 1. Example of Zelliga Tiles found in the Medersa Ben Youssef College (Heritage, 2022). Note. Image via Wikimedia Commons (Prince, 2010)

Teachers may also wish to have students explore careers in the tile industry by visiting the whytile.com careers page (Careers in the Tile Industry - Why Tile) and sharing one certification qualified tile installers can earn.

Ask students to create their own tile pattern or design using the hexagon, trapezoid, rhombus, and triangle pattern blocks on the hexagon grid paper (Appendix A). Students may wish to lay out the pattern initially using the pattern blocks and then color the grid paper for the repeated designs. See Figure 4 as an example. The students will use their designs to explore equivalent fractions in the next part of the lesson.

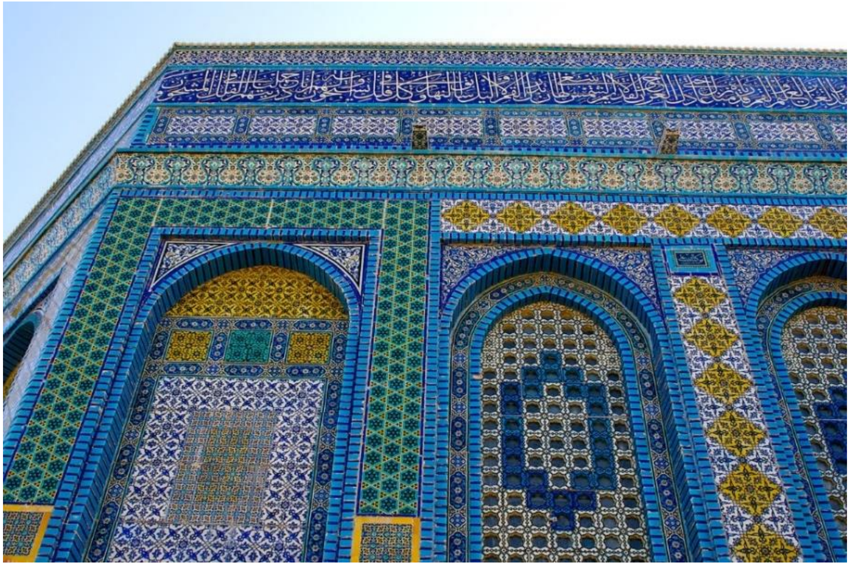

Figure 2. The Dome of the Rock Exterior from the Late Seventh Century. Note. Image via Wikipedia Commons (Ulin, 2007)

Figure 3. Encaustic Floor Tiles at St. Mary the Virgin's Church in Hatfield Broad Oak, Essex, England. Note. Image via Wikipedia Commons (Acabashi, 2015)

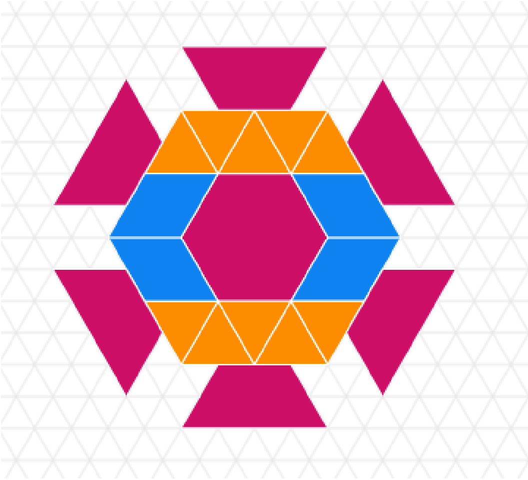

Figure 4. Example of Student Tiling Design. Note. Image from Polypad (Mathigon, 2022)

Explore

Part 1: The teacher will facilitate a discussion with the class about how the hexagon, trapezoid, rhombus, and triangle pattern blocks are related. Students should discuss how many of the smaller blocks it would take to make a bigger block. For example, it would take 2 triangles to make a rhombus. The teacher should question the students about how any of each block would be equivalent to another. Students should create a mini poster representing the pictorial relationships with their equivalent equations. For example, 1 hexagon = 6 triangles would be represented as $1 = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6}$ or 1 trapezoid = 3 triangles would be represented as $\frac{1}{2} = \frac{1}{6} + \frac{1}{6} + \frac{1}{6}$ or 1 rhombus = 2 triangles would be represented as $\frac{1}{3} = \frac{1}{6} + \frac{1}{6}$ (see Figure 5).

Figure 5. *Note*. Students may begin to express this relationship as $1\ rhombus = 1\ triangle + 1\ triangle$ or in fraction form $\frac{1}{3} = \frac{1}{6} + \frac{1}{6}$

Part 2: In small groups, using the four pattern blocks (hexagon, trapezoid, rhombus, and triangle), have the students complete the table (Appendix B). Based on the value of the given block, find the relationship to the other three blocks. The students will tell how many of one block it would take to make the "If" block. If students are struggling, have them place pattern blocks on top of each other and exchange blocks as needed to complete it. When they complete the table, have the students compare their table with the class. The students are looking for different representations of the comparisons. For example, a student might say the hexagon = 6/6 or one might say the hexagon = 1. This is where a discussion about equivalent fractions would be facilitated by the teacher.

The teacher will facilitate a discussion on the addition and subtraction of pattern blocks. Using the pattern blocks, students will complete Appendix C with equivalent equations. Students may use multiple blocks with both addition and subtraction. If students struggle, remind them to use the blocks to find equivalent fractions to answer their problems.

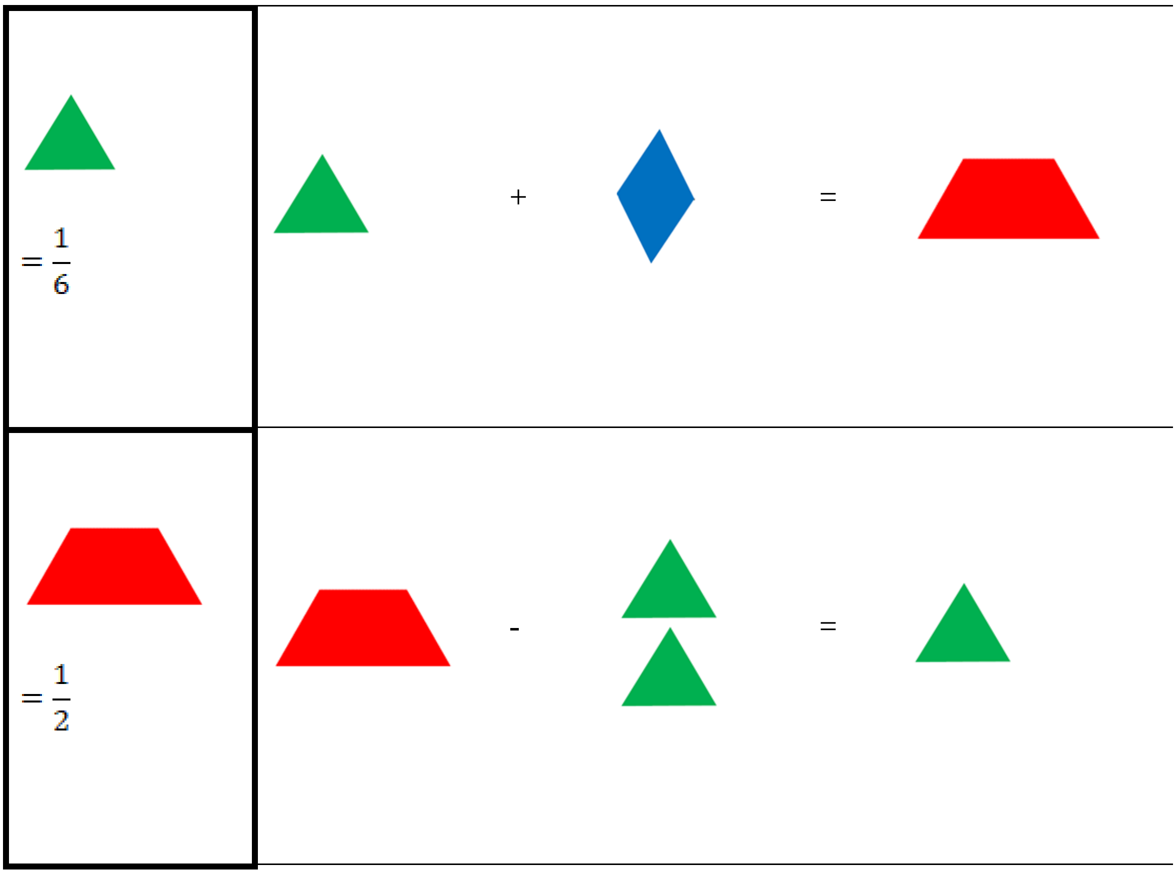

Figure 6

Part 3: After students have generated equations representing addition and subtraction from Part 2, ask students to get out their tile design from the Engagement part of the lesson. Ask students to write as many different equations as they can to represent the area of their design.

For example, in Figure 7, representing the hexagon as 1 unit, the trapezoids as $\frac{1}{2}$, the rhombus as $\frac{1}{3}$, and the triangles as $\frac{1}{6}$, the student may write

$$1+\left(\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}\right)+\left(\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}\right)$$
$$+\left(\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}\right)$$

$$=1+\frac{5}{2}+\frac{4}{3}+\frac{10}{6}$$

$$=\frac{6}{6}+\frac{15}{6}+\frac{8}{6}+\frac{10}{6}$$

$$=\frac{39}{6}$$

$$=6\frac{3}{6}$$

$=6\frac{1}{2}$ for the area of the tile design.

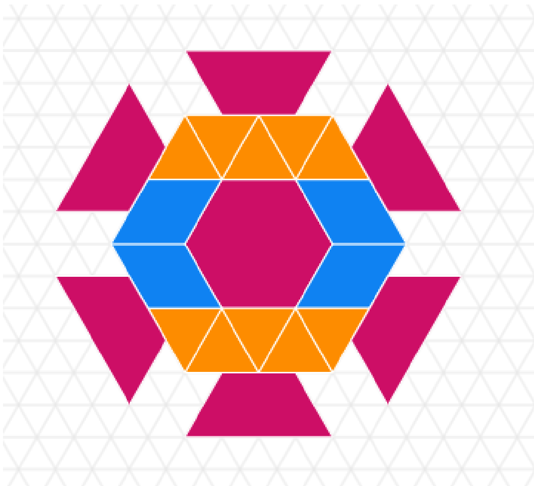

Figure 7

Explain

In part 1, the teacher should look for students that are not simplifying their expression and encourage students to write their expression another way. For example, when determining how many triangles are in a rhombus, a student might write $\frac{3}{6}$. The teacher can support this student by asking them to write this fraction in a simpler way or with fewer pieces. The teacher can ask the students to place the pieces on a hexagon and ask how much of the hexagon is covered to help students visualize this.

In part 2, the teacher should look for students to discover the relationship between the If Block and the other shapes. If students struggle to understand how many shapes "fit into" the If Blocks, the teacher can encourage students to physically manipulate the shapes. The teacher can further support the students by having students write the numerical representation. For example, 2 greens = 1 blue, which is another way of saying $\frac{1}{6}+\frac{1}{6}=\frac{2}{6}=\frac{1}{3}$.

In part 3, the teacher should look for what students choose as their unit. Because the hexagon was 1 unit in the previous parts, most students will probably continue with the hexagon as 1 unit in this example. As students find the area of the tile design, the teacher should monitor and select different approaches to finding the area of the tile design. The teacher should pay particular attention to the order students share their approaches. For example, the teacher may ask a student to share first who added all the shapes individually (similar to the example above) and then ask a student to share who looked for whole hexagons in the various shape without doing the formal operations. If a student used a different shape for the 1 unit, then the teacher could show this example at the end and then proceed to the What If? extension.

Extend

The teacher may choose to assign one extension to the whole class or assign different extensions to different groups based on interest, ability, and readiness.

Find Three– given two shapes from If Block, find three fractions that are greater than one If Block and less than the other If Block. The students should find equivalent fractions of the two If Blocks until they can find three fractions between them. For example, if the students choose $\frac{1}{2}$ and $\frac{1}{6}$, the students could write equivalent fractions of $\frac{6}{12}$ and $\frac{2}{12}$, resulting in students finding $\frac{1}{4}$, $\frac{1}{3}$, and $\frac{5}{12}$.

Add/Subtract Three– given three shapes from If Block, create an expression that contains both addition and subtraction, performing operations both visually and numerically. If all the shapes must be different, what is the largest value you can find? What is the smallest? How do you know?

What if? - Go back to Figure 7 and make the trapezoid 1 unit. What is the area of the tile design? How does that change the area? What if the hexagon is a half unit? What is the area of the tile design? How does that change? How does the original area compare with the new area? Could you predict what the new area would be if a different figure was 1 unit?

Evaluate

- In part 1, formative assessment for prior knowledge of the standard will occur as students are adding and subtracting fractions with common denominators. The teacher will also look for students using equivalent fractions, specifically writing fractions in their simplest form.
- In part 2, formative assessment for content occurs as students understand the idea of equivalent fractions. For example, the teacher could ask "How many triangles fit into the hexagon?" and encourage students to represent that with concrete objects as well as numerically. If students are having trouble seeing equivalent fractions, ask students how much of the hexagon is covered.
- In part 3, formative assessment for content and creativity occurs as students find the area of the tile design. The teacher is monitoring the class for both classroom trends in methods for solving as well as creative approaches that might be more efficient and/or conceptual in nature.
- In Find Three, formative assessment for content and creativity occurs as the students are writing equivalent fractions between two shapes. The teacher should ask students why they picked the specific equivalent fractions and look/listen for multiple ways students can represent these ideas. Students can also represent equivalent fraction with a tile.
- In Add/Subtract Three, formative assessment for content and creativity occurs as students are adding and subtracting three fractions with different denominators. The teacher could ask students about their approach to finding a common denominator. For example, while students might initially rely on trial and error as a strategy, the teacher could ask students to visualize the problem first in order to build in think time and encourage creativity. The teacher could also use questioning to connect the visual and numerical representations.

- In What If?, formative assessment for content occurs as students realize that changing the unit of the shape will change the area of the tile.

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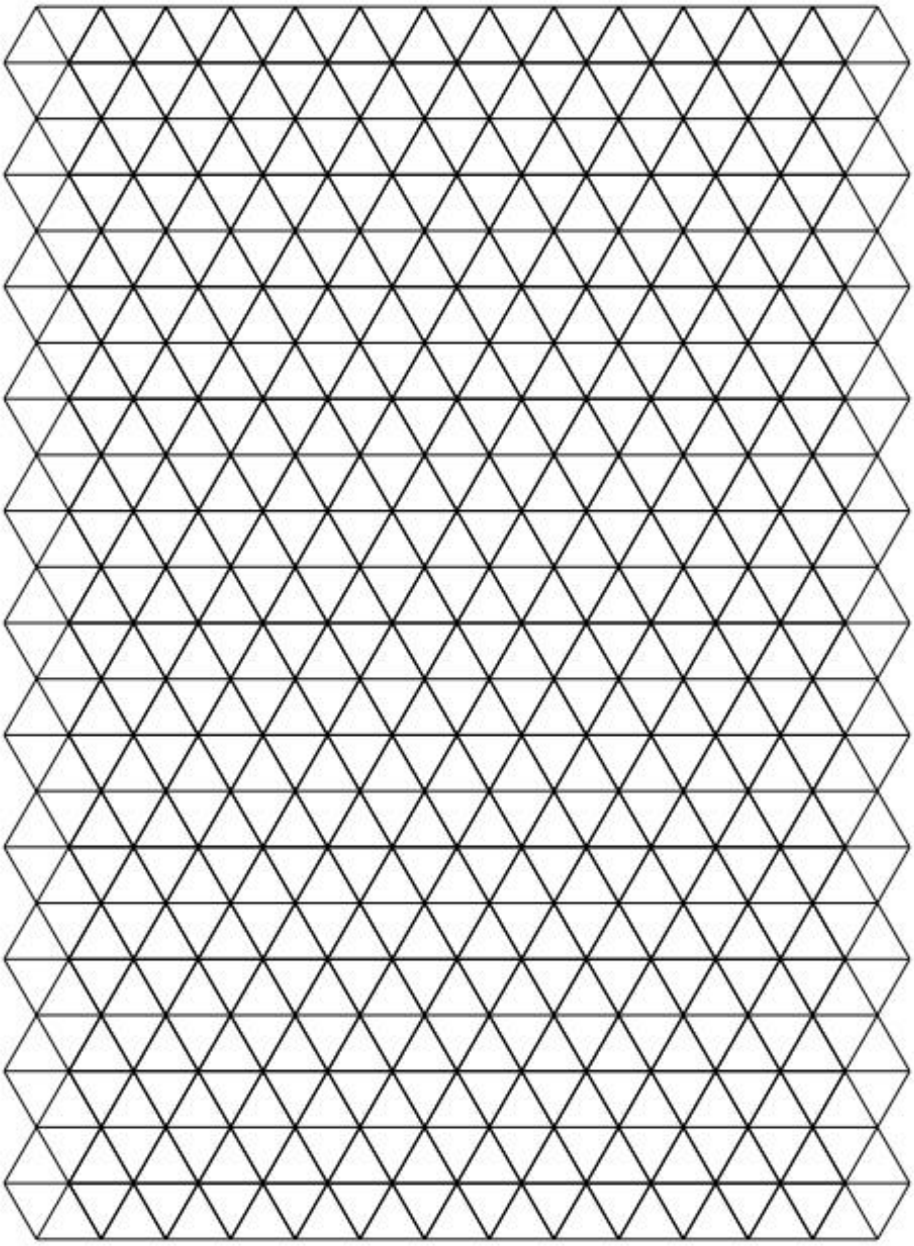
Appendix A



Appendix B

If:	 ____ reds = yellow	 __ blues = yellow	 __ greens = yellow
If: 	 ____ blues = red	__ blues = yellow	 ____ greens = blue
If: 	 ______ greens = red	 _____ greens = blue	____ greens = yellow

Appendix C

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Citation

Kellner, J., Kassel, A., & Butler, C. (2023). Tile Patterns. In A. Bicer (Ed.), *Creativity-Directed Mathematical Tasks for 5th Grade Common Core Classrooms* (pp. 267-280). ISTES Organization.

Task 20 - Let's Prepare for a Jog-A-Thon!

Helen Aleksani, Geoff Krall

Mathematical Content Standards

Focus Content Standard

CCSS.MATH.CONTENT.5.NF.A.1

Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)*

Supporting Content Standard(s)

CCSS.MATH.CONTENT.5.NF.A.2

Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.*

Mathematical Practice Standards

CCSS.MATH.PRACTICE.MP2

Reason abstractly and quantitatively.

CCSS.MATH.PRACTICE.MP4

Model with mathematics.

CCSS.MATH.PRACTICE.MP5

Use appropriate tools strategically.

CCSS.MATH.PRACTICE.MP7

Look for and make use of structure.

Vocabulary

Fraction addition, fraction subtraction, equivalent fractions

Materials

Fraction bars, fraction towers, grid paper, number line, color pencils.

Lesson Objective

In this lesson, students will be adding and subtracting fractions when denominators are different. Students describe how to find the sums of fractions in a way that makes sense to them. To enhance mathematical creativity, students will be using manipulatives such as fraction tower or bars, number line, and grid paper to discover different representations of commonly used fractions that students are familiar with from 3rd grade such as 1/2. In this task, students will be given the opportunity to experience a situation where it seems natural to add fractions with unlike denominators.

Engagement

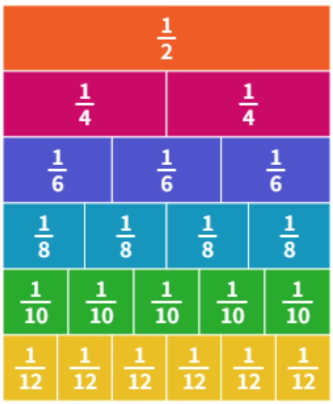
(15 minutes) Start the lesson by describing what a jog-a-thon is and how it is used by students to raise money for school. Show the students the following YouTube vide "Jog-A-Thon | Taylor Street Elementary School".
https://taylor.robla.k12.ca.us/apps/video/watch.jsp?v=167430

Then, ask students to speak to an elbow partner to explain their understanding of the video they watched. After that, call on a few students to share their observations with the class. Then, share with the students that they will be working with a scenario in which they will be involved in helping a friend prepare for a jog-a-thon. We will be adding and subtracting fractions using our understanding of fraction equivalence to help our friend meet the goal of practicing running one mile a day. Here is task to share with the class,

> Your friend is planning to train for the school's Jog-A-Thon to help raise money for the new benches. Your friend needs to run at least one mile a day to help prepare for this challenge. If your friend runs to your house, he/she is running $\frac{3}{8}$ of a mile. Then, he/she runs to school, which is $\frac{1}{2}$ mile away from your house. Will your friend have fulfilled the requirement of running a mile a day?

Explore

(30 minutes) First students will be in groups of two or three for this lesson. As a warm up, have students use the fraction bars and fraction towers to help remember the concept of equivalent fractions and adding fractions with same denominators. For example, have students pick a bar of $\frac{1}{2}$ a unit and try to find other equivalent bars that can be combined to equal the ½ a unit of a bar (see below).



Then, ask the students to write equivalent statements that would represent $\frac{1}{2}$. For instance,

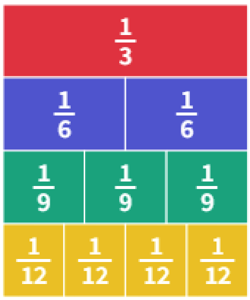
$$\frac{1}{2}=\frac{2}{4}=\frac{3}{6}=\frac{4}{8}=\frac{5}{10}=\frac{6}{12}$$

Ask the students to explain 2/4 by writing a statement that would show the sum of fractions. For instance,

$$\frac{2}{4}=\frac{1}{4}+\frac{1}{4} \quad or \quad \frac{3}{6}=\frac{1}{6}+\frac{1}{6}+\frac{1}{6}$$

This will help students remember their 4th grade standards on how to add fractions with the same denominators.

Then ask students to do the same but this time use $\frac{1}{3}$ bars of a unit to help students better understand the concept of equivalence. For example,



Then, ask students if they can use bars with different denominators to equal the $\frac{1}{2}$ bar of a unit and have them write a statement that would represent the sum. For example,

$$\frac{1}{2} = \frac{1}{4} + \frac{1}{8} + \frac{1}{8}$$

$$\frac{1}{2} = \frac{1}{4} + \frac{2}{8}$$

Allow students to create more relations similar to the one above working with their group members.

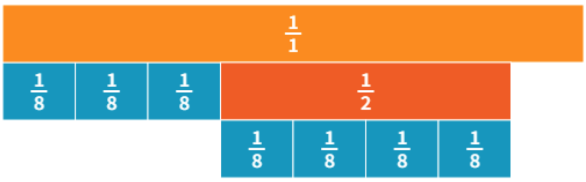
Now, ask the students to use the fraction bars to demonstrate the task shared with them earlier.

Your friend is planning to train for the school's Jog-A-Thon to help raise money for the new benches. Your friend needs to run at least one mile a day to help prepare for this challenge. If your friend runs to your house, he/she is running 3/8 of a mile. Then, he/she runs to school, which is $\frac{1}{2}$ mile away from your house. Will your friend have fulfilled the requirement of running a mile a day?

For example, a work similar to the one below is expected of each group. Then, ask students how far do they think their friend was able to run.

Students will most likely show that their friend was able to run $\frac{3}{8}+\frac{1}{2}$. Then, ask students if that would equal 1 mile. The expectation is for students to look at their fraction bar and immediately reply that no that is not the case. If they have a hard time seeing it, model it for them.

Then ask students, how many ⅛ are in a ½ and have them demonstrate using the fraction bars. Ask students to write the statement that would demonstrate the new sum $\frac{3}{8}+\frac{4}{8}=\frac{7}{8}$

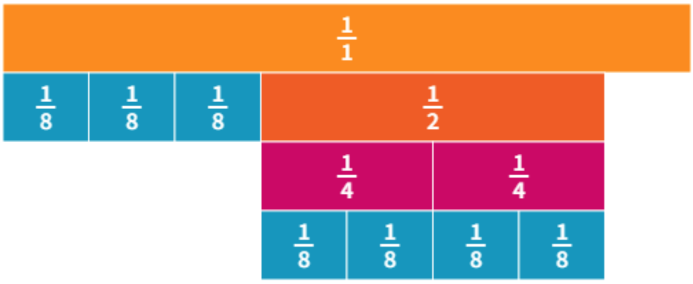


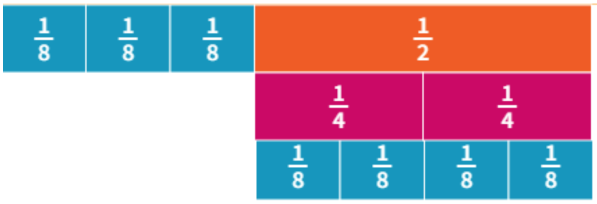
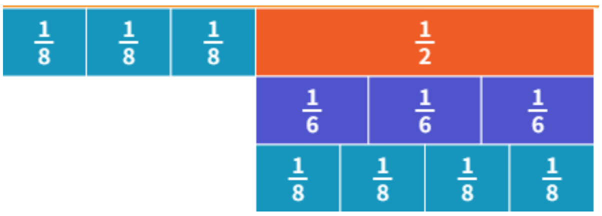
Ask students if they think running 3/8 of a mile then adding another half a mile will fulfill the requirements of practicing a mile a day. Have students write a statement that would show the sum they were able to demonstrate using the fraction bars.

$$1 > \frac{3}{8}+\frac{4}{8}$$

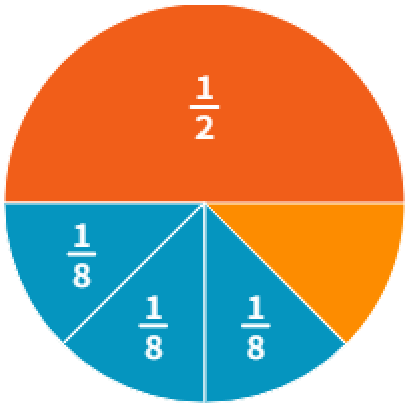
$$1 > \frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{2}=\frac{3}{8}+\frac{1}{2}$$

Have students try different fraction bars to demonstrate the scenario and record their answer in the table (Appendix A). For example,



Fraction bars image	statement
	$\frac{3}{8}+\frac{1}{2}=\frac{7}{8}$
	$\frac{3}{8}+\frac{1}{2}=\frac{7}{8}$ $\frac{3}{8}+\frac{4}{8}=\frac{7}{8}$
	$\frac{3}{8}+\frac{1}{2}=\frac{7}{8}$ $\frac{3}{8}+\frac{2}{4}=\frac{7}{8}$ $\frac{3}{8}+\frac{4}{8}=\frac{7}{8}$
	$\frac{3}{8}+\frac{1}{2}=\frac{7}{8}$ $\frac{3}{8}+\frac{3}{6}=\frac{7}{8}$ $\frac{3}{8}+\frac{4}{8}=\frac{7}{8}$

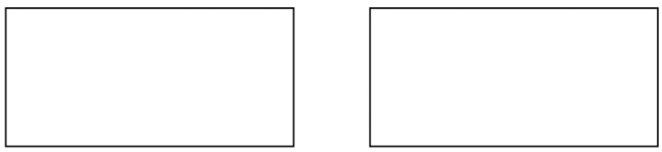
We can also have students demonstrate the scenario using circles. They can follow the steps provided above to demonstrate their understanding of the scenario using circles instead of bars.



Explain

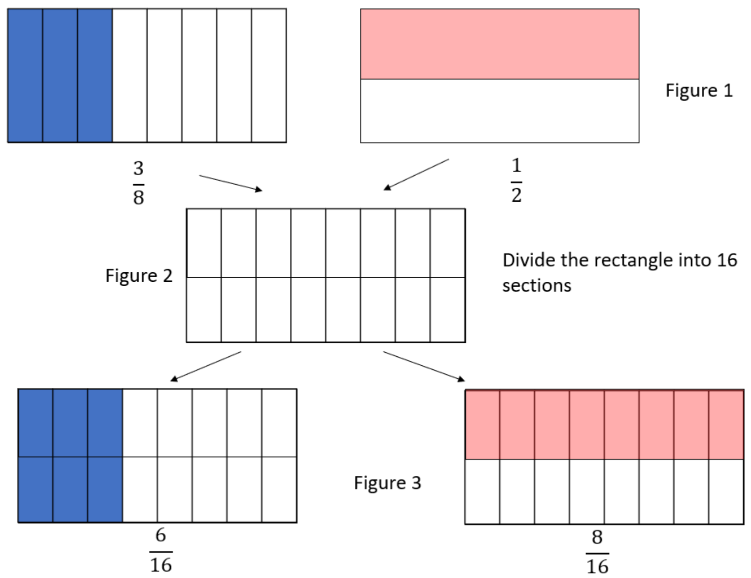
(30 minutes) Now, we will have students draw an area model to help understand the reason behind the fraction bars and why it works.

Start by providing students a sheet with two identical rectangles (Appendix B).



Students will now demonstrate $\frac{3}{8}+\frac{1}{2}$ using the area model. Divide the first rectangle into 8 equal sections using vertical lines and the second rectangle into 2 equal sections using a horizontal line (Figure 1). If students struggle with the terms vertical and horizontal, the teacher can model it for them following figure 1. Provide students with color pencils so they can shade the $\frac{3}{8}$ of the first rectangle and $\frac{1}{2}$ of the second rectangle (see Figure 1). Then, ask students if one shaded piece from the first rectangle is equal in area to one shaded piece from the second rectangle. Once students understand the area of the shaded pieces are not the same, then ask students if we can claim that we have shaded 4 equal sections combining the two rectangles. If students struggle with this concept, help students see the one blue section is not equal to the one red section by modeling it using a scenario such as this: if one blue shaded piece and the one red shaded piece represent a candy bar. Student A ate the blue candy bar and student B ate the red candy bar. Can we claim that both students ate the same amount of candy since they both ate one piece?

Once students understand the concept of area of each piece, ask students to help divide each rectangle so both rectangles have equal number of pieces (see Figure 2). Connecting it to the candy bar example, we can say that we want all the candy bars to be equal in size. Once students understand that they can use the first two rectangles to help divide the rectangles into equal numbers of parts, then ask students to count the shaded sections from each rectangle and write the fractions which are 6/16 for the blue one and 8/16 for the red one (Figure 3). Then have students use one rectangle divided into 16 sections that includes both the blue and red shadings (see Figure 4).



$$\frac{6}{16}+\frac{8}{16}=\frac{14}{16}$$

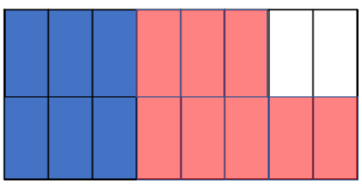


Figure 4

To help connect the lesson to equivalent fractions, ask students if the fractions in figure 3 are equivalent to fractions from Figure 1. We need the students to understand that shading $\frac{6}{16}$ is the same as shading $\frac{3}{8}$. Therefore, they represent equivalent fractions.

After this, have students compare the shaded rectangle in Figure 4 to a fully shaded rectangle (see Figure 5) and decide which is larger or smaller.

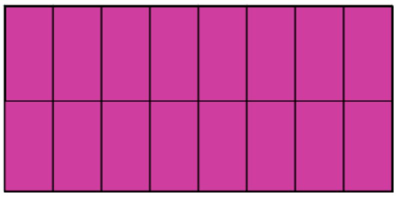


Figure 5

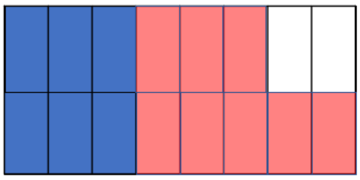
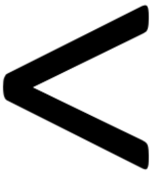


Figure 4



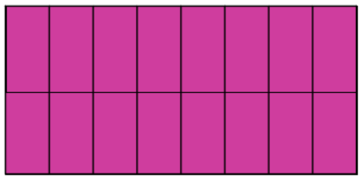


Figure 5

This rectangle has all the pieces shaded

$$\frac{16}{16} = 1$$

Extend

(20 minutes) Extend the activity with the following prompt:

"You are now helping your friend get ready for a 5 mile. She can currently run one mile a day. In order to build up her endurance she is going to run a little bit more each day. She is going to run a *little bit more* each day of the week, except Wednesday, starting on Sunday.

Sunday	Run $\frac{1}{2}$ additional mile compared to previous run
Monday	Run $\frac{1}{3}$ additional mile compared to previous run
Tuesday	Run $\frac{1}{4}$ additional mile compared to previous run
Wednesday	Rest day
Thursday	Run $\frac{1}{2}$ additional mile compared to previous run
Friday	Run $\frac{1}{3}$ additional mile compared to previous run
Saturday	Run $\frac{1}{4}$ additional mile compared to previous run

When will your friend build up the endurance to run 5 miles in a single day?"

If necessary, walk students through the first three days of this training regimen:

Day 1 (Sunday) = 1 mile + $\frac{1}{2}$ mile = 1 $\frac{1}{2}$ mile

Day 2 (Monday) = 1 $\frac{1}{2}$ mile + $\frac{1}{3}$ mile = 1 3/6 mile + 2/6 mile = 1 $\frac{5}{6}$ mile

Day 3 (Tuesday) = 1 $\frac{5}{6}$ mile + $\frac{1}{4}$ mile = 1 20/24 mile + 6/24 mile = 1 26/24 mile = 2 1/12 mile

Day 4 (Wednesday) = Rest day (stay at 2 1/12 miles)

Students may demonstrate mathematical creativity by solving this problem in multiple different ways. For example, some students may solve this linearly, going day by day. They could opt to make a table (Appendix C). Other students may bin the training into three-day chunks. For example, students may notice that each 3-day chunk adds $\frac{1}{2}+\frac{1}{3}+\frac{1}{4}$ =1 1/12 miles. They may then use fraction bars to determine while they will surpass 5 miles (see Figure 6).

Figure 6

Students may also notice that because 12 is a lowest common multiple of 2, 3, and 4, it might be most efficient to convert all fractions into denominators of 12:

1/2 = 6/12

1/3 = 4/12

1/4 = 3/12

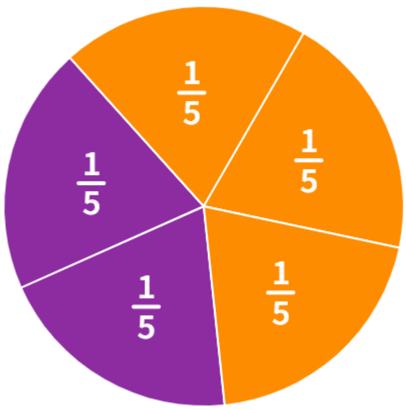
Group students that solve it visually with fraction bars with students who solve it using a table or other numerical method and have them discuss how their answers align. Use the following prompts:

- What is similar about your solutions? What is different?
- How does the visual solution show what is happening in the training?
- How does the numerical solution show what is happening in the training?
- Which solution might be easier to understand for your friend?

Evaluate

Look for students making connections with the various ways of solving the primary task and the extension task (numerical, visual, fraction bars, area model). Ask students to discuss the connections between the various models of solving adding fractions with unlike denominators. Students should have the tools to fluently add fractions with unlike denominators. However, this is a challenging topic and students may need additional support or individual practice. Appendix D contains three practice problems for individuals to solve independently to ensure understanding.

If students practice the following misconception, $\frac{1}{5}+\frac{1}{5}=\frac{2}{10}$, model the following representation.



In this example, students can clearly see that when adding two $\frac{1}{5}$ sectors, the results is 2 sectors out of 5 not 10.

Appendix A

Fraction bars image	statement

Appendix B

Appendix C

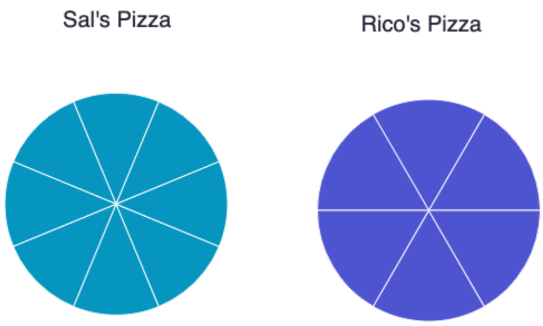
Day of training	Add miles	Miles Ran
Day 1 (Sunday)	Add ½ mile	1 ½ miles
Day 2 (Monday)	Add ⅓ mile	1 ⅚ miles
Day 3 (Tuesday)	Add ¼ mile	2 1/12 miles
Day 4 (Wednesday)	Rest (add 0 mile)	Rest day (stay at 2 1/12 miles)
Day 5 (Thursday)	Add ½ mile	2 7/12 miles
Day 6 (Friday)	Add ⅓ mile	2 11/12 miles
Day 7 (Saturday)	Add ¼ mile	3 2/12 miles
Day 8 (Sunday)	Add ½ mile	3 8/12
Day 9 (Monday)	Add ⅓ mile	4
Day 10 (Tuesday)	Add ¼ mile	4 3/12
Day 11 (Wednesday)	Rest (add 0 mile)	4 3/12
Day 12 (Thursday)	Add ½ mile	4 9/12
Day 13 (Friday)	Add ⅓ mile	5 1/12

Appendix D

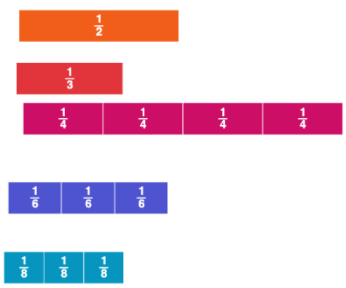
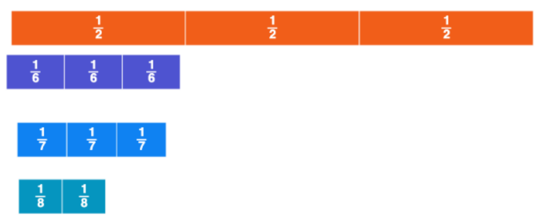
#1: Add the following fractions using two different methods.

2/7 + 5/6

#2: Sal sells pizza cut into eight slices. Rico sells the same size pizza, but cut into 6 slices. How much of a pizza would you have if you purchased 3 slices of Sal's pizza and 4 slices of Rico's Pizza?



#3: Solis has two sets of fraction strips below. Which set is bigger and how do you know?

Fraction Strip Set 1	Fraction Strip Set 2
	

Citation

Aleksani, H. & Krall, G. (2023). Let's Prepare for a Jog-A-Thon! In A. Bicer (Ed.), *Creativity-Directed Mathematical Tasks for 5th Grade Common Core Classrooms* (pp. 281-296). ISTES Organization.

SECTION 6 - APPLY AND EXTEND PREVIOUS UNDERSTANDINGS OF MULTIPLICATION AND DIVISION TO MULTIPLY FRACTIONS

Task 21 - Counting Cows

Melena Osborne, Michael Gundlach, Michelle Tudor

Mathematical Content Standards

CCSS.MATH.CONTENT.5.NF.B.4

Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

CCSS.MATH.CONTENT.5.NF.B.4.A

Interpret the product (a/b) × q as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations a × q ÷ b. For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = (ac)/(bd).

Mathematical Practice Standards

1. MP1 Make sense of problems and persevere in solving them
2. MP3 Construct viable arguments and critique the reasoning of others
3. MP5 Use appropriate tools strategically

Vocabulary

Partition, multiplication

Materials

Fraction tiles, snap blocks, copies of Appendix A

Lesson Objective

Students will use their knowledge of whole-number multiplication to develop the skills necessary to multiply a fraction by a whole number and a fraction by a fraction. Creativity will be emphasized by having students justify whether certain problems represent multiplication problems when the factors used are both fractions and whole numbers.

Students will focus on justifying why a context necessitates using multiplication and then using that context to understand the fraction multiplication process.

Engagement

(10 Minutes) Begin the class by engaging students in a number talk about the following situation:

> A new virus has been infecting local livestock. To protect their herd, a rancher will need to vaccinate their cattle. For the livestock to be protected, most of the livestock must be vaccinated. As more of the herd is vaccinated, the herd overall is safer. However, this particular vaccine is expensive, so vaccinating the entire herd is not possible with funds available. What fraction of the entire herd do you think the rancher should vaccinate? What additional information might help the rancher make this decision?

Let students talk about this situation in groups of 2-4 before giving any additional information. After students have had some time to discuss the situation, bring the class together and ask students what additional information they think would be useful. Possible questions include:

- How many cattle are in the herd?
- How much does one vaccine cost?
- Is there a “best fraction” of the herd that guarantees safety?
- How much money does the rancher have to pay for vaccines?

Explain A

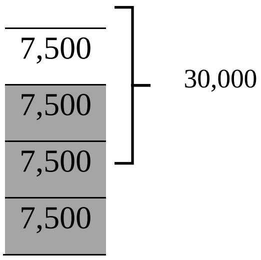
(15 minutes) At this time, share that the rancher has 30,000 cows and that vaccines cost $5 per shot. Have students briefly discuss with a partner how they would calculate the cost of vaccinating the entire herd. Monitor student conversations and pick a student to share a justification that discusses how the rancher needs 30,000 groups of $5 to pay for the shots, meaning the rancher needs $150,000 to vaccinate all of the cattle. If no such justification emerges, you may want to share it as an additional idea or have students share justifications about how the rancher would need to pay $5 per animal.

Explore A

(10 Minutes) After having students share explanations for why multiplication is needed to find the cost of vaccinating the entire herd, establish with the class that multiplication involves finding a number of groups of a certain size. At this time, invite groups to share their chosen fractions to vaccinate along with justifications for their fractions.

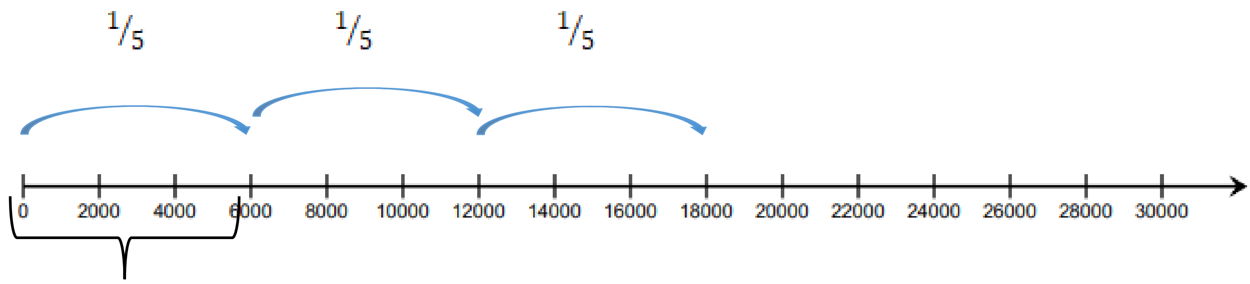
Choose a fraction with a denominator that is a factor of 30,000 (such as ⅔, ⅗, or ⅚), and challenge the entire class to find that fraction of 30,000. If students are struggling, encourage them to first find the unit fraction of 30,000. In other words, if students are struggling to find ⅚ of 30,000, ask them if they can find ⅙ of 30,000 first. Tell students to draw pictures or build models using fraction tiles to justify their answers. Some possible visualizations are shown below.

¾ of 30,000:



$7{,}500 + 7{,}500 + 7{,}500 = 22{,}500$

⅗ of 30,000:

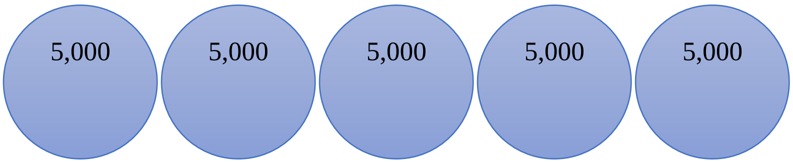


$30{,}000 \div 5 = 6{,}000$

Thus, ⅗ of 30,000 is 18,000.

$\frac{5}{6}$ of 30,000:

$30{,}000 \div 6 = 5{,}000$, so $\frac{1}{6}$ of 30,000 is 5,000



$5 \times 5{,}000 = 25{,}000$, so $\frac{5}{6}$ of 30,000 is 25,000.

Explain B

(10 minutes) Have at least one group present how they found the fraction of 30,000. Make sure their explanation includes some sort of visual model, such as an area model or a model with manipulatives. Students will likely talk about breaking down 30,000 into several groups of equal size then using some of those parts. Connect the word "partition" with the process of breaking 30,000 into groups of equal size. Next, have students brainstorm what operation they were doing when they found a fraction of 30,000. Have students share explanations for why this represents multiplication. If students struggle to generate an explanation, help them recognize that they are finding some fraction of a group of 30,000.

Explore B

(15 minutes) Give students copies of Appendix A. Tell students their job is **not** to solve these problems, but to determine whether they are multiplication problems. Encourage students to be prepared to explain their answers in multiple ways. Have students present justifications for at least three of the problems involving fraction multiplication to help reinforce the idea that fraction multiplication involves taking a group and partitioning it to help find the product. If students finish with this task quickly, have students determine, for each multiplication problem, the best model for representing the problem (out of the classic area, length, and set models).

Extend

(15 minutes) Once students have properly justified which problems represent fraction multiplication, challenge students to find the answer to the last two problems in Appendix A,

with the last involving the product of two fractions. Encourage students to use the "partitioning" method to determine the answer. Students may need help determining the size of one partition. Example solution methods can be found in appendix B.

Evaluate

Make sure to collect students' justifications on Appendix A as a summative evaluation. This will help you determine if students understand the conceptual meaning of fraction multiplication. You may also want to collect student justifications of finding a fractional amount of 30,000 as a summative evaluation as well, for similar reasons. As a formative evaluation, monitor student conversations throughout the lesson. Make sure students understand the definition of multiplication as finding the total number in sets of equal groups, and fractional multiplication as an extension where we are finding a part of a group. This understanding is critical to their ability to correctly use and interpret the results of fraction multiplication problems. If students are not grasping how fraction multiplication is an extension of whole number multiplication, considering reteaching the concept with an emphasis on creating models of fraction multiplication problems.

Appendix A

1. A coffee cup can hold 7/9 of a pint of liquid. If Emily pours 2/3 of a pint of coffee into a cup, how much milk can a customer add?

2. Each large cookie is 5/6 oz and each small cookie is 4/9 oz. What is the total weight of 2 large cookies and 1 small cookie?

3. Josh took out 8 glasses and poured juice from the pitcher. The capacity of each glass is 3/10 liter. If there was enough juice for 6 glasses, how much juice was there?

4. Pam baked some cupcakes for her friends. She baked 24 cupcakes. Each cupcake is 2/15 pound. If she packed 6 cupcakes in each box, what is the weight of each box?

5. There were two cheesecakes in the fridge. The first cheesecake was cut into 12 slices and there are 3 slices left. The other cheesecake was cut into 8 slices and 4 slices were sold. How much cake is left in the fridge?

6. According to a recipe, each batch of pancake mix can make 12 pancakes. Jose is making 3 batches for a brunch party. If each batch needs 7/12 cups of milk, how much milk does he need in total?

7. There is a bag of sugar in the storage room. The bag contained ⅘ pounds of sugar. The chef filled up an empty can with 1/10 pound of sugar and then used 3/20 of a pound of sugar for a cake. How much sugar was left in the bag?

8. According to a recipe, 9/20 oz of sugar is needed to make 6 cookies. Ashley decided to use only 1/3 of the sugar to make it healthier. How much sugar did Ashley use?

9. A local aquarium has a staff of 56 people. 5/8 of the staff works full-time. What fraction of the staff works part-time?

10. Rocky finished a 200-meter race in 5/12 of a minute. The winner of the race used 21/25 of Rocky's time to finish the race. What was the winning time? Give your answer as a fraction of a minute.

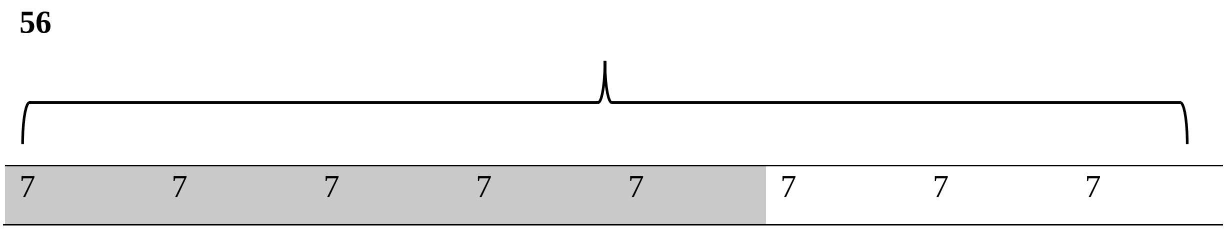
Appendix B

Example solutions

Problem: A local aquarium has a staff of 56 people. 5/8 of the staff works full-time. What fraction of the staff works part-time?

Solution: We need to partition 56 into 8 groups, and then use 5 of those groups to find the answer.

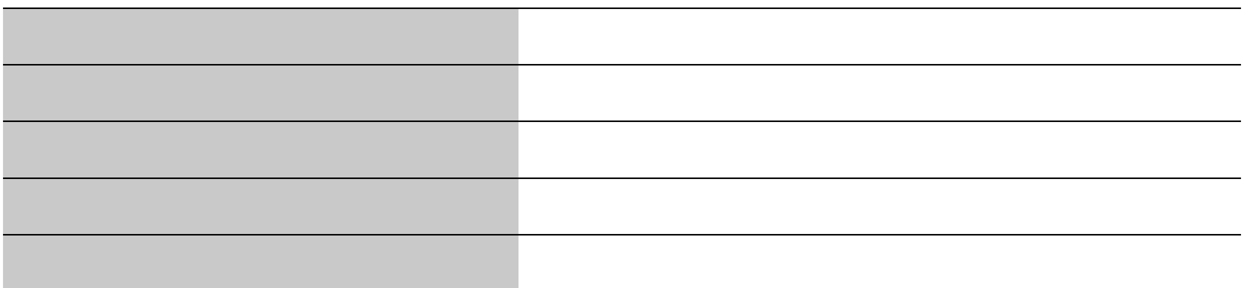
$56 \div 8 = 7$



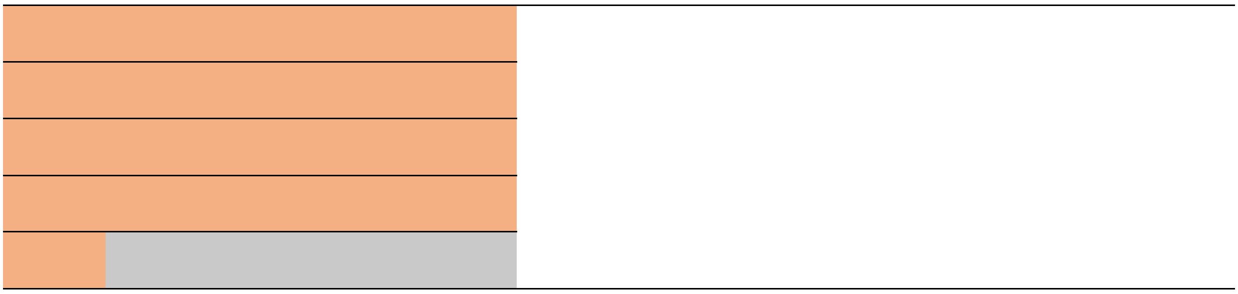
5 groups of 7 gives up 35 people, so 35 people work full-time. This means 21 people work part time.

Problem: Rocky finished a 200-meter race in 5/12 of a minute. The winner of the race used 21/25 of Rocky's time to finish the race. What was the winning time? Give your answer as a fraction of a minute.

Solution: We need to split 5/12 into 25 pieces to figure this out. We can do that by splitting each twelfth into 5 pieces (since $5 \times 5 = 25$).



We then need to highlight 21 of these 25 pieces.



Since each twelfth was cut into 5 pieces, each of the smaller boxes represents $\frac{1}{60}$ of a minute. Thus, the winning time was $\frac{21}{60}$ of a minute or 21 seconds.

Alternative solutions: $\frac{5}{12}$ of a minute is $\frac{5}{12}$ of 60 seconds. If 60 is divided into 12 groups, each group has 5 seconds. 5 groups of 5 seconds yield 25 seconds. Since the winning time was $\frac{21}{25}$ of this time, we can find $\frac{21}{25}$ of 25 seconds, which is 21 seconds.

Citation

Osborne, M., Gundlach, M., & Tudor, M. (2023). Counting Cows. In A. Bicer (Ed.), *Creativity-Directed Mathematical Tasks for 5th Grade Common Core Classrooms* (pp. 297-306). ISTES Organization.

Task 22 - My Flag

Fay Quiroz, Traci Jackson, Aylin S. Carey

Mathematical Content Standards

CCSS.Math.Content.5.NF.B.4

Extend the concept of multiplication to multiply a fraction or whole number by a fraction

CCSS.MATH.CONTENT.5.NF.B.4

Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

CCSS.MATH.CONTENT.5.NF.B.6

Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

Mathematical Practice Standards

CCSS.MATH.PRACTICE.MP3

Construct viable arguments and critique the reasoning of others.

CCSS.MATH.PRACTICE.MP4

Model with mathematics.

CCSS.MATH.PRACTICE.MP5

Use appropriate tools strategically.

CCSS.MATH.PRACTICE.MP6

Attend to precision.

CCSS.MATH.PRACTICE.MP7

Look for and make use of structure.

Vocabulary

Multiplication, numerator, denominator, product, fraction

Materials

Make a copy of appendix B and C for each student.

Lesson Objective

Students will investigate fraction multiplication by exploring examples of area models and length models to multiply whole numbers by a fraction, and to multiply a fraction by a fraction. Mathematical creativity will be represented in how students create their own flags, and then represent their unique designs as equations involving multiplication of fractions. Students will also explain their thinking when they are multiplying a fraction by a fraction. This lesson will be covered in 2 to 3 class times.

Engagement

(15 minutes) Begin the lesson by asking students what they know about flags. Mention that the origin of flags may not be known, however create a discussion on the purpose of flags and what they represent. Can students point out the flags they observe in their city, state, or country? Choose a common flag that students are familiar with and engage them in a conversation. Ask students what they observe. For example, if the flag choice represents the United States of America, look for answers such as the number of stripes and colors, or history and meaning of the stripes and colors.

Show students the following image of flags around the world (see Figure 1). Ask students what they notice about the flags. Some comments to look for are the number of colors (e.g., Some flags have two or three colors while some flags have one color), number of stripes, shapes, or patterns, such as “Some flags have designs in the middle”, “Some flags include shapes like rectangles, triangles, circles or stars”, and “Some flags have a repeating pattern, while some flags have horizontal, vertical, parallel or perpendicular lines.”

Figure 1. Flags from around the World

Note: Retrieved from https://wallpapercave.com/fags-of-the-world-wallpaper. In the public domain.

As students share out, ensure that students are paying attention to not only the designs, but also the colors. Remind them again that designs and colors are purposefully chosen to represent ideas, standards, values, etc.

Tell students in this lesson they will be designing a flag that represents them. Ask students to think about how designs and colors can be integrated into their own flags to help tell the story about who they are, where they have come from, or what their ideologies are.

Explore-Part A

(20 minutes) Students will design flags with their choice of two colors and write multiplication expressions to multiply a fraction by a whole number. Have each student choose two colors and label them as Color 1 and Color 2. Students then choose a flag and

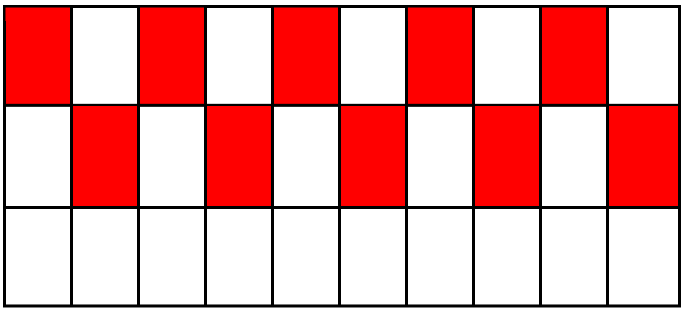
color it to represent themselves (see Appendix B). For this, have students write a short story about the choice of a flag, colors, and their representations. Students select the design and number of sections that they would like to color. For example, students may select Flag 4, and the two colors of Red and Yellow (see Figure 2).

Then students can write the fraction of the flag that is colored red and the equation to match, $\frac{1}{30} \times 10 = \frac{10}{30} = \frac{1}{3}$. Teachers should look for whether students can simplify $\frac{10}{30}\, into\, \frac{1}{3}$. Similarly, students can write the uncolored section in fraction form, $\frac{1}{30} \times 20 = \frac{20}{30} = \frac{2}{3}$. Next students choose how many sections to color with Color 2. Then they write the fraction of color 2 of the uncolored squares. Next they write the fraction of Color 2 of the whole flag. As students are working on their flags, ask what if you used color 2 first? How would that change your equation?

When students are drawing flags, ask if there is a different way they could draw the flag to represent the same equation**.**

Students choose Flag 4 and Color 1 is Red and Color 2 is Yellow.

Student chooses 10 sections for Color 1 (Red)



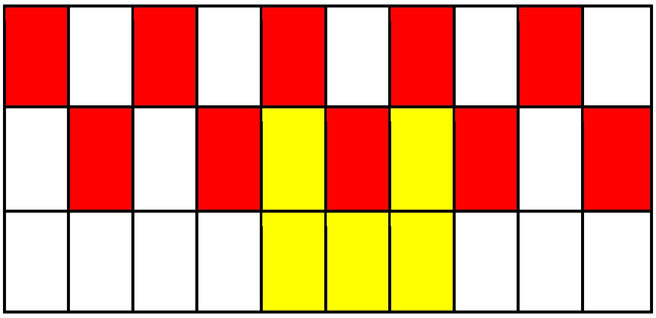
What fraction is red? $\frac{10}{30}\, or\, \frac{1}{3}$

How could you write this as an equation? $\frac{1}{30} \times 10 = \frac{10}{30} = \frac{1}{3}$

What fraction is uncolored? $\frac{20}{30}\, or\, \frac{2}{3}$

How could you write this as an equation? $\frac{1}{30} \times 20 = \frac{20}{30} = \frac{2}{3}$

Next the student chooses 5 sections for Color 2 (Yellow)



What fraction of the uncolored was changed to yellow? $\frac{5}{20}$ *or* $\frac{1}{4}$

What equation could you write to represent the fraction of uncolored that was changed to yellow?

$$\frac{1}{20} \times 5 = \frac{5}{20}$$

What fraction of the flag is yellow? $\frac{5}{30}$ *or* $\frac{1}{6}$

What equation represents the fraction of the yellow part of the flag?

Equation $\frac{5}{20} \times \frac{20}{30} = \frac{100}{60} = \frac{5}{30} = \frac{1}{6}$

Or $\frac{1}{4} \times \frac{2}{3} = \frac{2}{12} = \frac{1}{6}$

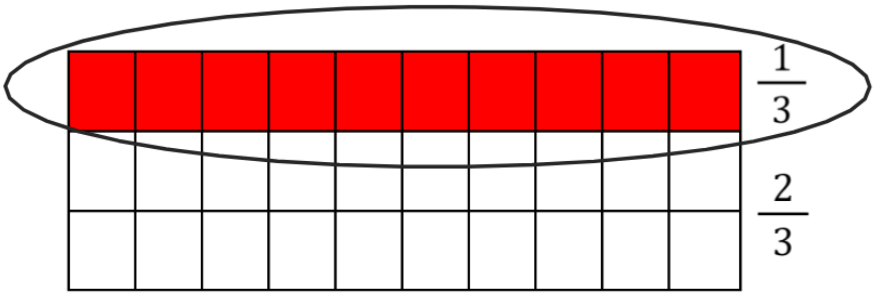
Explain in context. Coloring $\frac{1}{4}$ yellow of the $\frac{2}{3}$ uncolored squares gives me $\frac{1}{6}$ of the whole flag yellow.

Figure 2. Sample Student Process for Flag 1

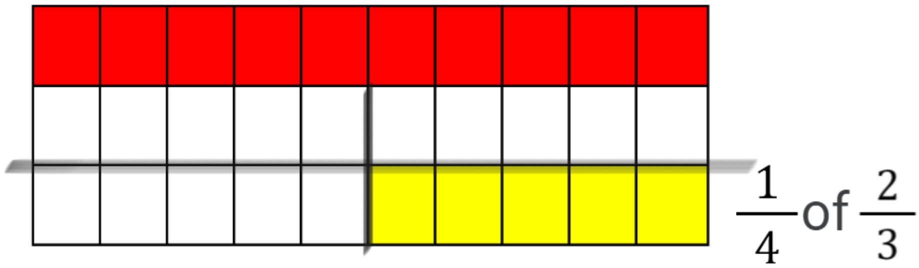
Bring the class together for the teacher to pick a few students with a mixture of Flags 1-6 to share their stories about the flags.

Explain-Part A

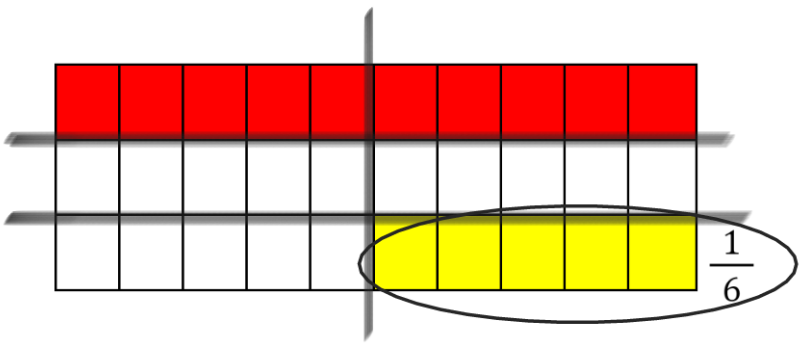
(20 minutes) Teachers can use the example from Figure 2 or another student flag. Have students share how they can visualize the simplified fractions of Color 1 and Color 2. Be sure to connect that when students are coloring the flag with Color 2 (yellow in example), they are coloring a fraction of the uncolored squares. When coloring a fraction (yellow) of a fraction (uncolored) this is equivalent to the fraction of the yellow in the whole flag (see Figure 3).



Shift the red from the second row up to make $\frac{1}{3}$ red and $\frac{2}{3}$ uncolored.



Color 5 yellow of the uncolored. Shift them to see that this is $\frac{1}{4}$ of the $\frac{2}{3}$ uncolored.



The yellow is $\frac{1}{6}$ of the whole flag.

Figure 3. A Student Explanation of how to Visualize $\frac{1}{4} \times \frac{2}{3}$

Use another flag design to have students write the equation for yellow (Color 2) (see Figure 4). Ask students what fraction of the flag is Red (Color 1) $\frac{5}{15}\ or\ \frac{1}{3}$. What equation would represent this? $3 \times \frac{1}{15} + \frac{2}{15}$. Ask what fraction is uncolored before the Yellow (Color 2) was added $\frac{10}{15}\ or\ \frac{2}{3}$. Take time to listen to how students visualize this. Then ask students what fraction of the uncolored portion of the flag was colored yellow, $\frac{3}{10}$. Connect this idea to the overall yellow color by asking the fraction of the flag that is yellow $\frac{3}{15}$ or $\frac{1}{5}$. Ask students to write an equation representing $\frac{2}{3} \times \frac{3}{10} = \frac{2}{10} = \frac{1}{5}$. Continue to ask how students visualize the fractions.

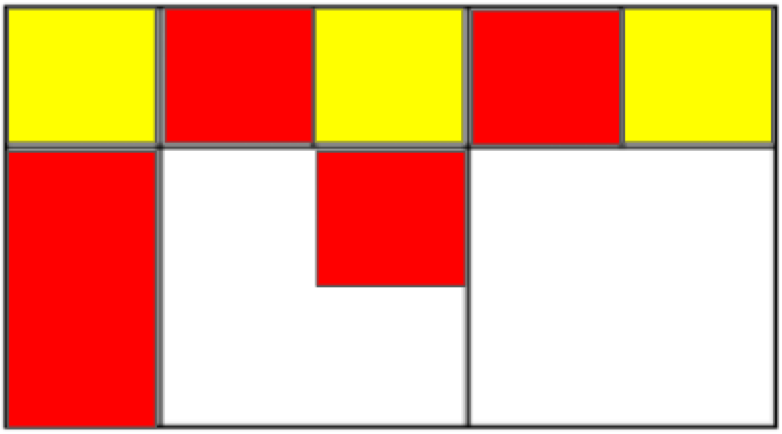


Figure 4.A Sample Design of Flag 6

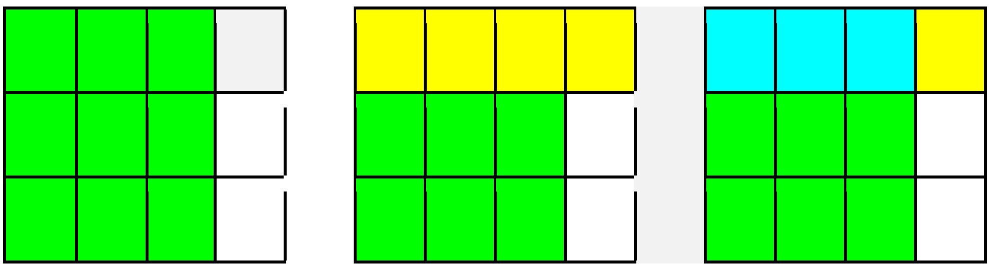
Explore-Explain-Part B

(30 minutes) After students understand how to design a flag, write an equation that includes a whole number multiplied by a fraction, and a fraction multiplied by a fraction using an area model. Now students will be given equations and will create a design for their flags using rectangular area models (see Appendix C).

To begin the class discussion about multiplying a fraction by a fraction using a rectangular area model, focus on Flag 1 and ask students to draw a flag that represents the equation $\frac{3}{4} \times \frac{1}{3}$where students will read the equation as $\frac{3}{4}$ of $\frac{1}{3}$. Students can continue to use the two colors they used from the Explore and Explain- Part A that represented themselves. Tell students Color 1 will represent $\frac{3}{4}$ of their flag and to draw and shade this on Flag 1. Then, tell students $\frac{1}{3}$ will be represented by Color 2. Ask students to think about how they can shade $\frac{1}{3}$ of the $\frac{3}{4}$? A class discussion may be necessary to help students understand how $\frac{1}{3}$ can be drawn and shaded to represent $\frac{1}{3}$ of the $\frac{3}{4}$ (see Figure 5). Teachers may need to give extra guidance to students if they are struggling to see how $\frac{1}{3}$ of only the $\frac{3}{4}$. Teachers should ask students to explain what the overlapping part represents? Hopefully students will see that the overlapping part or $\frac{3}{12}$ is the answer because it shows $\frac{3}{4}$ of the $\frac{1}{3}$. Teachers may connect back to the students first flag from Explore-Explain Part A and connect multiplication can be represented in more than one way, like the rectangular area model.

Flag 1,

Color 1 is green and represents $\frac{3}{4}$ of the flag. Color 2 is yellow and represents $\frac{1}{3}$ of the flag. The blue is the overlapped part or the answer to $\frac{3}{4}\, of\, \frac{1}{3}$.



Color 1 is green= 3/4 Color 2 is yellow= 1/3 Blue is the overlapped part or answer 3/12

Figure 5. An Example of Flag 1 Equation $\frac{3}{4} \times \frac{1}{3}$

As a last step, ask students to write to explain how they knew what to draw based on the equation. Teachers should look for answers that may look like this: “I knew $\frac{3}{4}$ was my first color and that I should split my flag into fourths and color in 3 of the fourths. Then, I looked at how I could split the fourths into thirds, and shade one of the thirds. The overlapping part is $\frac{3}{12}$which represents $\frac{3}{4}\ of\ \frac{1}{3}$.

Now pair students up and ask them to pick three additional equations, out of the six equations (see Appendix C), and draw the flags based on the equation. Students should write or explain how they decided their drawings matched the equation. At the end of this section choose two or three students to volunteer to share their drawings and their explanations about how the different drawings matched the equations. Students can respond to other students by critiquing and responding to their classmates' answers. Teachers can also listen for explanations that demonstrate student understanding about multiplying fractions by a fraction with phrases like $\frac{3}{4} of \frac{1}{3}$. Or, I colored $\frac{3}{4}$ first and then I shaded $\frac{1}{3}$ of the $\frac{3}{4}$ and I knew the overlapped part was $\frac{3}{12}$.

Extend

(40 minutes) It is time for students to explore problems with different models. For example, assuming that one of the students has decided to turn their flag creation to merchandise and would like to calculate the number of flags or prints, cost, and amount of paint/color. The students first have to decide on a pattern and color and then make a plan. Here are some problems to explore with students:

Problem One: $\frac{1}{8}$ *of the flag designed t-shirts are striped.* $\frac{3}{7}$ *of the remainder are plain colored. The rest of the t-shirts are printed. If the student has 72 plain colored t-shirts, how many more printed t-shirts than plain colored t-shirts does the student have?*

For this problem, have students draw a length model (see Figure 6). Look for students' ability in drawing the model with the correct number of parts and labels.

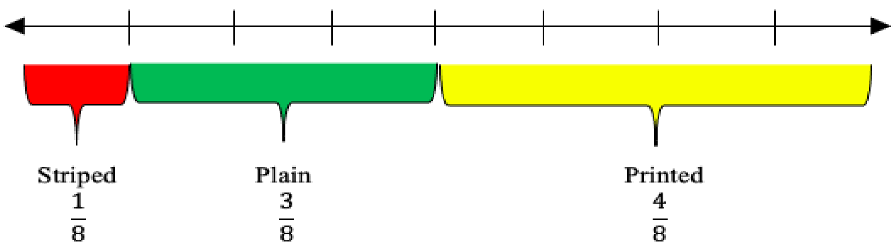


Figure 6. An Example of a Length Model

Math Talk: One part of the number line represents striped t-shirts. Out of the remaining 7 parts: 3 parts represent the plain colored t-shirts and 4 parts represent printed colored t-shirts. If 3 units are equivalent to 72 plain colored t-shirts, one unit must be, $72 \div 3 = 24$. Printed t-shirts have one part more than plain t-shirts. Therefore, the student has 24 more printed t-shirts than plain colored t-shirts.

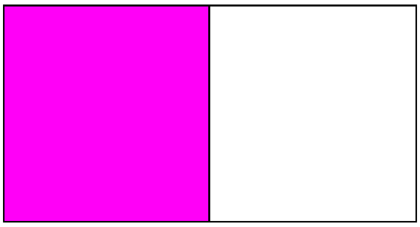
Ask students how they could algebraically translate $\frac{3}{7}$ of the remainder. Listen to their responses to make generalizations for $\frac{3}{7} \times \frac{7}{8} = \frac{3 \times 7}{7 \times 8} = \frac{21}{56}$ and observe their ability to simplify $\frac{21}{56} = \frac{3}{8}\frac{4}{8} = \frac{1}{2}\frac{4}{8} = \frac{1}{2}$.

While it is easier to visualize the calculation on the length model, encourage students to calculate algebraically whenever it is possible, especially if the students are ready to move from concrete to abstract thinking. If students are not ready to make a generalization based on the length model or the previous models they created with the explore problems, have students investigate with more area and length models and ask them share their observations.

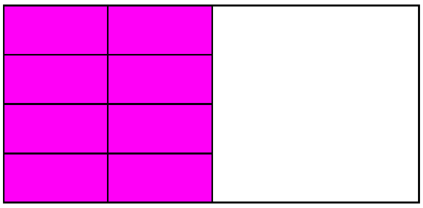
Problem Two: *The student had painted half of the flag pink when their initial idea was to paint in blue. The student immediately started painting over the pink portion of the flag. After 10 minutes, the student had repainted* $\frac{7}{8}$ *of the pink portion blue. What fraction of the entire flag is painted blue after 10 minutes?*

Teachers may need to emphasize that the task is asking for what portion of the total flag is blue, it is not asking what portion of the pink has been repainted. However, by now students should be able to multiply the fractions together to see what fraction of the flag is blue without any given explanation. Algebraically, students can multiply the fractions as $\frac{7}{8} \times \frac{1}{2} = \frac{7}{16}\frac{7}{16}$ of the flag is blue.

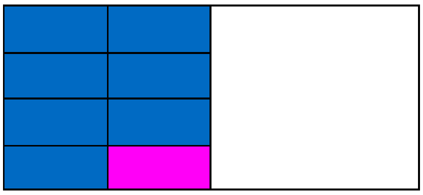
Students who are not ready to solve the problem algebraically can use an area model to present the solution (see Figure 7). The area model helps students see that after 10 minutes, $\frac{7}{16}$ of the flag is blue since 7 pieces of the total 16 are blue. In this problem, teachers can assess students' comfort level with the meaning of fraction multiplication if students easily solve it by multiplying $\frac{7}{8} \times \frac{1}{2}$ or not.



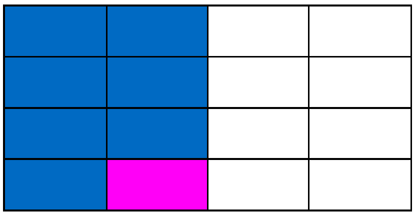
The flag before the student realized they made a mistake.



The student can break up the pink portion into 8 equally sized parts.



The student now can see what the flag looked like after 10 minutes by shading 7 out of these 8 parts blue.



Finally, it is easier to visualize if the student had broken up the flag into 16 equally sized pieces from the beginning, and from there the student can find the fraction of the flag that is blue by counting the number of blue pieces and compare them to the total.

Figure 7. An Area Model Example of a Student's Work

Problem Three: *The student would like to raise money for the classroom to help with the cost of the materials to make flags. A total of 128 flags are sold for* $\frac{4}{5}$ *dollars each and 4 bottles of colors cost* $4\frac{1}{10}$ *each. How much money was raised from the flag sale?*

In this problem, students need to rely on their previous knowledge and be able to multiply mixed numbers. They first need to calculate the total cost of 128 flags while taking the cost of paint into consideration. Look for students' understanding of fraction multiplication such as converting the mixed number to an improper fraction, multiplying numerator/denominator, or finding a common denominator. This problem requires students' knowledge of all fraction operations. Encourage students to solve this problem algebraically without relying on any visualization and emphasize the use of parentheses to group each calculation such as $\left(128 \times \frac{4}{5}\right) - \left(4 \times 4\frac{1}{10}\right)$. Observe how students do the calculation and address any misconceptions such as multiplying $5 \times 4 =$ and then adding 1. Teachers should look for a similar calculation after converting the mixed number to an improper fraction and then finding the common denominator: $\left(\frac{128 \times 4}{5}\right) - \left(\frac{4 \times 41}{10}\right) = \left(\frac{2 \times 512}{2 \times 5}\right) - \left(\frac{164}{10}\right) = \frac{1024 - 164}{10} = \frac{860}{10} = 86$. The first group represents the total cost of 128 flags ($102.4) and the second group represents the cost of 4 bottles of colors ($16.40). Students should be able to understand the task and to reason their findings: The student was able to raise $86.00 for the classroom.

Evaluate

When representing any given fractions, students should be flexible in representation by using manipulatives or drawings. To be able to apply and extend previous understanding of multiplication to multiply a fraction or whole number, students must have fluency with fraction operations. Because students build upon their prior knowledge, they must have a foundation with whole number operations. Fraction operations are not as intuitive as whole number operations; therefore, students must be provided with more opportunities with models or visuals to find different ways of representing the fractions. For example, when students are asked to multiply 4 by 12, do they envision an array or area model? Do they skip count by 10 four times, then skip count by 2 four times? Or do they visualize the calculation in an abstract way: stack and calculate? When students can visualize whole number operations, they can rely on their understanding of division to imagine a whole cut into four parts. They can apply and extend that idea to multiply a fourth by six. Teachers should regularly check these conceptual foundations, including multiplication facts. Teachers should also look for whether students feel comfortable with applying several models to solve a problem and eventually begin solving problems algebraically. Finally, look for students' ability in applying and extending their previous understandings of multiplication to multiply fractions and mixed numbers.

Appendix A

References

Flags of the world wallpapers. (2014-2022). WallpaperCave. Retrieved from https://wallpapercave.com/flags-of-the-world-wallpaper

Appendix B

Flag 1

Fraction of Color 1______

Equation Color 1___________________

Uncolored fraction_________

Uncolored equation ____________

Fraction of Color 2 of uncolored squares

____

Fraction of Color 2 to whole flag______

Equation for fraction of Color 2 of whole

Flag ___________________

Flag 2

Fraction of Color 1______

Equation Color 1___________________

Uncolored fraction_________

Uncolored equation ____________

Fraction of Color 2 of uncolored squares

____

Fraction of Color 2 to whole flag______

Equation for fraction of Color 2 of whole

Flag ___________________

Flag 3

Fraction of Color 1______

Equation Color 1___________________

Uncolored fraction_________

Uncolored equation ____________

Fraction of Color 2 of uncolored squares

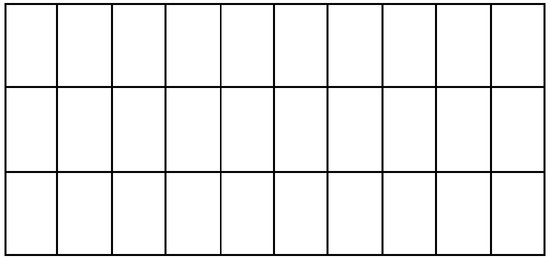
____

Fraction of Color 2 to whole flag______

Equation for fraction of Color 2 of whole

Flag ___________________

Flag 4



Fraction of Color 1______

Equation Color 1____________________

Uncolored fraction_________

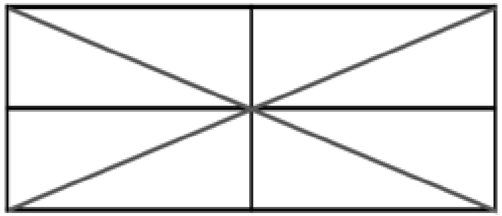
Uncolored equation ____________

Fraction of Color 2 of uncolored squares _________

Fraction of Color 2 to whole flag______

Equation for fraction of Color 2 of whole Flag ____________________

Flag 5



Fraction of Color 1______

Equation Color 1____________________

Uncolored fraction_________

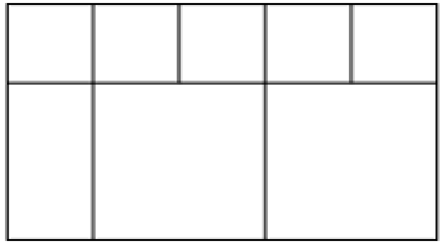
Uncolored equation ____________

Fraction of Color 2 of uncolored squares _________

Fraction of Color 2 to whole flag______

Equation for fraction of Color 2 of whole Flag ____________________

Flag 6



Fraction of Color 1______

Equation Color 1____________________

Uncolored fraction_________

Uncolored equation ____________

Fraction of Color 2 of uncolored squares _________

Fraction of Color 2 to whole flag______

Equation for fraction of Color 2 of whole Flag ____________________

Appendix C

Flag 1	Flag 2	Flag 3
Equation: $\frac{3}{4} \times \frac{1}{3} =$ ________	**Equation:** $\frac{4}{6} \times \frac{2}{5} =$ ________	**Equation:** $\frac{5}{6} \times \frac{3}{4} =$ ________
Explain how you know your drawing matches the equation.	**Explain how you know your drawing matches the equation.**	**Explain how you know your drawing matches the equation.**
______________________________	______________________________	______________________________

Flag 4	Flag 5	Flag 6
Equation: $\frac{3}{5} \times \frac{1}{4} =$ ________	**Equation:** $\frac{2}{4} \times \frac{1}{2} =$ ________	**Equation:** $\frac{3}{8} \times \frac{4}{5} =$ ________
Explain how you know your drawing matches the equation.	**Explain how you know your drawing matches the equation.**	**Explain how you know your drawing matches the equation.**
____________________	____________________	____________________
____________________	____________________	____________________
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Citation

Quiroz, F., Jackson, T., & Carey, A. S. (2023). My Flag. In A. Bicer (Ed.), *Creativity-Directed Mathematical Tasks for 5th Grade Common Core Classrooms* (pp. 307-324). ISTES Organization.

Task 23 - Chocolate Bars

Amy Kassel, Chuck Butler, Jennifer Kellner

Mathematical Content Standards

CCSS.MATH.CONTENT.5.NF.B.3

Interpret a fraction as division of the numerator by the denominator (a/b = a ÷ b). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?

Mathematical Practice Standards

CCSS.MATH.PRACTICE.MP4

Model with mathematics.

CCSS.MATH.PRACTICE.MP5

Use appropriate tools strategically.

CCSS.MATH.PRACTICE.MP7

Look for and make use of structure.

Vocabulary

Native, division, fraction, division equation, fractional representation, numerator, denominator, whole, part

Materials

Whiteboard, number lines, whiteboard markers

Lesson Objective

Students will interpret fractions as division by using context to create models and recognize patterns to show the relationship between division and fractions. This lesson will promote creative thinking by having students create multiple representations that can be used to represent division as a fraction. They will further use their creativity to generate their own story problems to represent division as a fraction.

Engagement

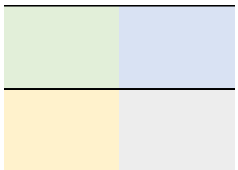
(20 minutes) To engage the students, show them a picture (Figure 1) of the small tropical cacao tree from the website Where Does Chocolate Come From? (Facts about Chocolate, 2014). Tell them the cacao tree will produce nearly two thousand pods per year. It is native to Central and South America, but about 70% of the world's chocolate is grown in Africa.

Figure 1. Picture of a Small Tropical Cacao Tree

Engage students in a discussion about the history of chocolate, such as chocolate dates back nearly 4000 years (Mathkind, 2021). Have the students explore the website and discuss how chocolate is made. The students can share their favorite chocolate food or drink with their classmates.

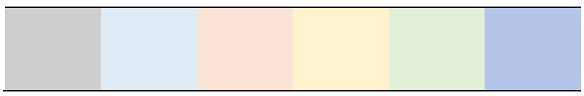
Explore

(20-30 minutes) Part 1: Teachers may start the lesson with a Number Talk by asking "You have a chocolate bar to share with some friends. You want to split it equally among your friends. How much of the chocolate bar will each person get?" Have students draw their models on whiteboards. As students are working on their models, teachers should select different models for students to present. For example, a student may choose to share a chocolate bar among four people and draw the following model (see Figure 2). While another student may choose to share a chocolate bar among six people and draw the following model (see Figure 3). Yet another student may choose to share a chocolate bar among three people and draw the following model (see Figure 4). Another student may choose to share a chocolate bar among eight people and draw the following model see (see Figure 5).



Students may represent 1 chocolate bar with 4 friends.

Figure 2

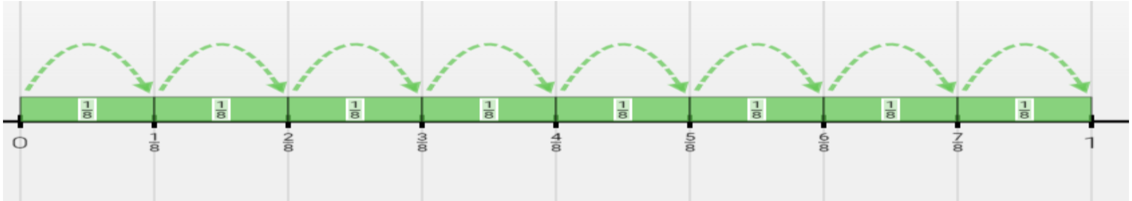


Students may represent 1 chocolate bar with 6 friends.

Figure 3

Students may represent 1 chocolate bar with 3 friends.

Figure 4



Students may represent 1 chocolate bar with 8 friends (The Math Learning Center, 2022).

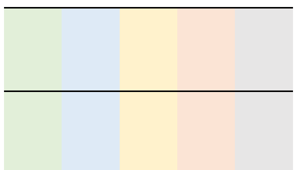
Figure 5

As students share their models, teachers should ask students to share what portion of the chocolate bar each person would get. For example, students may say $\frac{1}{4}$ for Figure 2, $\frac{1}{6}$ for Figure 3, $\frac{4}{12}$ or $\frac{1}{3}$ for Figure 4, and $\frac{1}{8}$ for Figure 5.

Part 2: Teachers have students explore more complex problems by doing another Number Talk by asking "What if you had more than 1 chocolate bar to share with some friends. What fraction of the chocolate bar would each person get?" If students struggle finding an example, teachers may suggest that there are 2 chocolate bars and 5 friends to get them started.

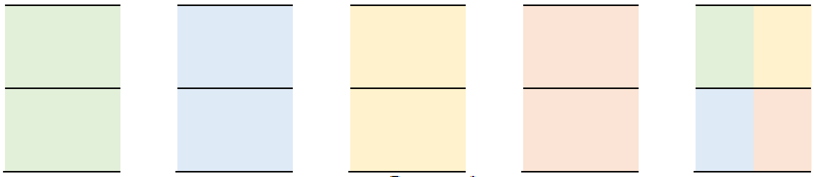
Students may have a variety of examples and models for this part. For example, a student may model 2 chocolate bars and 5 friends as Figure 6. Each bar represents a chocolate bar. Each color represents a friend. Another student may model 5 chocolate bars and 4 friends as Figure 7. Here teachers may remind students that $\frac{5}{4} = 1\frac{1}{4}$ as shown by the colors in the chocolate bars. Yet another student may model 3 chocolate bars and 5 friends as Figure 8. Another student may represent 6 chocolate bars with 4 friends as Figure 9.

As the students are working on their models, teachers should ask students to also write their division equation represented by their model. Teachers should also prepare for the explain section by selecting a variety of models for students to present.



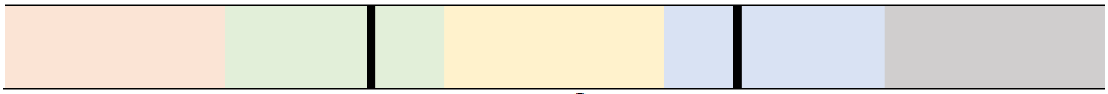
Students may write $2 \div 5 = \frac{2}{5}$ to represent their model.

Figure 6



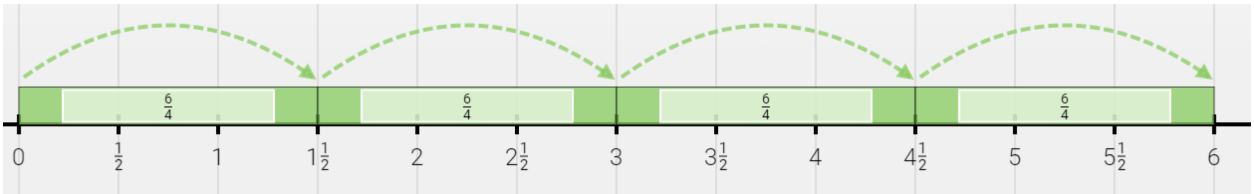
Students may write $5 \div 4 = \frac{5}{4} = 1\frac{1}{4}$ to represent their model.

Figure 7



Students may write $3 \div 5 = \frac{3}{5}$ to represent their model.

Figure 8



Students may write $6 \div 4 = \frac{6}{4} = 1\frac{1}{2} = \frac{3}{2}$ to represent their model (The Math Learning Center, 2022).

Figure 9

Part 3: Teachers may wish to reinforce the connection between models, division, and fractions through different contexts. Teachers may group students and challenge the group to create a story problem where they must share more than one item with multiple people. Then students should create as many different models as possible to represent the scenario and write the division equation to represent their problem. Students can create a mini poster for the classroom display.

Explain

(20-30 minutes) Part 1: As students are presenting their models, teachers may ask students what operation could be used to represent the amount of chocolate bar each person would receive. Students may recognize the operation as division or as a fraction. As students are presenting their models, teachers may record the models on the classroom display along with students' responses to the operation, relating the division expression and the fraction. For example, teachers may record a table such as the following (see Figure 10).

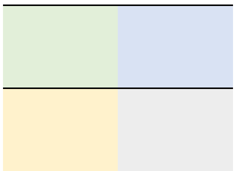
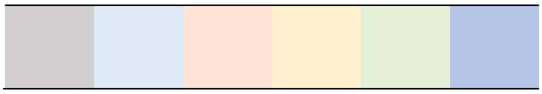
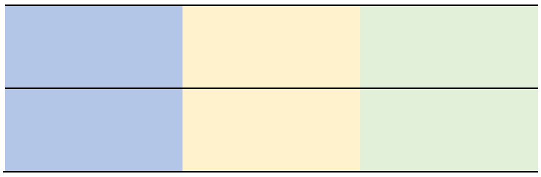
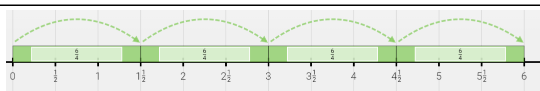
Scenario	Model	Division Equation
1 chocolate bar, 4 friends		$1 \div 4 = \frac{1}{4}$
1 chocolate bar, 5 friends		$1 \div 5 = \frac{1}{5}$
1 chocolate bar, 3 friends		$4 \div 12 = \frac{4}{12}$ or $1 \div 3 = \frac{1}{3}$
1 chocolate bar, 8 friends		$1 \div 8 = \frac{1}{8}$

Figure 10

After students present, teachers may ask students to describe any patterns they noticed. For example, students may observe that all the answers are fractions, or that the numerators represent one chocolate bar, or that the denominators represent the number of people sharing the chocolate bar. Teachers can capitalize on this thinking by asking students to explain why they think this is happening and connecting it back to the context of the problem. Teachers can further extend students' thinking by asking them how much of a chocolate bar each person would get if there were 10, 15, or 18 friends. Teacher should encourage students to generalize the pattern without drawing a model.

Part 2: As students are presenting their models, teachers may record their work on the classroom display so that all data is represented for students to analyze for patterns (see Figure 11).

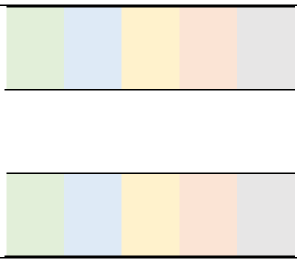
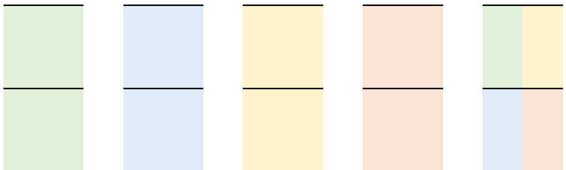
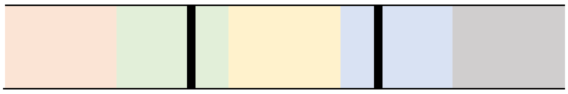
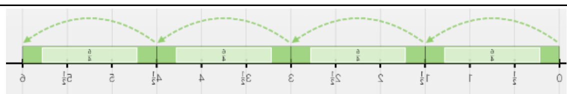
Scenario	Model	Division Equation
2 chocolate bars, 5 friends		$2 \div 5 = \frac{2}{5}$
5 chocolate bars, 4 friends		$5 \div 4 = \frac{5}{4}$
3 chocolate bars, 5 friends		$3 \div 5 = \frac{3}{5}$
6 chocolate bars, 4 friends		$6 \div 4 = \frac{6}{4} = 1\frac{1}{2} = \frac{3}{2}$

Figure 11

After students present their models and the data is recorded, teachers may ask students what patterns they see. Students may state that the number of chocolate bars is represented in the numerator, and the number of friends is represented in the denominator. Teachers may restate this as the number of wholes is represented by the numerator, and the number of parts that you divided the whole into is represented by the denominator.

After students have made the connection of the fractional representation of wholes and parts in context, teachers should then help students interpret fractions as division. To make the connection of interpreting fractions as division within the context of the problem, teachers should ask students questions such as "Is dividing 2 wholes by 5 parts the same as $2 \div 5$?" and "Can you show that in your model?".

Part 3: Teachers may select representations from the mini posters for student presentations asking students to share their story problem, their representations, and how their representation connects to the division equation.

Extend

Teachers may choose to assign one extension to the whole class or assign different extensions to different groups based on interest, ability, and readiness.

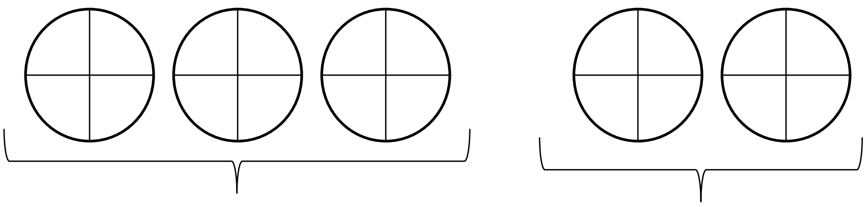
Two More – After reviewing other student's methods at the end of part 2, have students return to their scenario and solve their scenario visually in two different ways.

How Much More? If 2 boys share 8 chocolate bars and 3 girls share 11 chocolate bars, who gets more? How much more do they get? Solve visually.

B1	**B1**	**B1**	**B1**	**B2**	**B2**	**B2**	**B2**

<table>
<tr><td rowspan="3">G1</td><td rowspan="3">G1</td><td rowspan="3">G1</td><td rowspan="3">G2</td><td rowspan="3">G2</td><td rowspan="3">G2</td><td rowspan="3">G3</td><td rowspan="3">G3</td><td rowspan="3">G3</td><td>G1</td><td>G1</td></tr>
<tr><td>G2</td><td>G2</td></tr>
<tr><td>G3</td><td>G3</td></tr>
</table>

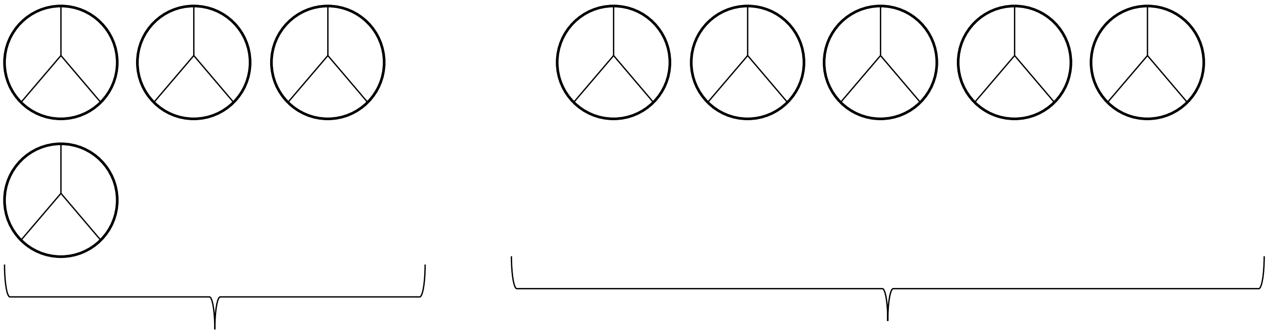
Two Cookies - 4 people share 3 chocolate chip cookies and 2 oatmeal cookies. How much did each person get of each cookie?



Chocolate Chip Cookies

Oatmeal Cookies

One Share - 9 people shared cookies. If one person's share was $\frac{1}{3}$ chocolate and $\frac{2}{3}$ oatmeal cookie, how many of each cookie did the group share? Solve using two different representations.



Evaluate

- In part 1, formative assessment for creativity and content occurs as students are visualizing the context, deciding on the number of friends to share one candy bar, and drawing their representation. Teachers will look for students to cut the candy bar into equal size pieces, according to the number of friends. If a student cuts the candy bar into different-sized pieces, then teachers can ask "What does it mean if you share the chocolate in this way?"
- In part 2, formative assessment for creativity and content occurs as students are visualizing the context, deciding on the number of friends and candy bars, and drawing their representation. Formative assessment for content occurs as students are writing their division equation represented by their model. For example, $5 \div 4 = \frac{5}{4}$, teachers can ask "What does 5 mean in our story? What does $\frac{5}{4}$ mean in this context? If a student incorrectly writes the division statement as $4 \div 5 = \frac{4}{5}$, teachers can ask "What does 4 mean in this story?" or "What are we sharing in this story?"
- In part 3, formative assessment for creativity and content occurs as students are creating the story, deciding on the units, drawing several visual representations, and creating their division equation. For example, teachers could show a student's visual representation and ask "What could this story be about?" or "What could the division statement be for this representation?"
- In Two More, formative assessment for creativity and content occurs when students try to express the same context in two more visual representations. For example,

teachers could ask "How else can we cut the chocolate bars to share with our friends?"

- In How Much More, formative assessment for creativity and content occurs as students represent their thinking visually to decide who gets more chocolate bars. For example, teachers could ask "How do you see the boys sharing the chocolate?" or "How do you know how much chocolate to share with each girl?" Formative assessment for content also occurs as students are using equivalent fractions and subtraction to determine how much more one group gets than the other.
- In Two Cookies, formative assessment for content occurs as students are finding an equal share of two different units. Instead of sharing the chocolate and oatmeal cookie individually, a student might share the five cookies with four people by writing $5 \div 4 = \frac{5}{4}$. Teachers can use questioning to reveal what the student's current representation says about the context by asking "What does $\frac{5}{4}$ mean in this context?" or "How much did each person get of each cookie"
- In One Share, formative assessment for content and creativity occurs as students are using one share to find the total amount of cookies using two different representations. For example, teachers could ask "How do you visualize one share?" and "How can you represent one share using an equation? A student might write the story as $9 \div \frac{1}{3}$ or $\frac{1}{3} \div 9$. In either case, teachers could use questioning to reveal what the student's current representation says about the context by asking "What does $\frac{1}{3} \div 9$ mean?"

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Facts About Chocolate. (2014, January 5). *Where Does Chocolate Come From? Does it grow on trees?* Retrieved February 28, 2022, from Facts About Chocolate: https://facts-about-chocolate.com/where-does-chocolate-come-from/

Mathkind. (2021). *Switzerland A Chocolate Chronology*. Retrieved February 28, 2022, from Mathkind: https://mathkind.org/global-math-stories/switzerland/

The Math Learning Center. (2022). *Number Line*. Retrieved February 10, 2022, from https://www.mathlearningcenter.org/apps/number-line

Citation

Kassel, A., Butler, C., & Kellner, J. (2023). Chocolate Bars. In A. Bicer (Ed.), *Creativity-Directed Mathematical Tasks for 5th Grade Common Core Classrooms* (pp. 325-336). ISTES Organization.

Task 24 - Designing a Kitchen

Geoff Krall, Helen Aleksani

Mathematical Content Standards

Focus Content Standard

CCSS.MATH.CONTENT.5.NF.B.4.B

Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

CCSS.MATH.CONTENT.5.NF.B.6Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

Supporting Content Standard(s)

Mathematical Practice Standards

CCSS.MATH.PRACTICE.MP2

Reason abstractly and quantitatively.

CCSS.MATH.PRACTICE.MP6

Attend to precision.

CCSS.MATH.PRACTICE.MP7

Look for and make use of structure.

Lesson Objective

Students will use multiplication of fractional side lengths to determine the dimensions of a kitchen. Afterward, students will demonstrate mathematical creativity by designing a kitchen of their own using physical or digital tools. Student creativity will be used to craft the layout of the kitchen by drawing diagrams and creating new and unique shapes.

Engagement

(15 minutes) The day before the lesson, have students go home and sketch out the floor plan of their kitchen. If they can measure the dimensions with a ruler or meter stick, have them do so and create a drawing to bring to class the next day. Students may also estimate dimensions using their steps or the tiles on the floor (many tiles have lengths of one foot).

The day of the lesson, have students compare their drawings. Check to see if any students used fractions in their drawings. Use this time to highlight the brilliance of a student who could use a boost in confidence.

Once you choose a selection of student work to highlight, ask the student and the class some of the following questions:

- How many sides does the floorplan have? What shape does that make it?
- What kind of flooring does your kitchen have? Is it tile? Wood? Carpet? Concrete?
- How did you come about your dimensions? Did you measure or estimate?

Explore

(20 minutes) Show students the layout of the kitchen floor (see Figure 1) and ask them to describe the dimensions. You may start by asking "what do you notice about this shape?" Students may respond with the following:

- The layout has six sides (making it a hexagon).
- The shape has a lot of squares.
- There are nine tiles along the top.
- There are 6 ½ tiles along the left side.
- There are 42 whole tiles and nine partial tiles.
- There are 8 half-tiles and 1 one-quarter tiles.

Use the Think-Pair-Share routine to have students answer the questions: How many tiles will I need to tile the entire kitchen, if I can cut and use tiles (i.e., I can cut one tile into two half-tiles and use both)?

Think: Silently think about the question for 1 minute.

Pair: Discuss your ideas with your neighbor for 2 minutes.

Share: Share your ideas with a small group.

Once students have engaged in conversation about their strategies, have them decide on the best strategy with a solution.

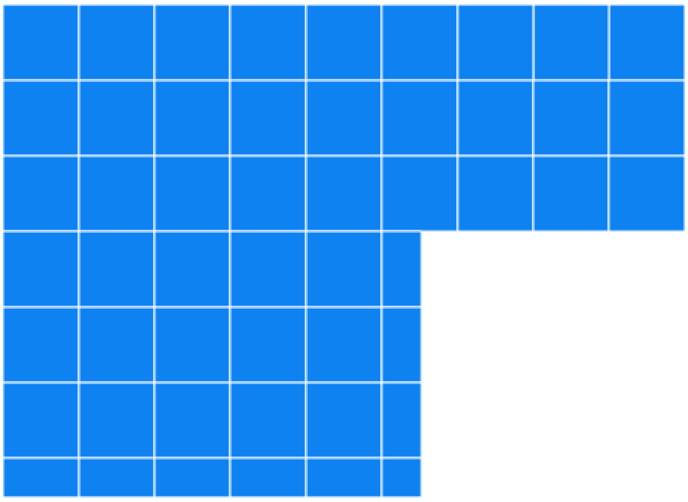


Figure 1. Layout of the Kitchen

Explain

(30 minutes) Tell students we are going to do the same process, but with a slightly more complicated shape. Show students Figure 2. Once again, ask students how many tiles we would need to cover this floor.

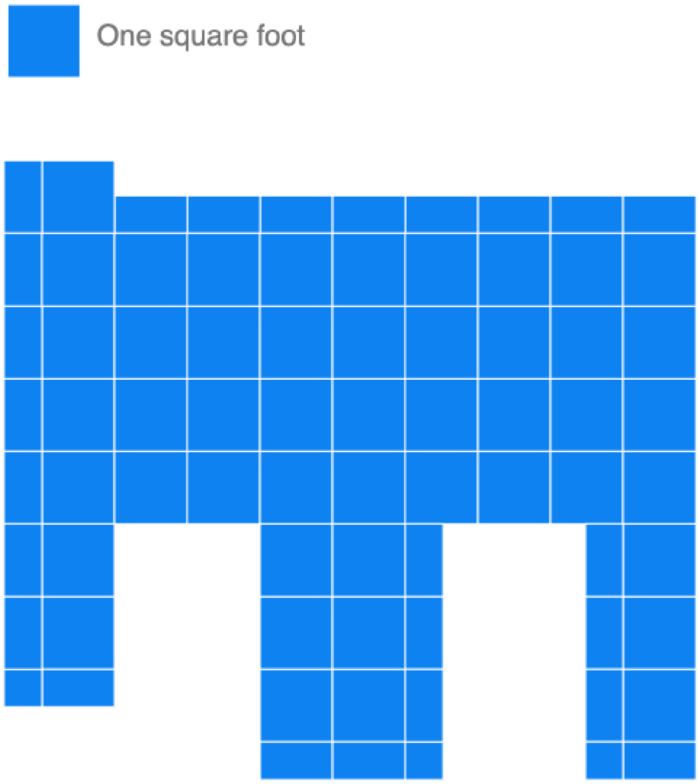
Then tell students the following:

"Uh oh! The owner of this kitchen has decided NOT to use tiles, but instead to PAINT the floor! Instead of individual tiles, we must find the AREA of the shape! Use your previous work, your knowledge of areas to find the following:

1) Each of the side lengths of the floor.

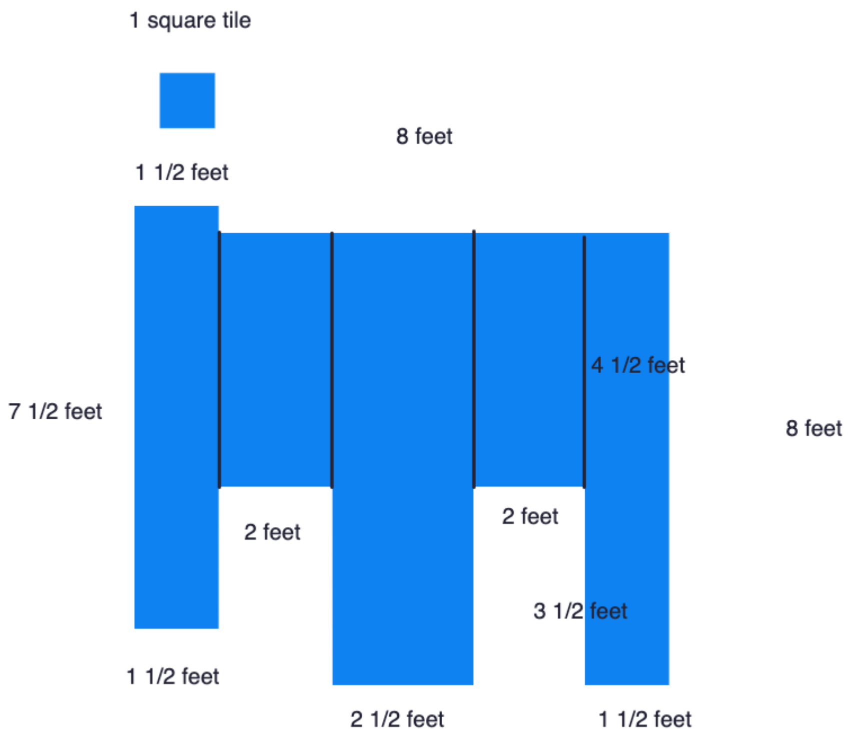
2) The entire area of the floor

It might help to cut the floor into several pieces to create a compound area."


One square foot

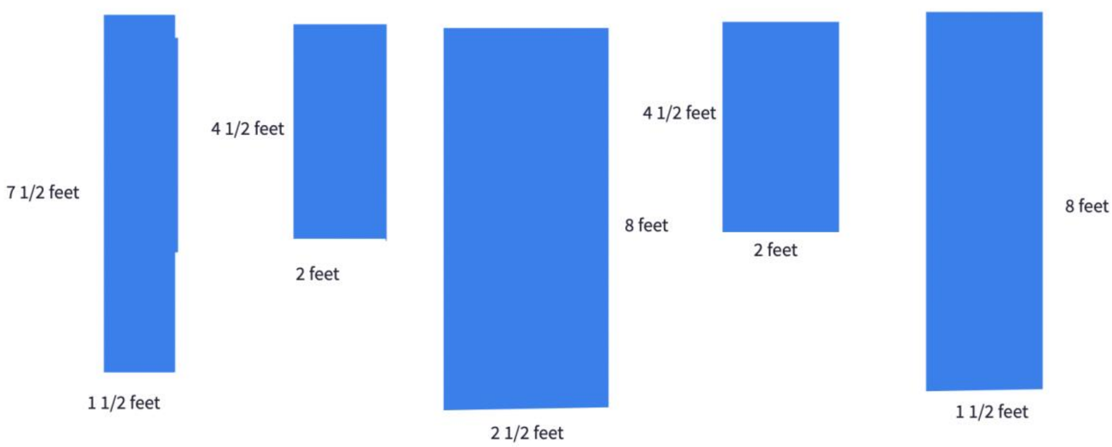

One square foot

Students will have to break up the shape and identify the lengths of the sides. Here is an example:

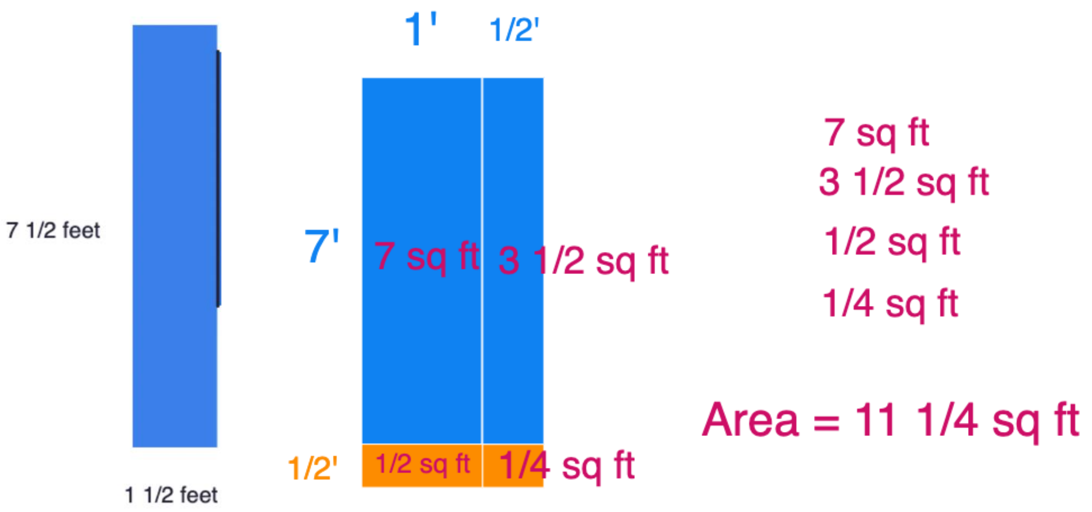


In order to help students organize their ideas, use the recording sheet in appendix A. Students will need to find the area of rectangles that include fractional side dimensions.

Using the diagram above, a student may wish to draw and calculate the areas of each individual rectangle.



Work through an example with students on how to find the area of a rectangle with fractional side length. For example, work through the leftmost rectangle in this kitchen diagram, with the aid of an area model.



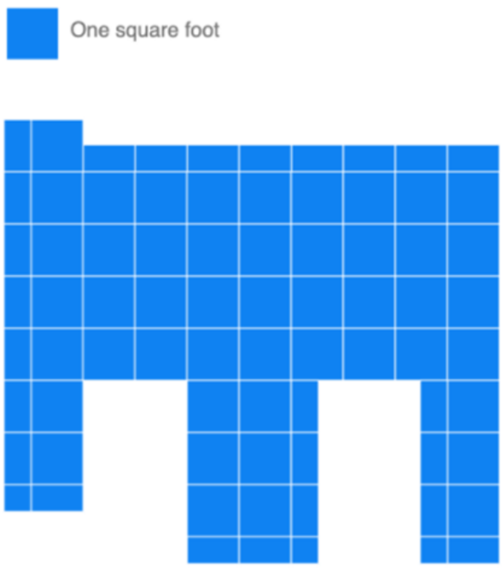
Extend

(20 minutes) In order to allow students to demonstrate mathematical creativity, have students design *their own* kitchen floor with fractional side lengths. Students may wish to use a digital tool such as polypad (mathigon.org/polypad), pencil and paper, or grid paper. Give students the following prompt:

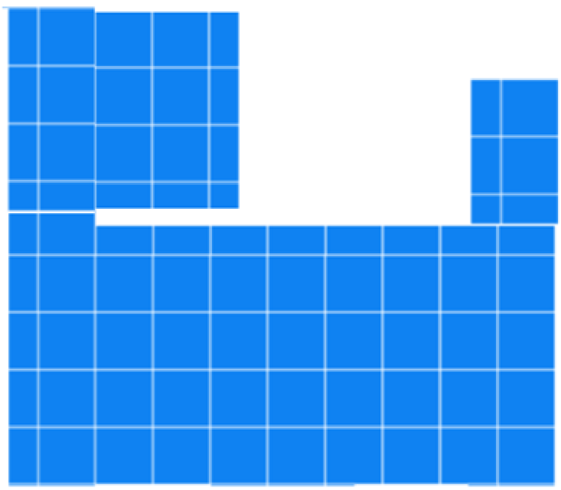
> Now you will be designing your very own kitchen layout. Using a tool of your choice, draw a floorplan of your ideal kitchen. The only requirements are as follows:
>
> - The kitchen must have at least seven sides.
> - The kitchen must have at least two side lengths that are not integers.
>
> Once you design your kitchen, use another recording sheet to calculate the area of the individual sections as well as the entire kitchen.

To encourage students to extend their understanding of the concept further, have them think of remodeling the kitchen. Ask the students to draw a model of the remodeling idea they have on paper. Students can work with their groups or a partner to get some ideas and share their thoughts. Once students finish drawing their new model, have them repeat the steps and the process explained throughout the lesson. Once they find the area of the new model, ask students to compare the two areas, the current model and the new one. Have them share their findings with the class (see Figure a &b).

Current model



New model



Then, share with the students the cost of one tile: $5. Ask students to calculate the cost of tiling the current kitchen floor and record their answer. Then, ask the students to calculate the cost of tiling the new kitchen floor and record their answer. Ask the students to compare the two models and share their findings with the class.

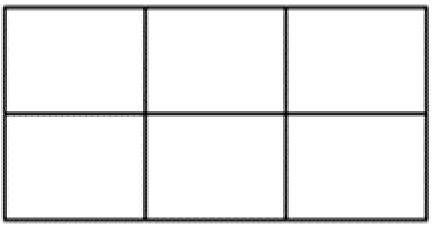
Evaluate

While students are working through the second shape, look for students who are struggling multiplying by fractional side lengths. Evidence of struggling may include the following:

- Multiplying by whole numbers instead of including the additional fraction (i.e. a student wants to multiply 2' x 3 ½' and only multiplies 2' x 3').
- Truncating (leaving off) the fractional side length in the answer (i.e. a student writes the answer 3' x 4 ½' = 12 sq ft.
- Not partitioning the kitchen in a way that is helpful for the final outcome.

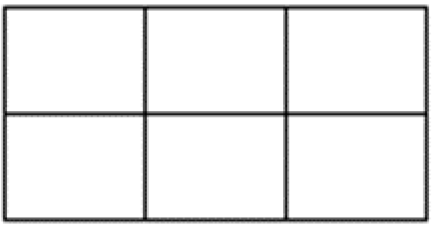
Teachers should draw upon the learning students experienced when working only with the tiles in the first kitchen diagram.

When working on the extended part where students are asked to find the area of the kitchen floor, look for students who might struggle with finding the area. We can review finding the area of a shape before having students start working on this section. For instance, we can ask students to find the area of a rectangle by counting the number of titles.



Area is 6 units squared

Also, students might struggle finding the total cost when titling the kitchen floor. To help eliminate some confusions, model how to find the cost when reviewing the area of a rectangle. For example, if the cost of one tile is 3$, what is the cost of tiling this model?



Cost = area x 3= 6 x 3 = 18$

Appendix A

Area recording sheet

Name: ________________

Kitchen Area Recording Sheet

Sketch a picture of the kitchen floor here. Be sure to include all important side lengths.

Draw additional lines to help you find the area of each section of the kitchen.

Record the dimensions and areas here. (You may not need to use all rows of the table).

Section Dimensions	Area calculation	Area of section

Total area of the kitchen:

Citation

Krall G. & Aleksani, H. (2023). Designing a Kitchen. In A. Bicer (Ed.), *Creativity-Directed Mathematical Tasks for 5th Grade Common Core Classrooms* (pp. 337-348). ISTES Organization.

SECTION 7 - APPLY AND EXTEND PREVIOUS UNDERSTANDINGS OF MULTIPLICATION AND DIVISION TO DIVIDE FRACTIONS

Task 25 - Where's the Beef?

Melena Osborne, Michael Gundlach, Michelle Tudor

Mathematical Content Standards

CCSS.MATH.CONTENT.5.NF.B.7

Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.

CCSS.MATH.CONTENT.5.NF.B.7.A

Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3.*

CCSS.MATH.CONTENT.5.NF.B.7.B

Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4.*

Mathematical Practice Standards

1. MP 2 Reason abstractly and quantitatively.
2. MP 3 Construct viable arguments and critique the reasoning of others.
3. MP 4 Model with mathematics.
4. MP 8 Look for and express regularity in repeated reasoning.

Lesson Objective

Students will explore and learn how to divide whole numbers by unit fractions and then extend this to whole numbers divided by any fraction. Students will then explore and learn how to divide unit fractions by whole numbers and then extend this to any fraction divided by a whole number. Creativity will be emphasized by letting students explore the relationship between fraction and whole number division and multiplication. Additionally, creativity will be emphasized by challenging students to develop an algorithm for fraction division. They can use models and/or manipulatives to help develop the algorithm.

Vocabulary

Fraction division, unit fraction

Materials List

Graph paper, colored pencils, fraction tiles, plain paper for folding

Engagement

To engage students, have them watch this short news video clip from December, 2021: https://www.youtube.com/watch?v=4Hz_0CJZZKY

Tell students that since retail beef prices have increased by over 20%, they are going to butcher one, two, or three beef cows to either divide between their family or their family and friends to save some money. The average price of a butchered beef cow is $4.50-$7.00 a pound (this includes hamburger meat, steaks, roasts, etc.), which is much cheaper than what you pay at the grocery store. (Let students know this also supports their local farmer!) The average amount of meat from a whole beef is 750 lbs. Share this with students and have students discuss how much beef (how many lbs.) they think their family would eat. Ask them how they could divide the 750 lbs. between multiple families (or family and friends). Pick a few students to share their ideas with the class and their reasoning.

Explore A

Give students the following problem: “There are 2 lbs. of steak that need to be divided into ½ pound portions. How many $\frac{1}{2}$ pound portions can be made?” Give students 2 pieces of blank paper to represent the 2 lbs of steak. Have them fold each one in half. How many halves are in one whole? How many in two? How many $\frac{1}{2}$ pound portions are in 2 pounds? Ask students how many portions can be made if we want $\frac{1}{4}$ pound portions. Ask them to fold the paper again to find the answer. After students have folded the paper, ask them to show a way to solve the problem using a different model such as strips, sets or area models. This can be done for any of the numerical problems given above. Additionally, you could have students answer problems of the form, “2 lbs. of steak are $\frac{1}{2}$ of the portion given to one family. How many pounds of steak did they receive?” Have students share and explain their models to a partner and then discuss as a group some of the different strategies that were used.

Explain A

Have students discuss with a partner how they can extend what they learned in the Explore section to the following problems.

$2 \div \frac{1}{2}, 3 \div \frac{1}{3}, 4 \div \frac{1}{4}$

Ask students to think about how they could model/draw these examples. They could possibly think about the definition of multiplication to help them here if they are struggling. For example, the students could rewrite the division problems as multiplication problems and use the definition of multiplication to help model the examples. Once the students have their models (or you have done them as a whole class), give them several more examples $18 \div \frac{1}{2}, 18 \div \frac{1}{3}, 18 \div \frac{1}{6}$ and have students create models for these quotients. Some examples of the student models are found in Appendix A. Once they have their models, challenge students to rewrite the above examples as multiplication problems and look for a relationship. Have students share ideas and observations.

Explore B

Go back to the engagement problem. Have students, in small groups, discuss how much of the cow a family would get if half of the cow is divided among 4 families. Have students also determine how much of the cow a family would get if a third of the cow or a fourth of a cow is divided among four families. Encourage students to use models or manipulatives to help them find these quotients. Ask two or three groups to share their models. These groups could be volunteers or the teacher could select groups based on unique and creative models.

Explain B

Have students discuss with a partner how the following problems relate to their exploration.

$\frac{1}{2} \div 4, \frac{1}{3} \div 4, \frac{1}{4} \div 4$

Ask students to think about how they see these problems in their models from the exploration. They could possibly think about the definition of multiplication to help them here if they are struggling. Once the students have their models (or you have done them as a whole class), give them several more examples such as $\frac{1}{2} \div 12, \frac{1}{3} \div 18, \frac{1}{4} \div 24$ and have students create models for these fractions. Some examples of the student models are found in Appendix B. Challenge students to rewrite the above examples as multiplication problems and look for a relationship. Have students share ideas and observations.

Extend

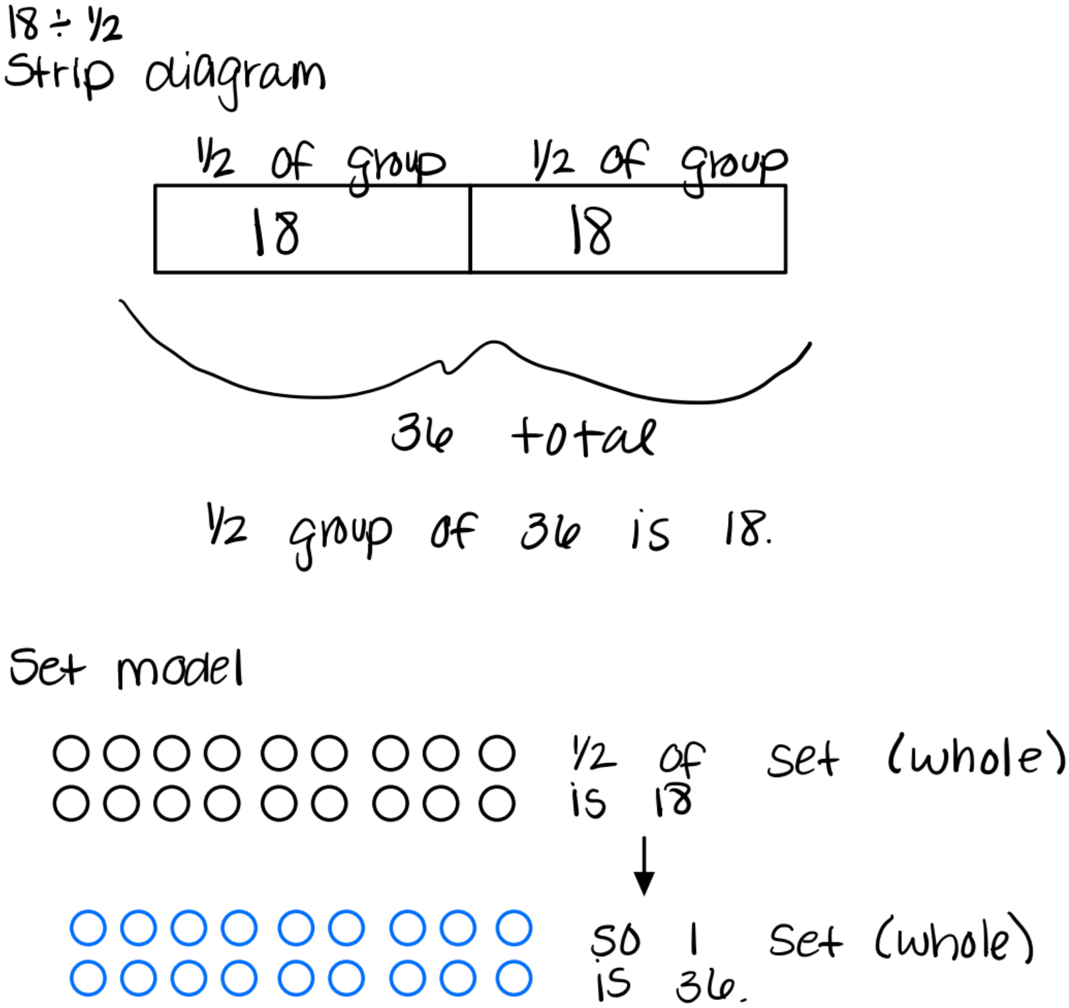
Have students discuss the following problem: "$\frac{3}{2}$ lbs of ground beef needs to be divided into $\frac{1}{4}$ lb hamburger patties. How many patties can be made?" Have students use models to determine an answer to this problem. Have students look at answers to problems where a number is being divided by $\frac{1}{4}$ and then see if they can find a pattern in the answers. Have students similarly look for patterns when dividing a fraction by 4, and have students write general rules for how we divide a fraction by 4 and by $\frac{1}{4}$. At this point, the students may notice that dividing by fractions is the same thing as multiplying by the reciprocal. For example, $\frac{3}{2} \div \frac{1}{4} = \frac{3}{2} \times \frac{4}{1}$. Once the students have developed this algorithm, go back to some of the previous examples in this lesson and have students "check" their answers using the standard algorithm.

Evaluate

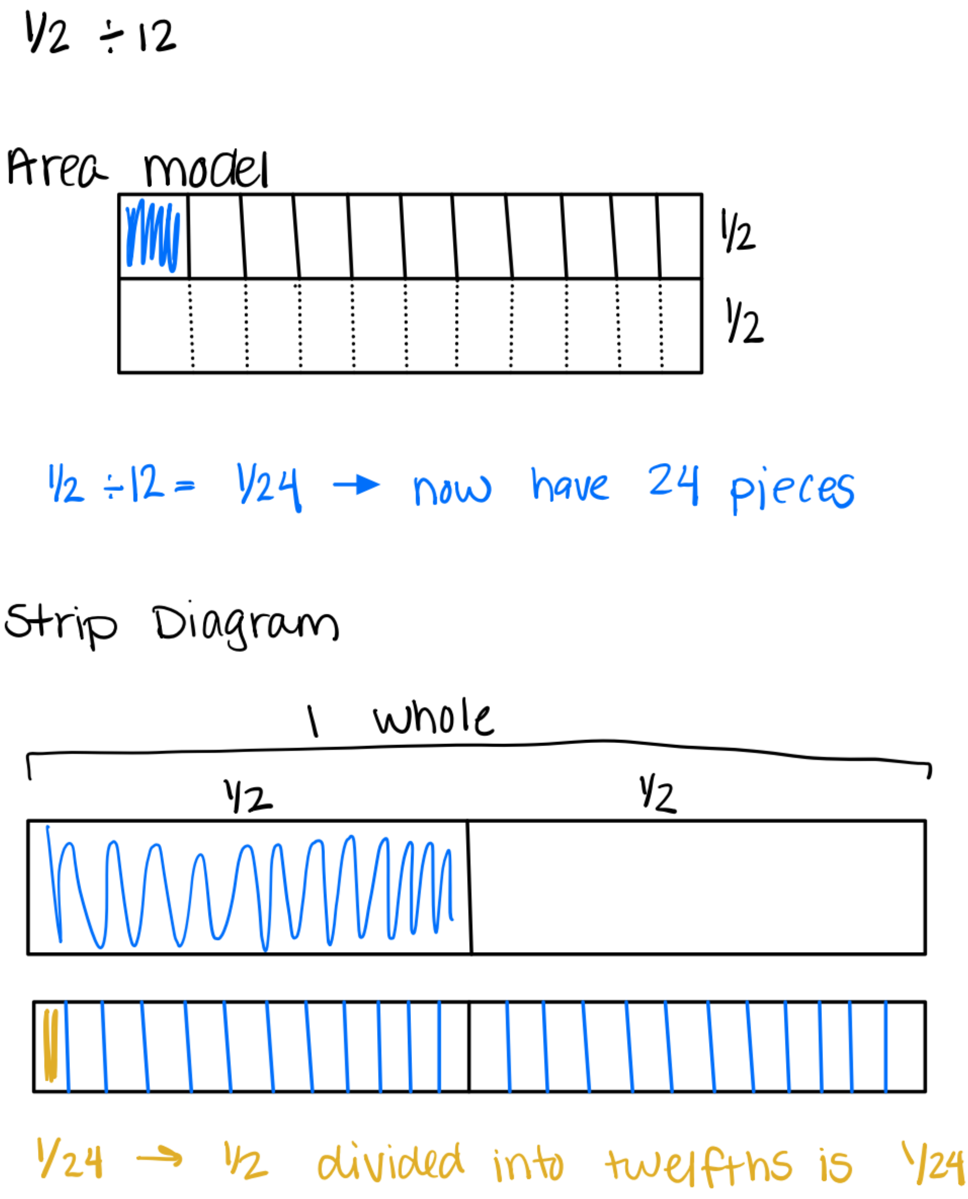
As a formative evaluation, monitor student work on generating solutions to the fraction division problems. Make sure they are using mathematically correct models and using the context of the problem or of the model to get a solution. Discourage students from trying to "shortcut" the problem-solving process by just using a calculator or some other method to quickly get an answer. The goal here is for students to develop a deep understanding of fraction division and to avoid just learning how to invert and multiply without questioning why.

As a summative evaluation, have students each select one model of a fraction division problem to turn in. This can be turned in through a paper copy of a model, taking a picture of a model, or even having the teacher evaluate a physical model before the manipulatives are put away. Make sure the emphasis in evaluation is on the use of the model, not just on the correct answer. As mentioned before, the goal of this lesson is to help students understand the meaning of fraction division through a context rather than just teaching the fraction division algorithm.

Appendix A


18 ÷ 1/2
Strip diagram
1/2 of group
1/2 of group
18
18
36 total
1/2 group of 36 is 18.
Set model
1/2 of set (whole)
is 18
so 1 set (whole)
is 36.

Appendix B



Citation

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Task 26 - Camping in the Woods with Fractions

Traci Jackson, Aylin S. Carey, Fay Quiroz

Mathematical Content Standards

CCSS.MATH.CONTENT.5.NF.B.7.B

Interpret division of a whole number by a unit fraction and compute such quotients. For example, create a story context for 4 ÷ (1/5), and use a visual model to show the quotient.

CCSS.MATH.CONTENT.5.NF.B.7.C (partial)

Solve real world problems involving division of unit fractions by non-zero whole numbers and whole numbers by unit fractions, e.g. by using visual fractions and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share 1/2 pound of chocolate equally? How many 1/3 cup servings are in 2 cups of raisins?*

Supporting Standards (extend)

CCSS.MATH.CONTENT.5.NF.B.7.A

Interpret division of a unit fraction by a non-zero whole number and compute such quotients. For example, create a story context for ⅓ divided by 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/4) ÷ 4 = 1/12 because (1/2) x 4 = 1/3

CCSS.MATH.CONTENT.6.NSA.1

Interpret and compute quotients of fractions and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d)= (ad)/(bc). *How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4 -cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?*

Mathematical Practice Standards

CCSS.MATH.PRACTICE.MP4

Model with mathematics.

CCSS.MATH.PRACTICE.MP5

Use appropriate tools strategically.

CCSS.MATH.PRACTICE.MP7

Look for and make use of structure.

Materials

Fraction bars, Number lines, Printouts of Appendix A, measuring cups (optional), and Virtual manipulatives can be found https://mathigon.org/polypad

Vocabulary

Area model, tape diagram, s'more, campsite, serving size¸ quartz crystals, feldspar

Lesson Objectives

Students will explore models to justify how dividing a whole number by a fraction is equivalent to multiplying by the reciprocal through various models and generalize dividing a whole number by a unit fraction.

Students' creativity is encouraged by using multiple representations, comparing and contrasting different methods, generalizing dividing a fraction by a whole number, and creating their own problems.

Engagement

(20 minutes) Ask students if they have ever been camping or to an organized camp.

The teacher then tells a story while drawing a few illustrations on the board (see Figure 1). A student had 12 pieces of chalk that they are going to use to create a design outside the camp to welcome all of the campers. All 12 pieces of chalk fell on the ground. 3 leaves fell from one of the trees nearby and each leaf covered an equivalent amount of chalk. However, the leaves didn't cover all the chalk, 4 whole pieces remained uncovered. Have students work together to decide what the chalk under each leaf looks like.

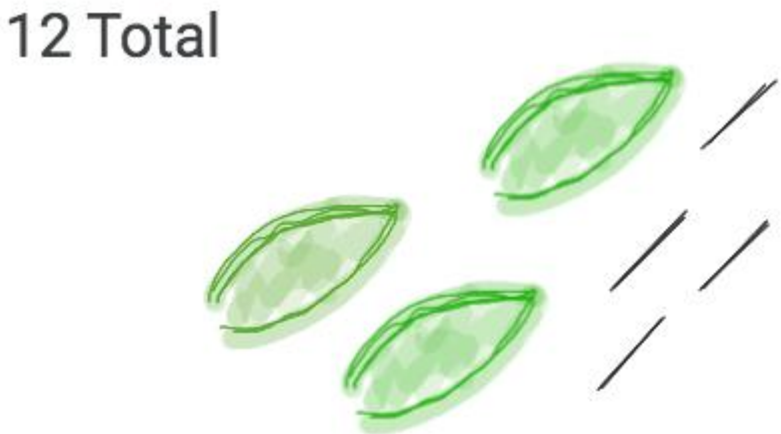
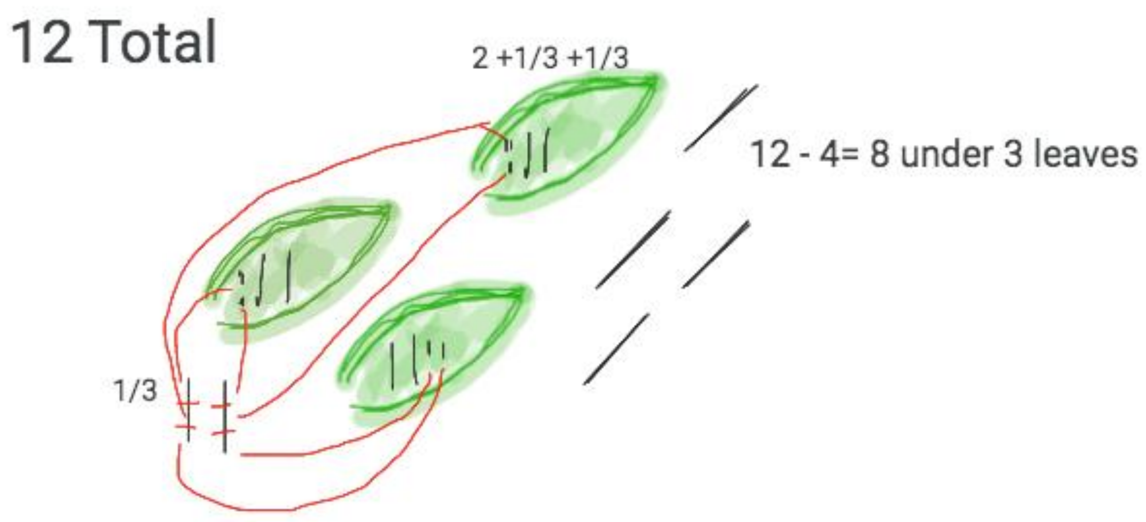


Figure 1. Illustration of the Chalk Story

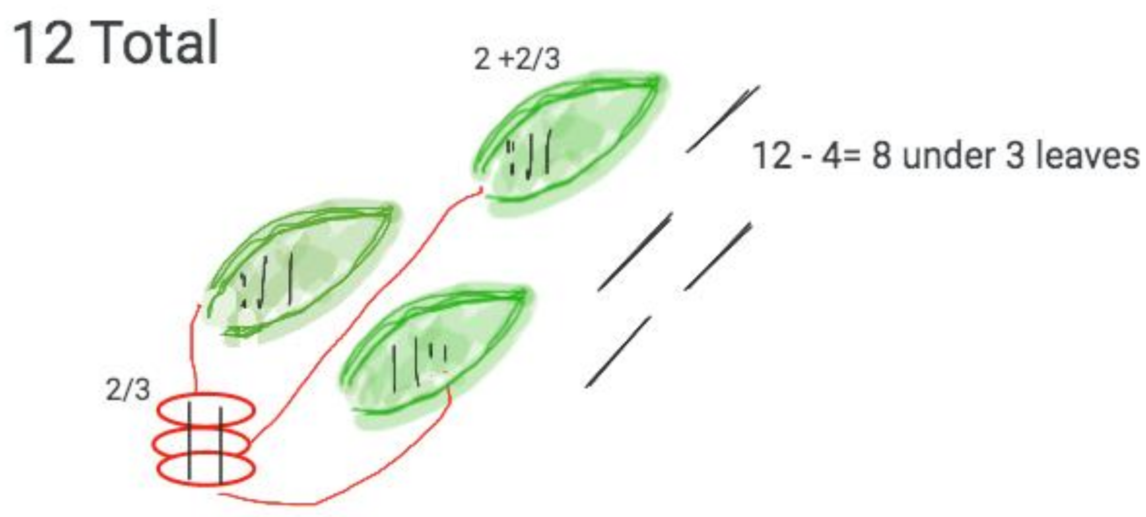
Students may ask if the chalk can break. Respond by asking "Can chalk break?"

Let students explore what could be under each leaf. Figure 2 has a few possible ways students may envision the solution.

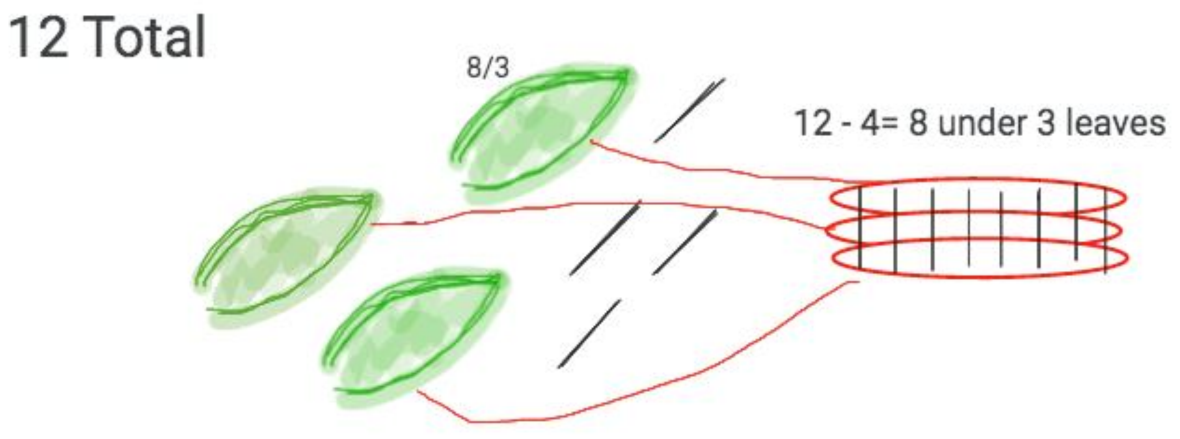
If groups only calculate using a decimal approximation $2.\overline{66}$ encourage students to visualize what that would look like to encourage the use of fractions. With any response, encourage students to talk about how the chalk would break, and what it would actually look like under each leaf. For example, if student say 2 and 2/3, ask what the 2/3 would look under each leaf. Look for students dividing all 8 pieces into thirds, putting 2 whole pieces in each leaf then dividing each of the 2 pieces into thirds (1 divided by 3 then multiply by 2) as in example 2 of Figure 2, or dividing both pieces into thirds together to 2 divided by 3. Lead a discussion on how these methods are connected by asking students if the amounts under each leaf are equivalent. Display a group's solution and ask the class how they think this group solved (or started solving) the problem.



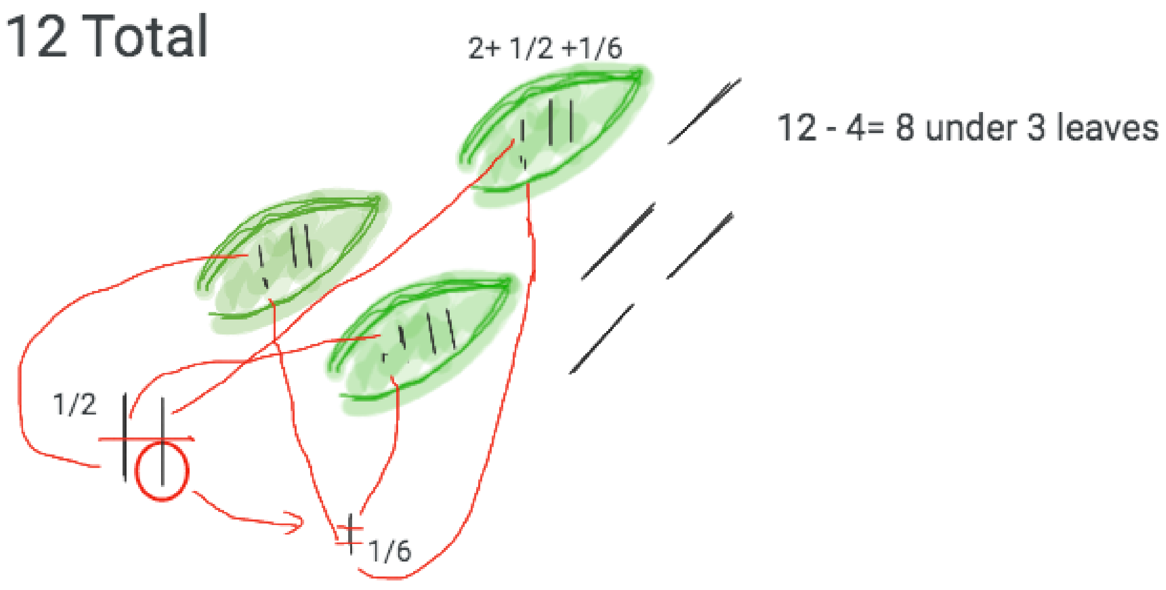
1. Students place 2 whole pieces of chalk under each leaf then divide each of the two remaining pieces into thirds and place 1/3 from each piece under the leaf.



2. Students place 2 whole pieces of chalk under each leaf then divide each of the two remaining pieces together into thirds and place ⅔ under each leaf.



3. Students divide all 8 pieces into thirds and then place 8 one third pieces of chalk under each leaf.



4. Students place 2 whole pieces of chalk under each leaf then divide each of the two remaining pieces into halves under each leaf. The remaining 1/2 is divided into 3 pieces (1/6 pieces of chalk) and then placed from each piece under the leaf.

Figure 2. Examples of How Students may Approach the Problem

Ask students if it is possible for each leaf to contain 2 whole pieces of chalk and 1/2 piece (this can be done in full class or with individual groups who have already shown a strong understanding in thirds.) Ask what would be left over that would still need to be divided?

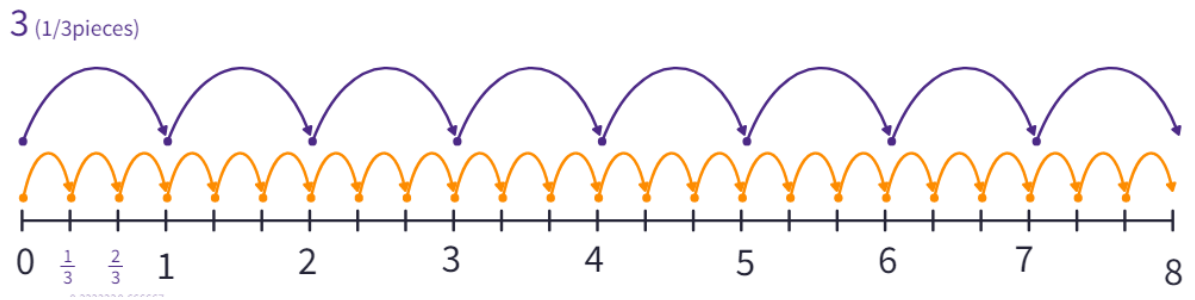
(1/2 piece of chalk). Ask how to take that 1/2 and put it under each leaf (1/2 into 3 pieces, 1/6 each). Encourage students to connect this to the 2 and 2/3 and 8/3 solution (see Figure 2 example 4). To encourage students to continue thinking after they have one way of representing the situation, ask, “Are there other ways to divide the chalk?” or “What about fourths, fifths, tenths?”

Tell students that at camp there is often a campfire where students can cook their own dessert, s’mores! Show them a quick video about how to make a s’more. https://www.youtube.com./watch?v=Sw5_E1TIQRQ After showing the video, ask what students noticed in the video. Ask why they think they are called s’mores (some more). If it doesn’t come up, ask if they use a whole chocolate bar for the s’more.

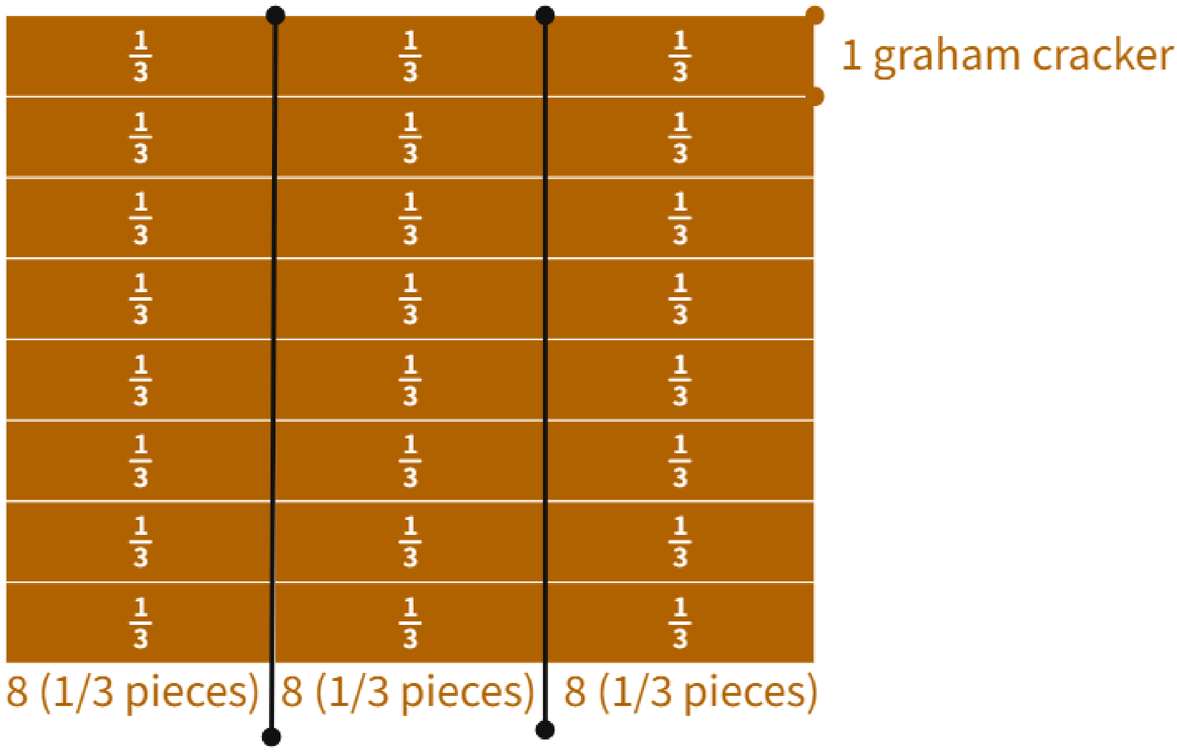
Explore

(1 hour) Have students work in groups of 4 and tell them that they are in charge of passing out supplies for s’mores to some of the campers. Tell students they have 8 plain graham crackers and are to give each camper 1/3 of the graham cracker for the bottom of the s’more. Have students illustrate how many students they can pass out a bottom to in as many ways as they can. See Figure 3 for some ways students may represent the problem.

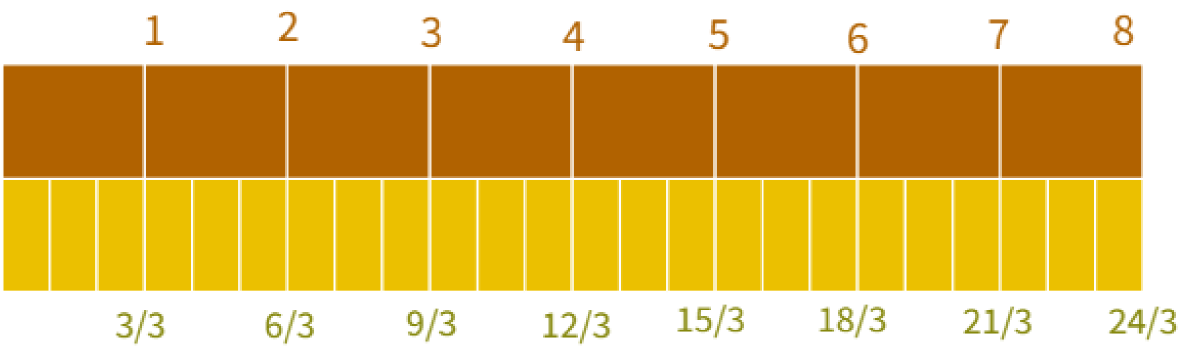
Bring the students together to discuss and show several different ways to think through the problem including the area model, tape diagram and number line (see Figure 3). Pass out the handout (Appendix A) for students to record their representations. This will encourage students to use these representations for the other scenarios.



Number line



Area Model: Students stack the graham crackers and break them into thirds giving them 8 stacks of 3 pieces.



Tape diagram showing that for every 1 graham cracker, 3 pieces are created.

Figure 3. Different Ways of Representing How Many Campers can 8 Graham Crackers Feed

Tell the students for the top cracker of the s'mores you have special chocolate graham crackers, but some were crumbled and you only have a stack of 6 of chocolate graham crackers. Your group decides to break them into 1/4 pieces, how many will you pass out to the campers? Teachers can ask, do you have enough or do you have too many for the students you passed the plain graham crackers out to? After students draw a model (area model, tape diagram, or number line) Teachers ask, how can we have the same amount of the regular and chocolate graham crackers, if we started with 8 of one kind and 6 of another?

Next students then work on representing passing out marshmallows. Tell students the camp accidently bought jumbo marshmallows and realized they will be too big for the graham crackers. The marshmallows will need to be cut in half. Based on how many campers you have given graham crackers to, how many marshmallows do you need? Ask the students to represent the situation in multiple ways. After students have had the chance to explore, emphasize understanding the relationship between multiplication. Students can figure out 24 x (1/2) =12 and therefore, 12 ÷ (1/2) =24.

Let students know we are now passing out the chocolate. We have 5 chocolate bars and each camper will get 1/6 of a bar. Ask teams if they have enough, too many, or the correct amount and to justify their answer with a model. After they determine they have too many, ask how much is left, how many additional campers could they give to.
Have students create their camping food own story to match 3 ÷ (1/5)?

Explain

(30 minutes) Teachers look for students who begin to make generalizations about dividing different types of numbers. Connect it to students' background knowledge from 4th grade that a whole number divided by a whole number will equal a whole number and some leftover, which can be a remainder or a fraction like in the leaf and chalk problem. This can be connected to their new knowledge that a whole number 8 ÷ (1/3) will also equal a whole number because we are looking for how many groups of 1/3 are in 8.

Teachers begin this next part by having a whole class discussion with students by showing the different types of models students can come up with for 8 ÷ (1/3) (see Figure 3) and comparing them to the engagement leaf example models of 8 divided by 3. Discuss the differences within each model and what the equations are asking. For example, teachers may ask: What do you notice about 8 pieces of chalk under 3 leaves versus 8 graham crackers divided into 1/3 pieces? To help students make this connection, teachers may break this problem up further by focusing on the leaf problem to ask what and how we were dividing. Then, focus just on the graham crackers and again ask what and how those are divided.

As the camping story continues, teachers share with students that the next day the campers wake up and make plans to move to a new campsite closer to the river. Campers pack up their one-man tents, food and hiking backpacks to set off to find a new campsite. After an hour of hiking the campers come upon a grassy meadow next to the river that could potentially work for a new campsite. Three campers get into a discussion about how to set up their tents, and if their tents will even fit in the meadow. As each camper describes their idea, they pick up a stick and draw in the dirt what they think is the best way to set up their tents in the meadow. Teachers should show each meadow design while reading the reasoning behind each camper's idea. Ask students to listen and look for how the ideas are similar or different and whose idea they think is the best. Remind students they need to give examples and explanations to support their reasoning of their ideas.

Show Figure 4. Camper 1 says, "I think we should set up the tents in the 7 spots without rocks, and we can split each spot into fifths to fit each of our tents. We will even have some spots leftover so we can play games!" The red shapes represent the rocks, and the rectangles represent the spots for the tents.

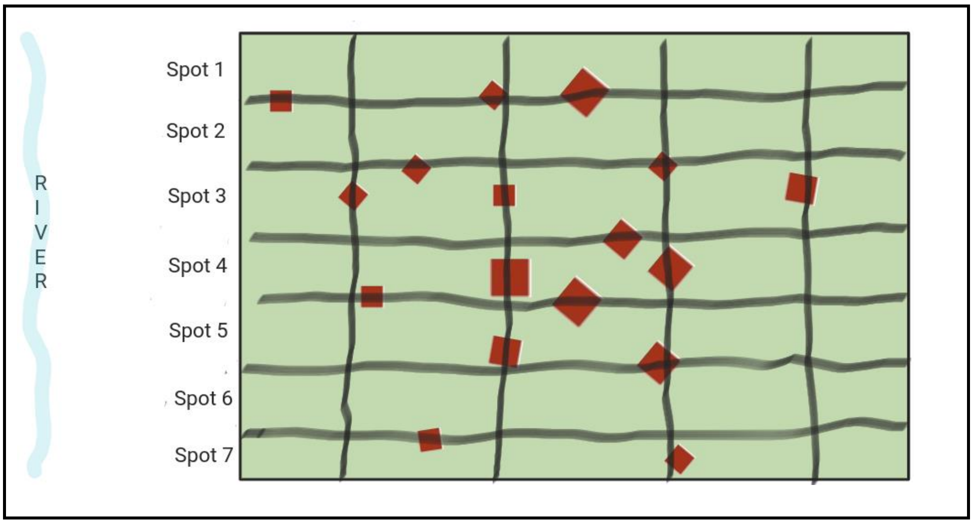


Figure 4. Camper 1 explaining their Reasoning where to put the Tents using an Area Model

Show Figure 5. Camper 2 says, "I think we should set up the tents in the 7 spots between the rocks too, but I think we should set them up in a circle so we can have 5 tents in each spot to build our own campfire." The red shapes represent the rocks and the green circles represent 7 large sections divided into fifths for tents.

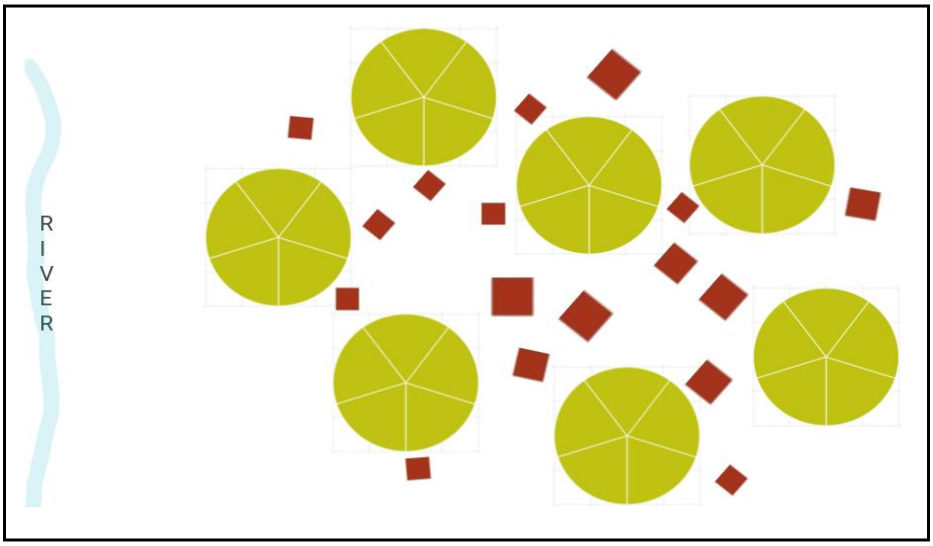


Figure 5. Camper 2 explaining their Reasoning where to put the Tents using a Set Model

Show Figure 6. Camper 3 says, "I think it would be better to be closer to the river. Let's spread out into the 7 spots between and in front of the bushes next to the river, so we can split each spot into 1/5 to fit each of our tents. I don't think we'll have any spots leftover, so we may want to draw straws to see who gets to pick first." The larger black rectangles represent the 7 spots divided into smaller fifths.

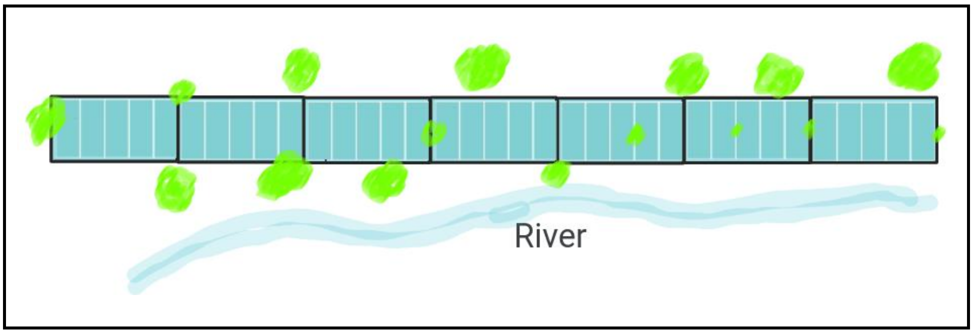


Figure 6. Camper 3 explaining their Reasoning where to put the Tents using a Length Model

Teachers look for and listen to similarities and differences that students may notice while discussing the different camper's ideas. For example, students may notice all of the ideas have 35 campsites, or there will always be several extra campsites or room to play games since there are 24 campers. Students can think back and use prior knowledge from how many s'mores they made in the explore part to know the number of campers. Students may also think Camper 2 is wrong or not understand his reasoning because he said there would be five tents around each campsite, but the other two campers said 1/5. Students may also notice Camper 3 is incorrect because there will be spots leftover, even if spread out differently next to the river. Lead a discussion surrounding student understanding of dividing a whole number of spots like 7 into 1/5 pieces, and how one tent will fit in each spot to have 35 tent spots focusing on the different representations and reasonings from each student. To enrich student understanding, teachers may ask questions like, what if campers had two-man tents, how would that change how many camp spots they need? Generalizations may be made by students that a whole number divided by a fraction will be a whole number.

Teachers continue to story and tell students after the campers spend 4 days fishing and hiking around this spot, they decide to move to one more campsite before heading home. Ask students to create a campsite story problem using the equation $5 \div (1/7)$.

Give students time to design their campsites, and then bring students back to a whole-class discussion for students to share their story problems. Student designs should demonstrate their understanding of a whole number divided by a fraction, and that the answer is a whole number.

Extend

(Flexible) It is time for students to apply and extend previous understandings of division to divide whole numbers by unit fractions or fractions by fractions. Encourage students to write their own problems and have them investigate situations that they have created. Provide some ideas without overloading students with information. Students already have explored situations at a campsite. Therefore, there could be some ideas that involve camping such as food, activity, game, etc. Teachers should remind students that the problem must include division of whole numbers by unit fractions.

Example Problem 1: Assume that students created a problem about sharing breakfast. It is the last day of camping, and there is not much oatmeal left in a package. Students would like to know how many servings of wheat cereal there are in the package if a package contains 6 cups of cereal and there is 1/3 cup of cereal in each serving. Encourage students to explain and draw a picture to illustrate their solution.

For this type of a problem, have students explain and draw a picture to illustrate their solution.

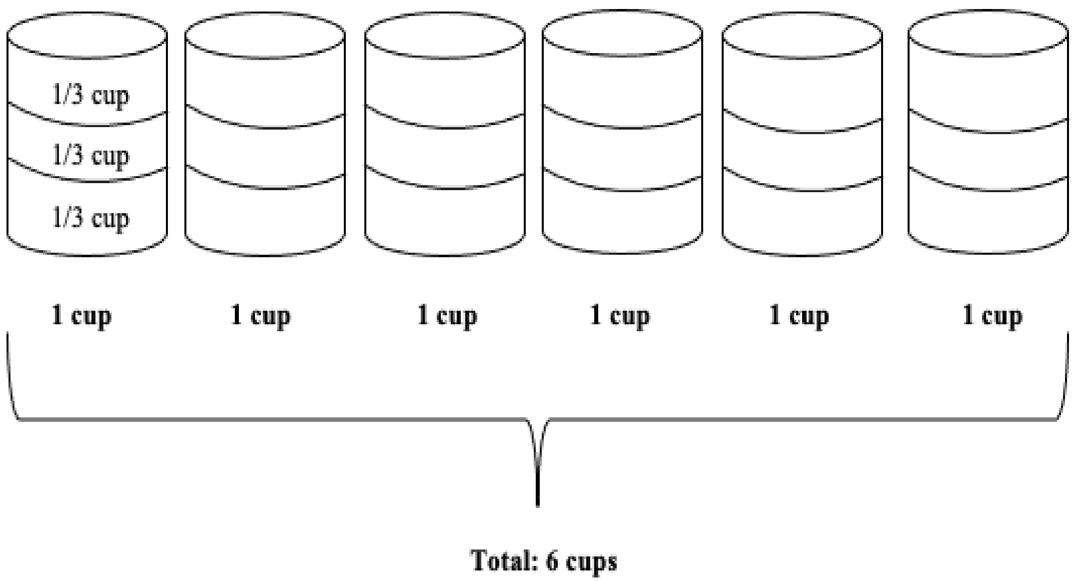


Figure 7. Solution of Example Problem 1

Students are familiar with division problems that use "How many groups?" examples. The "groups" in this case are the servings of cereal and the task is finding how many servings (or groups) there are in the package. Look for students' ability to interpret how much in one group and pay attention to students' models of "How many groups?" and "How much in one group?" since both cases are different. Some students, particularly English Language Learners, may not be familiar with the vocabulary and serving size. Therefore, it would be ideal to demonstrate a measuring cup. For this problem, show 1/3 and 1 cup on a measuring cup. Students should be able to understand that it takes 3 of the 1/3 cups to make 1 cup.

Figure 7 shows the six cups of cereal. Since there is 1/3 of a cup of cereal in a serving, students should be able to make a connection that each of the six cups contains 3 servings and the total number of servings would be $6 \times 3 = 18$ (6 groups of 3 servings). Once students understand the solution to this problem, listen for generalization of why the "invert and multiply" or "multiplying by the reciprocal" algorithm for division works in this setting. Students may need to see some examples and understand why multiplying by the reciprocal algorithm works in order to build abstract thinking.

Example Problem 2: Assuming that students would like to collect some rocks at the campsite and write a report on their collection for their Earth Science class. They are fortunate to find some quartz crystals and feldspar (Note: For the purpose and simplicity of this lesson, the rock choices are from Wyoming, the United States of America. However, rocks can be any type depending on the location and/or teachers' decision.) Some students get competitive and would like to calculate who has the most rocks based on this problem: The student A has 8 quartz crystals. If one third of Student A' rocks are quartz crystals, how many rocks does Student A have? For this problem, have students draw a diagram and explain. Based on their readiness, have students solve the problem algebraically as well.

Math Talk: If one third of Student A's rocks are quartz crystals, this means that 2/3 of the student's rocks are feldspar. This means that Student A has two times as many rocks other than quartz crystals. Since Student A has 8 quartz crystals, this means that the student has 16 other rocks and so the student has $16 + 8 = 24$ rocks total.

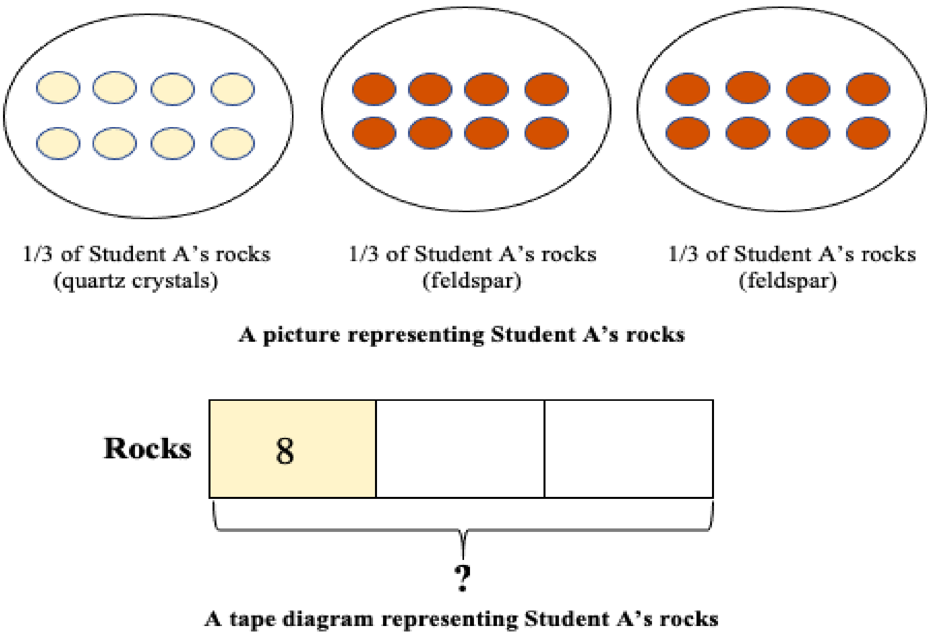


Figure 8. Solutions of Example Problem 2

The picture and tape diagram representing Student A's rocks (see Figure 8) can be interpreted as a missing factor problem: $\frac{1}{3} \times \_\_ = 8$. However, the missing factor problem is also equivalent to the division problem: $8 \div \frac{1}{3} = \_\_$. Regardless, both cases result in 24 rocks. In the tape diagram, the number of rocks is represented by 1 unit. As the diagram shows, this is equivalent to solving: $8 \div \frac{1}{3} = 8 \times 3 = 24$. Student A has 24 rocks.

Example Problem 3: After collecting rocks, students may want to celebrate with hot chocolate. Before making hot chocolate, students need to calculate whether they have enough ingredients to make hot chocolate for all students. If one thermos of hot chocolate uses 2/3 cup of cocoa powder, the task is to calculate the number of thermos Student B can make with $3\frac{1}{2}$ cups of cocoa powder? For this problem, have students solve by drawing a picture and explain how they can see the answer to the problem in their picture. Also, teachers should provide these two questions to encourage for algebraic thinking:

a. Which of the following multiplication or divisions equations represents this situation? Explain your reasoning.

$3\frac{1}{2} \times \frac{2}{3} = ?$ $\qquad$ $3\frac{1}{2} \div \frac{2}{3} = ?$ $\qquad$ $\frac{2}{3} \div 3\frac{1}{2} = ?$

b. Solve the problem you chose in part (a) and verify that you get the same answer as you did with your picture.

Students may have difficulty with transitioning to division problems involving fractions even if they understand this type of division context. To help them successfully make this transition and ensure a solid understanding, provide students with multiplication and division with whole numbers and multiplication with fractions problems. Language also plays a significant role when moving from whole number to fraction division when the quotient is less than 1. Therefore, point out the wording such as "Fraction of a group unknown" or "What fraction of a group?" or "How much in each group?".

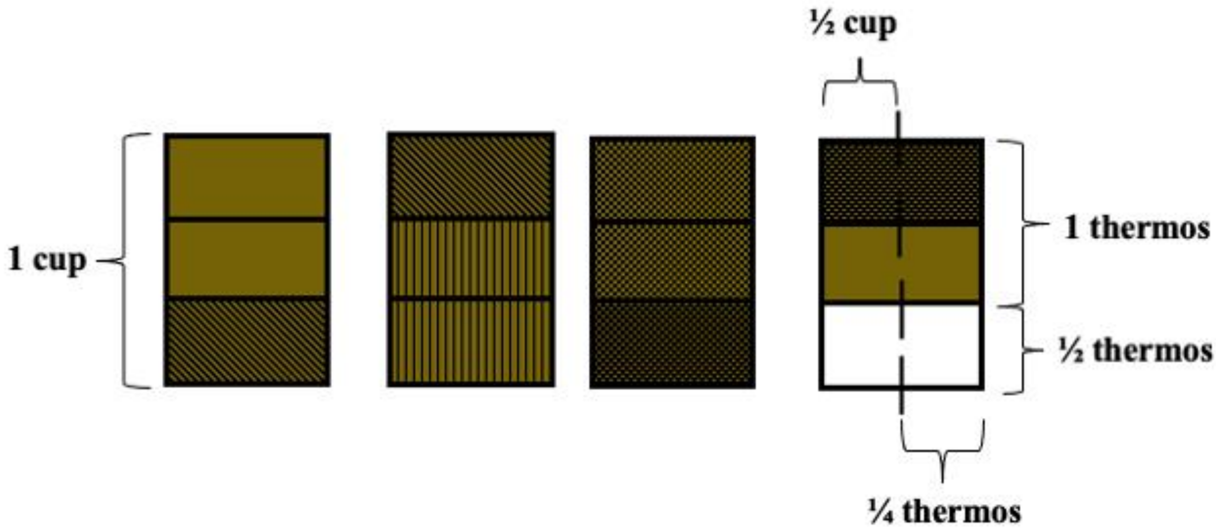


Figure 9. Solutions of Example Problem 3.

Math Talk:

The picture (see Figure 9) shows four rectangles that each represent 1 cup of cocoa powder. Each cup is horizontally divided into thirds. Since one thermos requires 2/3 cup, 2 thirds are shaded to show a single thermos of cocoa. The fourth rectangle is also vertically divided to show 1/2 cup and 1/4 thermos since the calculation involves working with three- and one-half cups of cocoa powder. As shown in the picture, there are five whole groups of 2/3 cups of cocoa and 1/4 of a group of 2/3 cups of cocoa. Student B can make $5\frac{1}{4}$ thermoses of hot chocolate.

For part a, observe whether students are able to choose the correct equation, $3\frac{1}{2} \div \frac{2}{3} = ?$, after solving the problem with a diagram. Listen for students' explanations such as "We have divided the $3\frac{1}{2}$ cups of cocoa powder into groups of size 2/3, so we are finding out how many groups of 2/3 there are in $3\frac{1}{2}$. So, the correct equation is $3\frac{1}{2} \div \frac{2}{3} = ?$" For part b, look for students' ability to solve the arithmetic problem correctly and verify that they get the same answer as they did with their picture. Correct calculation would involve first converting the mixed number, $3\frac{1}{2}$, to an improper fraction, 7/2, and then re-writing the equation: $\frac{7}{2} \div \frac{2}{3} = ?$. From here, students should be able to apply their previous knowledge to multiply 7/2 by the reciprocal of 2/3, $\frac{7}{2} \times \frac{3}{2}$, and correctly find the result. Once students have the result: $\frac{7}{2} \times \frac{3}{2} = \frac{21}{4}$, they should be able to simplify and convert the improper fraction to a mixed number to reason their findings: $\frac{7}{2} \times \frac{3}{2} = \frac{21}{4} = 5\frac{1}{4}$, and conclude with a statement such as "So the calculation gives the same answer as we see in the picture."

Evaluate

Teachers should look for students' ability of dividing a whole number by a fraction through multiple representations, comparing and contrasting different methods, and finally making a generalization of dividing a whole number by a fraction is equivalent to multiplying by the reciprocal. Students' mathematical creativity is measured by their ability of designing and solving their own real-world scenarios. These problems can be re-used with other students. Another observation is to check students' ability of arriving at a reasonable answer such as "How many 1/3 cups are in one cup?" and "If there are three 1/3 cups in one cup, how many 1/3 cups will be in 6 cups?" This line of reasoning can also lead students into ideas of proportionality that they would be introduced in future grades. Teachers should also regularly check and help their students build stronger fraction conceptual foundations since students will work with more complex fractional concepts when they are in higher grades.

Listen for the algebraic arguments for understanding the "invert and multiply" or "multiplying by the reciprocal" algorithm after students have done work with equations of the form as seen with the extension word problem two. The equation $\frac{1}{3} \times \_\_ = 8$ can be solved by either dividing both sides by 1/3 or multiplying both sides by 3. The goal is to have students understand the meaning of "invert and multiply" or "multiplying by the reciprocal" algorithm rule. The extension word problem three encourages students to interpret and compute quotients of fractions, and solve problems involving division of fractions by fractions. Look for students' flexibility in using visual fraction models and equations to represent a problem. Overall, the goal is to have students move away from drawing diagrams while dividing fractions by whole numbers since some problems would be more complex to draw pictures, and students should be able to generalize fraction divisions.

Appendix A

Representations	
8 Plain Graham Crackers, each camper gets 1/3 of a cracker.	
6 Chocolate Graham Crackers, each camper gets 1/4 of a cracker.	
Jumbo marshmallows must be cut in half.	
5 Chocolate Bars, each camper gets 1/6 of a bar.	
Your story 5 ÷ (1/3)	

Citation

Jackson, T., Carey, A. S., & Quiroz, F. (2023). Camping in the Woods with Fractions. In A. Bicer (Ed.), *Creativity-Directed Mathematical Tasks for 5th Grade Common Core Classrooms* (pp. 357-374). ISTES Organization.

Task 27 - Trail Mix

Chuck Butler, Jennifer Kellner, Amy Kassel

Mathematical Content Standards

CCSS.MATH.CONTENT.5.NF.B.7

Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.

CCSS.MATH.CONTENT.5.NF.B.7.B

Interpret division of a whole number by a unit fraction and compute such quotients.

For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. *Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4.*

CCSS.MATH.CONTENT.5.NF.B.7.C

Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem.

For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?

Mathematical Practice Standards

CCSS.MATH.PRACTICE.MP4

Model with mathematics.

CCSS.MATH.PRACTICE.MP5

Use appropriate tools strategically.

CCSS.MATH.PRACTICE.MP7

Look for and make use of structure.

Vocabulary

Ingredients, division, maximize, relationship, operation

Materials

Pictures of pancakes, number lines, grid paper, individual whiteboards, markers, colored pencils, pattern blocks, fraction bars, virtual manipulatives

Lesson Objective

Students will apply their understanding of division of fractions and whole numbers to solve real-world problems. This lesson will promote creative thinking by having students use the 3 Read Method and create their own questions for a story. They will model real-world situations to explain their solutions.

Engagement

(20 minutes) The students will have interesting pictures of pancakes to look at to start the class. This lesson will use the 3 Read method to ensure sense-making and encourage problem-posing. The teacher will read the students the following story aloud."Chef Larry is cooking pancakes this morning for his restaurant. He makes 8 cups of pancake mix at the start of his shift. One order of pancakes requires ¼ of a cup of pancake mix."

Ask the students "What is this situation about?". Have the students discuss with their partner and fill out the first section under Read #1 in Appendix A. (Give them 1-2 minutes.) Now, the teacher will read the same story aloud again. Ask the students, "What quantities do you see?". Have them discuss with their partner and fill out the second section under Read #2. (Give them 3-5 minutes.) Finally, the teacher will ask the students, "What math questions can you ask about this situation?" Some example questions could be: "How many customers can Chef

Larry serve?", "How many orders can be filled?", "What happens if a customer orders more than one pancake?", "What happens if we use less than $\frac{1}{4}$ of a cup in our recipe?", or "What happens if we use more than $\frac{1}{4}$ in our recipe?". For the questions, the teacher should provide independent think time first in order to encourage divergent thinking. After 1-2 minutes, the students will share with their partner and then the whole class.

After writing all the questions on the board, the teacher will thank the students for their interesting math questions and then select one problem for the students to answer.

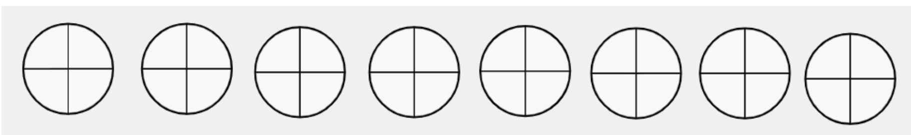
The teacher will say, "The first question we are going to discuss is how many orders can we make with this batch of pancake mix?" or select another question they asked. Have the student fill out the solution section with the question and work. The other student-generated questions can be added to the extend portion of the lesson.

Explore

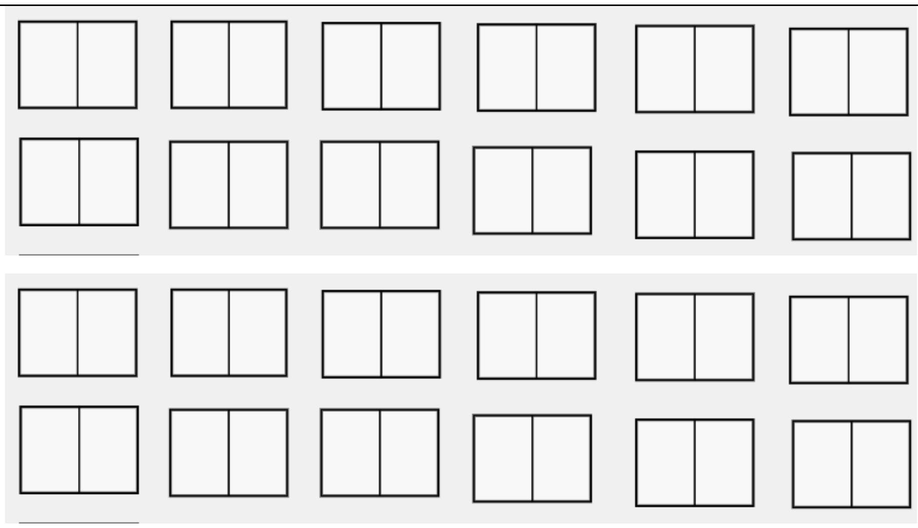
(30 minutes) Part 1: Teachers will put students in groups of 3 to 4 students and give each group one of the statements about breakfast (see Appendix B). Teachers will give the problem stem and one of the breakfast items for each group to solve.

Teachers will ask students to create a model to represent their problem. For example, students may create the following models:

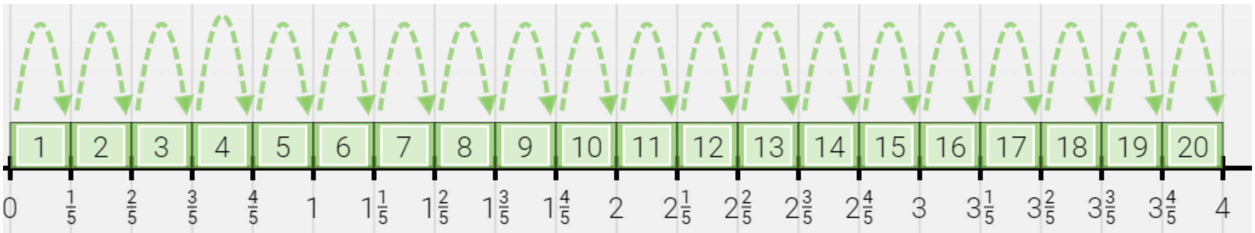
The chef made 8 cups of pancake mix this morning. An order of pancakes is made from $\frac{1}{4}$ of a cup of pancake mix.



Scrambled eggs require $\frac{1}{2}$ cup of egg mixture. The chef has 24 cups of scrambled egg mixture made.



The chef made 4 cups of donut mix this morning. Each donut requires $\frac{1}{5}$ of a cup of mix.



Note. Images from The Math Learning Center (The Math Learning Center, 2022)

Figure 1

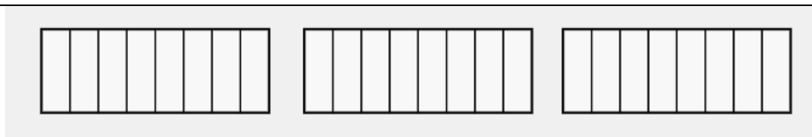
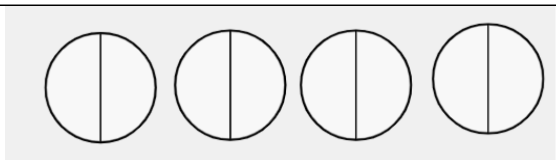
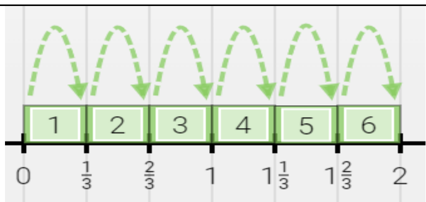
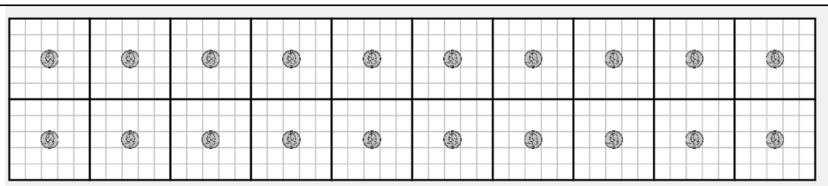
As students share their models, teachers should ask students to use their model to explain how many people could order the food item in their problem.

Part 2: Teachers will give students the Trail Mix Recipe Problem (see Appendix D).

Students may create a recipe for their Trail Mix as shown in Figure 2.

Trail Mix Recipe

Ingredient	**How many pounds?**	**One serving size is what fraction of a pound?**	**Number of servings**
Peanuts	3	$\frac{1}{8}$	24

Model			
Equation	$3 \div \frac{1}{8} = 24$		
M&Ms	4	$\frac{1}{2}$	8
Model			
Equation	$4 \div \frac{1}{2} = 8$		
Pretzels	2	$\frac{1}{3}$	6
Model			
Equation	$2 \div \frac{1}{3} = 6$		
Dried Fruit	5	$\frac{1}{4}$	20
Model			
Equation	$5 \div \frac{1}{4} = 20$		

Note. Images from The Math Learning Center (The Math Learning Center, 2022)

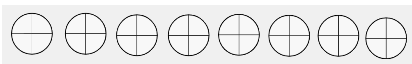
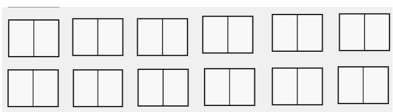
Figure 2

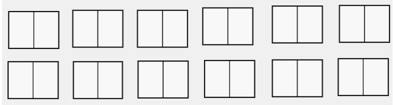
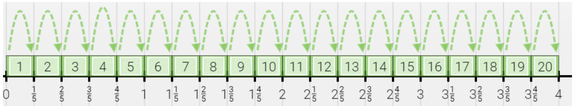
In this example, students would list the ingredients from greatest number of servings to the least number of servings as peanuts, dried fruit, M&Ms, and pretzels. Students would be able to make at most 6 servings of this recipe.

Explain

(30 minutes) Part 1: As students are creating models to solve their problem, teachers should listen and observe students working. If students are struggling to recognize the problem as division, teachers may support them by asking them to model a problem with whole numbers. For example, if there are 8 cups of coffee and each person drinks 2 cups, how many people can drink coffee? Teachers may ask students to model this problem and help them recognize this as equal groups and division. Teachers may then ask students how their problem is similar or different to the coffee problem.

Teachers should ask students to share their model with another group by pairing groups that have used different models. As groups finish, teachers should ask one representative from each group to draw their model on the classroom display (Appendix C). See Figure 3 for sample student answers.

Problem	**Model**	**Equation**
The chef made 8 cups of pancake mix this morning. An order of pancakes is made from $\frac{1}{4}$ of a cup of pancake mix.		
Scrambled eggs require $\frac{1}{2}$ cup of egg mixture. The chef has 24 cups of scrambled egg mixture made.		

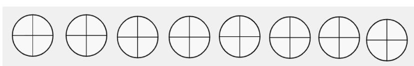
		
The chef made 4 cups of donut mix this morning. Each donut requires $\frac{1}{5}$ of a cup of mix.		

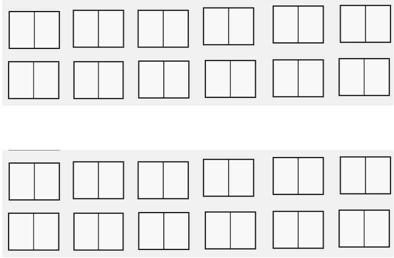
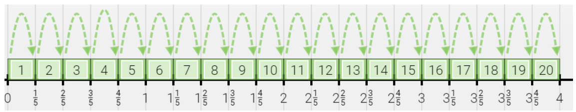
Note. Images from The Math Learning Center (The Math Learning Center, 2022)

Figure 3

Teachers should ask students to look at the table generated and ask students to do a quick write about what connections they notice between the problems and the models. Teachers may ask students to share with a partner and then ask several pairs of students to share whole-class what they noticed. For example, students may share that there are 8 wholes and each whole is broken down into 4 parts so there are 32 parts.

After students explore the relationships between the problems and the model, teachers may ask if students have used an operation before that allows them to determine equal groups or how many groups of. Students should say division. Ask students if they could write an equation to represent one of the problems on the board on the paper they used for a quick write. Ask students to discuss their equation with their partners. After students have had time to discuss, teachers may ask students to share their equations and record them in the table as below (Figure 4).

Problem	**Model**	**Equation**
The chef made 8 cups of pancake mix this morning. An order of pancakes is made from $\frac{1}{4}$ of a cup of pancake mix.		$8 \div \frac{1}{4} = 32$

Scrambled eggs require $\frac{1}{2}$ cup of egg mixture. The chef has 24 cups of scrambled egg mixture made.		$24 \div \frac{1}{2} = 48$
The chef made 4 cups of donut mix this morning. Each donut requires $\frac{1}{5}$ of a cup of mix.		$4 \div \frac{1}{5} = 20$

Note. Images from The Math Learning Center (The Math Learning Center, 2022)

Figure 4

Teachers may ask students what commonalities they see in the equations. For example, students may recognize that there are 8 wholes broken into 4 parts each and equals 32 total parts ($8 \times 4 = 32$). Teachers should use this opportunity to help students begin to recognize the relationship between multiplication and division.

Part 2: Teachers may select groups to share their Trail Mix recipes specifically asking them to connect their model to their equation. Students can explain how they determined the maximum number of servings of Trail Mix they could make. Teachers may wish to have students create a recipe that maximizes the number of servings from the given ingredients.

Extend

The teacher may choose to assign one extension to the whole class or assign different extensions to different groups based on interest, ability, and readiness.

Cutbacks – At the end of part 1, the students are told that we need to change the recipe in order to produce more food from the same mix. For example, the Chef needs 8 cups of pancake mix to produce more pancakes. What should the team do? Create a model and equation.

No More Division – At the end of part 1, students are told to write an equation but do not use division. Can students write a rule for writing division statements as multiplication statements?

Pick 3 – If you only need to include three ingredients in the trail mix, which three ingredients would you pick if you were going to eat it? If you wanted to produce the most trail mix? If you wanted to produce the least trail mix?

No More Unit Fractions – For some strange reason, a customer requests that unit fractions be banned from his trail mix. Change the serving size of two of the trail mix ingredients and find the number of servings of trail mix and draw a visual model.

Evaluate

- In part 1, formative assessment for creativity and content occurs as students are visualizing the context, asking questions about their context, drawing their representation, and finding the answer to their question. If a student struggles to produce a mathematical question, the teacher could encourage students to represent their situation using manipulatives (pattern block, fraction bars, or visuals).
- In part 2, formative assessment for creativity and content occurs as students are deciding the serving size, determining the total number of servings, and then creating a visual for the context. As students complete deciding on the serving sizes for each ingredient, the teacher can ask students to compare two serving sizes. For example, if a student has $\frac{1}{8}$ lb of peanuts and $\frac{1}{11}$lb. of M & M's, the teacher could ask "What does this mean in our context?" or "What could be a few reasons for this?" By asking questions about the meaning of fractions in their context, the teacher hopes to encourage sense-making.
- In Cutbacks, formative assessment for content and creativity occurs as students understand the context and change the serving size to produce more whole pancakes. If a student has a fraction for an answer, the teacher can ask "What does this mean in this story?"

- In No More Division, formative assessment for content occurs as students write division as a multiplication statement. If students struggle to do this with unit fractions, the teacher can ask questions about writing division statements with whole numbers. For example, “How can you write $6 \div 2 = 3$ as a division statement?”

- In Pick 3, formative assessment for creativity and content as students create different mixtures based on student preference and maximum or minimum trail mix servings.
- In No More Unit Fractions, formative assessment for creativity and content occurs as students pick two ingredients, determine the serving sizes, create a visual model, and then find the number of servings of trail mix they can make with these two ingredients. If a student picks a serving size that results in a fraction of a serving of trail mix, then the teacher asks “What does this mean in this story?” For example, if the serving for Dried Fruit is $\frac{2}{7}$ lb, then the teach can ask “What does $5 \div \frac{2}{7} = \frac{35}{2}$ mean

  in this context?

References

The Math Learning Center. (2022). *Number Line*. Retrieved February 10, 2022, from https://www.mathlearningcenter.org/apps/number-line

Appendix A

Problem:		
Read #1 What is the story about?	Read #2 What are the quantities and units?	Read #3 What questions do you have?
What do you know about this situation? Write a sentence or draw a picture.	What numbers do you notice? What is being used to express the amounts?	What questions can you ask about the situation?
Solution: Choose a question related to the situation. Write it here and solve below:		

Appendix B

You are a manager at a local restaurant and must determine the number of people you can serve today.

1. The chef made 8 cups of pancake mix this morning. An order of pancakes is made from $\frac{1}{4}$ of a cup of pancake mix.
2. The chef made 10 cups of waffle mix this morning. An order of waffles requires $\frac{1}{3}$ of a cup of waffle mix.
3. Scrambled eggs require $\frac{1}{2}$ cup of egg mixture. The chef has 24 cups of scrambled egg mixture made.
4. Omelets require $\frac{1}{4}$ cup of egg mixture. The chef prepared 10 cups of the omelet egg mixture.
5. Breakfast potatoes require $\frac{1}{8}$ of a pound of diced potatoes. The chef diced 4 pounds of potatoes.
6. Hashbrowns require $\frac{1}{6}$ of a pound of shredded potatoes. The chef shredded 5 pounds of potatoes.
7. There are 5 pounds of sausage to make sausage patties. Each sausage patty is made from $\frac{1}{7}$ of a pound of sausage.
8. Each serving of bacon is $\frac{1}{4}$ of a pound. There are 12 pounds of bacon.
9. The chef made 4 cups of donut mix this morning. Each donut requires $\frac{1}{5}$ of a cup of mix.
10. Muffins are made from $\frac{1}{6}$ cup of batter. The chef made 4 cups of muffin batter.

A full breakfast order at your restaurant includes pancakes or waffles, scrambled eggs or an omelet, one kind of potatoes, one kind of meat, and either a donut or a muffin. How many people can order a full breakfast this morning so that you can guarantee that you will always have any combination they may choose?

Appendix C

Problem	Model	Equation
The chef made 8 cups of pancake mix this morning. An order of pancakes is made from $\frac{1}{4}$ of a cup of pancake mix.		
The chef made 10 cups of waffle mix this morning. An order of waffles requires $\frac{1}{3}$ of a cup of waffle mix.		
Scrambled eggs require $\frac{1}{2}$ cup of egg mixture. The chef has 24 cups of scrambled egg mixture made.		
Omelets require $\frac{1}{4}$ cup of egg mixture. The chef prepared 10 cups of the omelet egg mixture.		
Breakfast potatoes require $\frac{1}{8}$ of a pound of diced potatoes. The chef diced 4 pounds of potatoes.		
Hashbrowns require $\frac{1}{6}$ of a pound of shredded potatoes. The chef shredded 5 pounds of potatoes.		
There are 5 pounds of sausage to make sausage patties. Each sausage patty is made from $\frac{1}{7}$ of a pound of sausage.		
Each serving of bacon is $\frac{1}{4}$ of a pound. There are 12 pounds of bacon.		
The chef made 4 cups of donut mix this morning. Each donut requires $\frac{1}{5}$ of a cup of mix.		
Muffins are made from $\frac{1}{6}$ cup of batter. The chef made 4 cups of muffin batter.		

Appendix D

Trail Mix Recipe

You want to make some trail mix. You look in your pantry and find the following ingredients:

3 pounds of peanuts
1 pound of raisins
4 pounds of chocolate-covered candies
2 pounds of pretzels
2 pounds of chocolate chips
5 pounds of dried fruit
1 pound of peanut-butter chocolate candies
3 pounds of honey nut cereal
4 pounds of rice cereal

You want to make the trail mix with your favorite four ingredients by mixing different amounts of each of the four ingredients. You want to make your trail mix by using the most of your favorite ingredient followed by decreasing amounts of each of the remaining ingredients so that your least favorite ingredient has the least amount in your trail mix.

- Complete the table to show how many servings of each ingredient you have in the pantry.

Trail Mix Recipe			
Ingredient **Model** **Equation**	**How many pounds?**	**One serving size is what fraction of a pound?**	**Number of servings**
Model			
Equation			

Model

Equation

Model

Equation

Model

Equation

- Write the ingredients in order from the greatest number of servings to the least number of servings.
- How many servings of Trail Mix can you make with this recipe?

Citation

Butler, C., Kellner, J., & Kassel, A. (2023). Trail Mix. In A. Bicer (Ed.), *Creativity-Directed Mathematical Tasks for 5th Grade Common Core Classrooms* (pp. 375-390). ISTES Organization.

Task 28 - Mom's Recipe

Helen Aleksani, Geoff Krall

Mathematical Content Standards

CCSS.MATH.CONTENT.5.NF.B.7

Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.1

CCSS.MATH.CONTENT.5.NF.B.7.A

Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3.

CCSS.MATH.CONTENT.5.NF.B.7.B

Interpret division of a whole number by a unit fraction, and compute such quotients. For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4.

CCSS.MATH.CONTENT.5.NF.B.7.C

Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?

Mathematical Practice Standards

CCSS.MATH.PRACTICE.MP2

Reason abstractly and quantitatively.

CCSS.MATH.PRACTICE.MP3

Construct viable arguments and critique the reasoning of others.

CCSS.MATH.PRACTICE.MP4

Model with mathematics.

CCSS.MATH.PRACTICE.MP8

Look for and express regularity in repeated reasoning.

Vocabulary

Dividing a fraction by a whole number, dividing a whole number by a fraction, connecting multiplication and division in fractions, mathematical modeling, generalizing**.**

Materials

1, $\frac{1}{4}$ and $\frac{1}{3}$ measuring cups. Kids crafting sand. Small bucket. Fraction bars. Fraction circles.

Lesson Objective

Students are going to explore modeling by dividing a whole number by a unit fraction using a real-life application. They will be practicing the connection between the visual representation and multiplication. Students will be practicing creativity by getting the opportunity to investigate the relationship between fraction division and fraction multiplication by applying it to a real-life application.

Engagement

(20 minutes) Begin the lesson by telling students about helping their mother baking her brownie recipe since they accidentally broke mom's one cup measure. She needs help with

using her $\frac{1}{3}$ and $\frac{1}{4}$ measuring cups to make the recipe work. To help those students who haven't helped cook or bake, the teacher can bring a $\frac{1}{4}$ and $\frac{1}{3}$ measuring cup so students are familiar with the concept. The teacher can also use images to demonstrate the purpose of the story (Figure 1). Since the teacher is going to require students to write mathematical expressions to model the scenario, it will be helpful to also revisit writing mathematical expressions before starting the activity. For example, 10 x $\frac{1}{5}$ = 10/5 = 2 and 10 ÷ $\frac{1}{5}$ = 50.

Figure 1. Sample Measuring Cups

Show the ingredients needed for the recipe so the students can help their mother start measuring.

Mom's Brownie Recipe Ingredients

4 cups sugar

3 cups flour

3 cups cocoa powder

2 cups chocolate chip

1 cup olive oil

1 teaspoon vanilla extract

1 teaspoon salt

2 large eggs

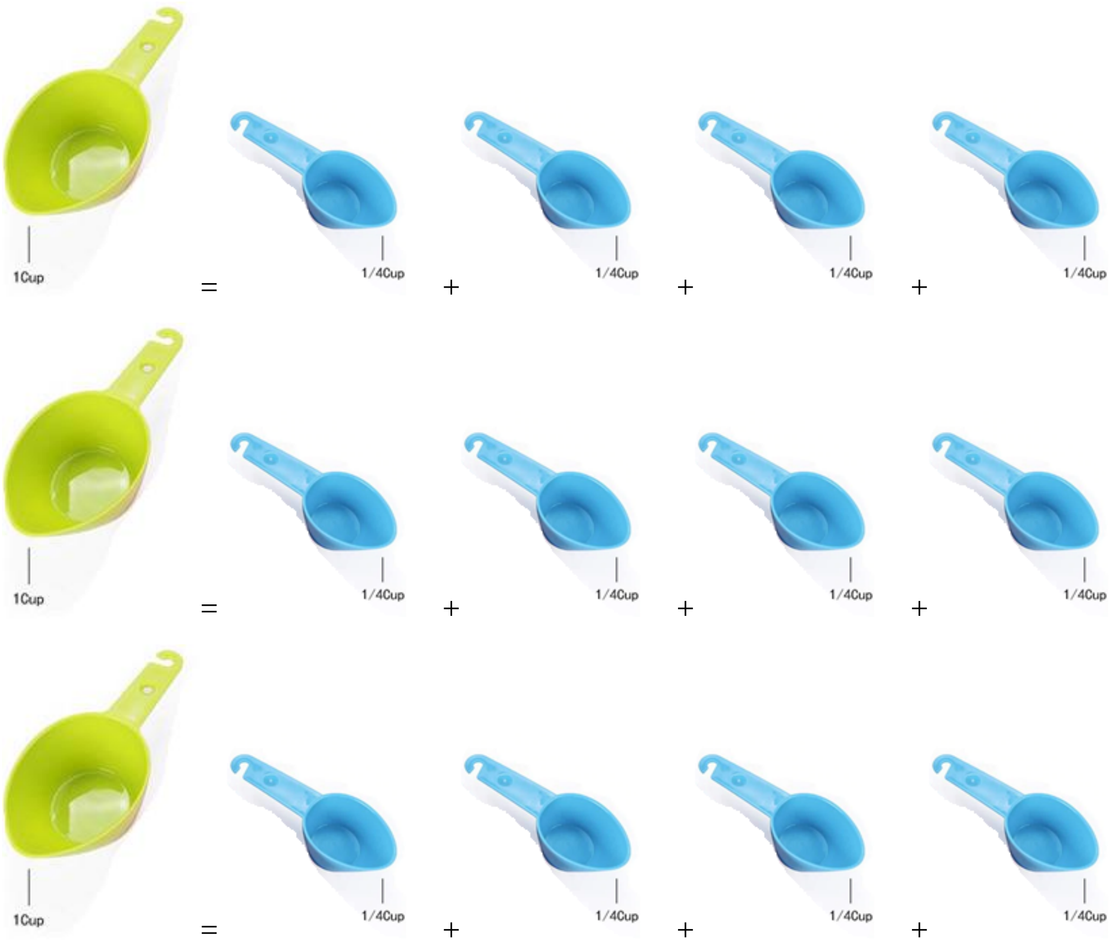
Clarify with your students that they are going to learn dividing a whole number by a unit fraction using the baking recipe. Students will be working with groups, and every group will receive a set of 1, $\frac{1}{4}$ and $\frac{1}{3}$ measuring cups as well as kids crafting sand to help visualize measuring the ingredients. Before releasing the students, show them how to use the sand, bucket and measuring cup. For example, use a $\frac{1}{4}$ measuring cup to fill the 1 cup measuring with sand and count how many $\frac{1}{4}$ was needed to fill the 1 measuring cup.

Explore

(30 minutes) Students will be working with a group of three. Tell the students to use the tools given to answer the following question. How many quarter cups or $\frac{1}{3}$ cups does she need for the sugar? Draw a picture and write a mathematical statement to demonstrate your reasoning and write a mathematical statement. Encourage students to use both $\frac{1}{4}$ and $\frac{1}{3}$ measuring cups to answer the question.

Also, provide a table to groups to keep track of their progress (see Appendix A). Clarify with students that they need to be using multiplication and division to demonstrate the scenario rather than just using addition and subtraction by modeling some examples to help them understand the usefulness of using multiplication and division. For example, we can show this problem in two different ways using addition and multiplication.

$\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}=\frac{4}{3}$ is the same as $4\times\frac{1}{3}=\frac{4}{3}$



Once students see the number of groups needed for 4 cups of sugar, have them draw a picture using fraction bars, which can also be considered an area model, and write an expression for the number of $\frac{1}{4}$ cups needed in a one cup (see Figure 2).

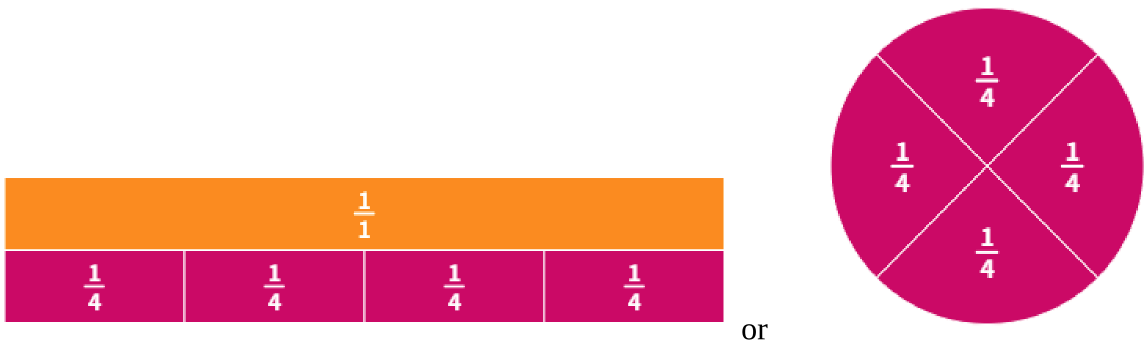


Figure 2. Length and Area Model of Four of 1/4

This can help students see that it is a division problem since it is asking how many quarter cups are in a cup? Students can write:

1÷1/4=4 or 4x1/4=1Students can verify that 4 is the correct answer by doing:

$4 \times \frac{1}{4} = 4/4 = 1$

Then clarify that, for this recipe, we need 4 cups of sugar instead of one. Therefore, how many total 1/4 cups of sugar is needed to make 4 cups of sugar.

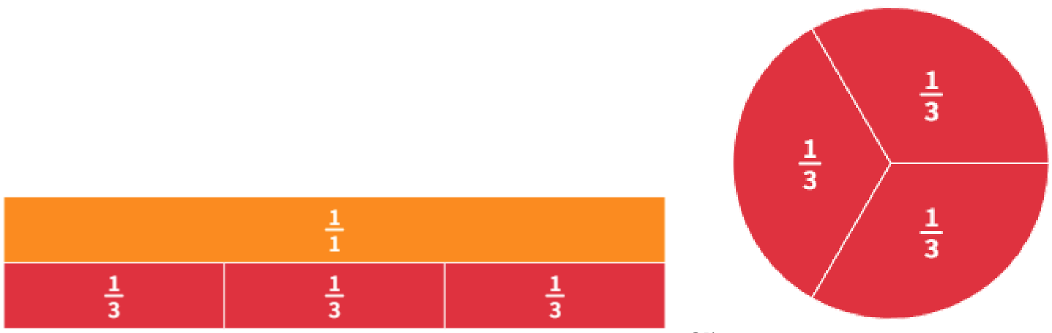
4 sets of 4 which equals 16

Table 1. Total Number of Cubs for Given Measures

Ingredients	Total number of cups needed	Using ⅓ measuring cup for every 1 cup	Total number of ⅓ measuring cups needed	Using ¼ measuring cup for every 1 cup	Total number of ¼ measuring cups needed
1 cups sugar	4 cups	3	12	4	16

Have students repeat the same process measuring 4 cups of sugar using the $\frac{1}{3}$ measuring cup (see Figure 3).

Figure 3. Sample Use of 1/3 Cup to measure a One Cup



or

Figure 4. Length and Area Model of 3 of 1/3

This can help students see that it is a division problem since it is asking how many $\frac{1}{3}$ cups are in a cup? Students can write (Figure 4):

1÷1/3=3 or 3x1/3=3

Students can verify that 3 is the correct answer by doing:

3 x 1/3 = 3/3 = 1

Then clarify that, for this recipe, we need 4 cups of sugar instead of one. Therefore, how many total 1/3 cups of sugar is needed to make 4 cups of sugar.

4 sets of 3 which equals 12.

To practice more, repeat the above process figuring out the measurements for the rest of the ingredients in the recipe.

Table 1. Total Number of Cubs for Given Measures

Ingredients	Total number of cups needed	Using $\frac{1}{3}$ measuring cup for every 1 cup	Total number of $\frac{1}{3}$ measuring cups needed	Using $\frac{1}{4}$measuring cup for every 1 cup	Total number of $\frac{1}{4}$ measuring cups needed
1 cups sugar	4 cups	3	12	4	16
1 cup flour					
1 cup cocoa powder					

Explain

(20 minutes) When students finish completing the table, ask them if they notice a pattern. Give students 1 to 2 minutes to think about their answer individually. Then, have them share their ideas with their group members. Encourage students to use area models to demonstrate their understanding of the concept (Figure 5).

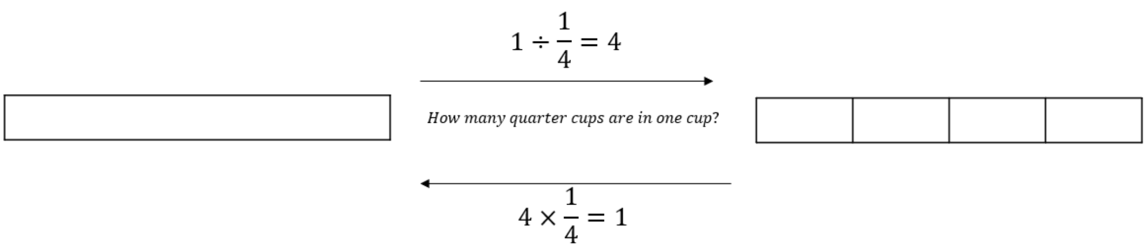


Figure 5: demonstrating the area model for then number of quarter cups in one cup

Using the set model, student can argue that 16 quarter cups are needed to measure 4 cups of sugar (Figure6)

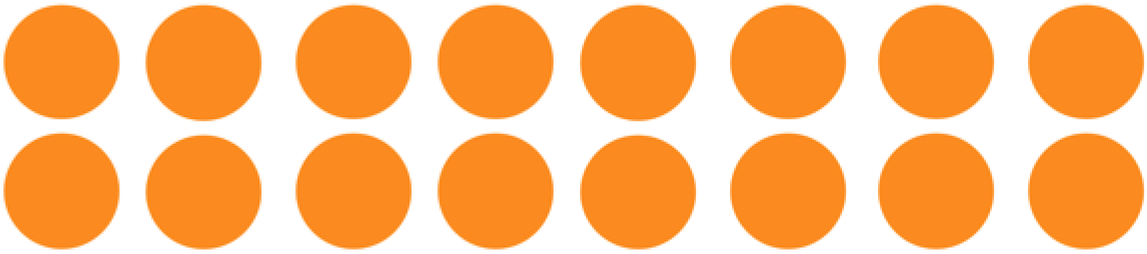


Figure 6. 4 Cups of Sugar measured using 4 Quarter Cups

Students can circle the 4 cups of sugar measured using 4 quarter cups (Figure 7)

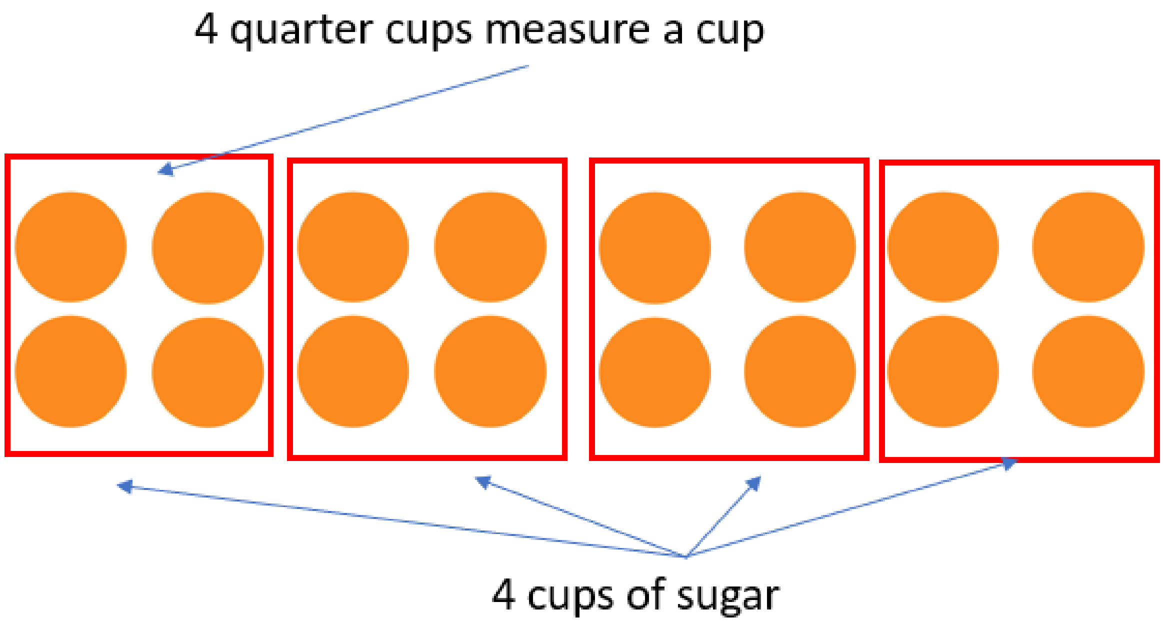


Figure 7. Demonstrating the Use of 4 Cups of Sugar measured using 4 Quarter Cups

Extend

(20 minutes) Once students have practiced modeling dividing whole numbers by a unit fraction, extend their knowledge to work on problems such as the one below:

Your mom doesn't remember the order in which she combined the ingredients. However, she does remember that the last ingredient added was three 1/3 cups. Can you help your mom to find the last ingredient added to the mixture? Draw a picture to help explain your reasoning.

$$1 \div \frac{1}{3} = 3$$

How many $\frac{1}{3}$ cups are in one cup?

$$3 \times \frac{1}{3} = 1$$

$$1 \div \frac{1}{3} = 3$$

How many $\frac{1}{3}$ cups are in one cup?

$$3 \times \frac{1}{3} = 1$$

$$1 \div \frac{1}{3} = 3$$

How many $\frac{1}{3}$ cups are in one cup?

$$3 \times \frac{1}{3} = 1$$

$$9 \div 3 = 3$$

Since the answer is 3, going back to the ingredients, we know that 3 cups of cocoa powder as well as 3 cups of flour is needed to make their recipe. Thus, we can't really know what one was added last.

Evaluate

When going over this task, teachers must look for students' understanding of modeling dividing whole numbers by a unit fraction. If there are students who are considered beginners in understanding the meaning of division by a unit fraction, the teacher can bring actual ¼ and ⅓ measuring cups and model the number of $\frac{1}{4}$ or $\frac{1}{3}$ cups needed to fill a one cup. In order for students to be successful working on the task provided in the Extend section, they need to be able to successfully complete the first question asked in the task. For that, students may need more practice seeing the connection between the visual representation and the more abstract equations. Since this task emphasizes mainly on the relationship between multiplication and division, therefore teachers should bring students' attention to it if they fail to see the connection.

Appendix A

Ingredients	Total number of cups needed	Using $\frac{1}{3}$ measuring cup for every 1 cup	Total number of $\frac{1}{3}$ measuring cups needed	Using $\frac{1}{4}$ measuring cup for every 1 cup	Total number of $\frac{1}{4}$ measuring cups needed

Citation

Aleksani, H. & Krall, G. (2023). Mom's Recipe. In A. Bicer (Ed.), *Creativity-Directed Mathematical Tasks for 5th Grade Common Core Classrooms* (pp. 391-404). ISTES Organization.

SECTION 8 - CONVERT LIKE MEASUREMENT UNITS WITHIN A GIVEN MEASUREMENT SYSTEM

Task 29 - Deciphering Deworming Metric Measurements

Michelle Tudor, Michael Gundlach, Melena Osborne

Content Standards

CCSS.MATH.CONTENT.5.MD.A.1

Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05m), and use these conversions in solving multi-step, real-world problems.

Practice Standards

CCSS.MATH.PRACTICE.MP1

Make sense of problems and persevere in solving them.

CCSS.MATH.PRACTICE.MP6

Attend to precision

CCSS.MATH.PRACTICE.MP8

Look for and express regularity in repeated reasoning.

Lesson Objective

Students will learn how to convert commonly used metric units and customary units to each other. The focus will initially be on units of volume conversions between milliliters and liters, and then the students will extend this knowledge to length conversions. Creativity will be emphasized as students engage in problem solving to develop innovative solutions to authentic problems.

Vocabulary

Unit conversion, metric system, customary units, metric units, milliliter, deciliter, centiliter, liter, dekaliter, hectoliter, kiloliter

Materials List

Container of water (a large bowl), a 1 ml syringe (from a baby Tylenol box or similar), a graduated cylinder and an empty 1-liter bottle, copies of MSU article, appendices A, B, and C

Anticipatory Set

On a table in the front of the room, display the following items. A container of water, a 1 ml syringe, a graduated cylinder, and an empty 1-liter bottle (like a soft drink bottle). Ask students to write down an estimate about the number of 1 ml syringes of water it would take to fill the 1-liter bottle. Ask one student to come up and put 10 syringes of water in the graduated cylinder. Compare it to the liter and allow students to talk to a partner to see if they want to adjust their estimate. Ask 9 more students to place 10 syringes of water in the graduated cylinder. At this time talk about how many ml of water are in the cylinder (100). Pour this amount into the liter bottle and ask students to estimate how many more 100 ml will it take to fill the bottle. Have students keep track as you pour the amounts into the liter bottle. Ask students what they notice about the relationship between the amounts of water in each container (powers of 10). Then introduce the prefixes milli-, centi-, and deci- and explain which prefix matched the amounts of water just demonstrated.

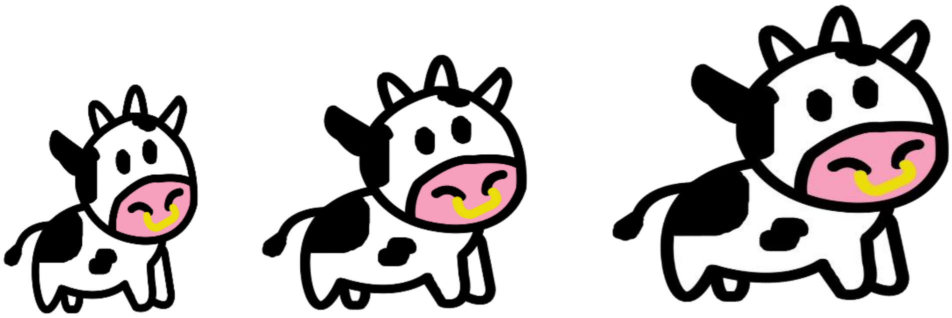
Engage

To engage students in this lesson, explain to them the importance of deworming cattle. Watch this short video to help students understand the concept of deworming a cow.
https://www.youtube.com/watch?v=VD1WmlMNTNE&t=49s

Read this short article to the students from Michigan State University to help with explaining the importance of deworming cattle: https://www.canr.msu.edu/news/beef_cattle_deworming_strategies. Tell students that there are two popular types of dewormer for cattle: a pour on dewormer (that you pour down their back) and an injectable dewormer. Each of the dewormers requires a different amount per cow based on body weight. The pour on dewormer requires 1mL per 22lbs, so on average 60 mL per cow. The injectable dewormer requires 1mL per 110lbs, so on average 12 mL per cow. Explain to the students that mL (milliliters) is a metric unit.

Explore

Give the students the chart attached in Appendix A (liters conversion chart). At this point, students already have prior knowledge of decimals. Ask students to study the chart and discuss anything they observe (patterns, division by 10, multiplication by 10, etc.). Now give the students the following problem to ponder: Suppose Rancher John wants to deworm his herd of cows. He has decided to use the pour on dewormer because he thinks it will be quicker than the injectable. Remember, the pour on dewormer requires 1mL per 22lbs, so on average 60 mL per cow. Rancher John has 3 cows that are not the average cow weight though, so he needs to calculate how much dewormer to give them. One cow is 800lbs, one is 1050lbs, and one is 1575lbs. How much dewormer does each cow need? (Round answers to the nearest whole number). Now, Rancher John has a problem: his pour on gun doesn't measure in mL; it measures in deciliters. Using the values you found previously, examine Appendix A, and help Rancher John determine how much dL he needs of the wormer for each cow. Let the students work in groups of 2-3 here. Give them time to explore and discuss different strategies and ideas.



800 lbs 1050 lbs 1575 lbs

Explain

Have 2-3 groups of students present their solution strategies to the class. Make sure to have different strategies presented. As a class, fill in the extended table found in Appendix B. Have students discuss and then share with the class why the extended table may be more useful at times, and how they can quickly create it. Use the extended table to help Rancher Heather determine how much injectable dewormer is needed for the cows weighing 800 lbs, 1050 lbs, and 1575 lbs. As a reminder, the injectable dewormer requires 1mL per 110lbs, so on average 12 mL per cow.

Extend

Give students a copy of appendix C, where they have tables, they can fill out for length units. Challenge students to come up with an object that would have a length appropriately measured by each unit. If students are stuck, appendix D has a filled in version of the table with ideas for the objects that work well with each unit.

Evaluate

During the Anticipatory Set, it is important to listen to student reasoning and ideas because they will probably have some great ideas since they (probably) don't know about these measurements yet. Possibly use some of the students' reasoning and ideas to build the rest of the lesson on.

During the Explore part, listen for use of prior knowledge. If students are having trouble activating prior knowledge, prompt them with questioning. For the problem, listen for ideas and reasoning about the conversions. Students should use the charts in Appendix A here. Listen to ensure that students are doing conversions correctly; if they are struggling, give them some guidance by pointing out the "pattern" in the chart in Appendix A. You could also give them a different conversion example (e.g., how would you convert 2 liters to dekaliters? What about 14 liters to deciliters?)

During the Explain part, ask questions and have students share out answers to check for understanding of the Explore problem. Circulate the room as students work on the second part of the problem (using the injectable dewormer) and make sure students are on the right path. If students are struggling, ask guiding questions and refer back to the first part of the problem that they did. Have students share out answers and discuss the problem as a whole class.

During the Extend part, check to see that students are using and extending what they learned about volume conversions for length conversions. If students are struggling, go over an example or two with the whole class and then have them continue working in groups of 2-3.

Appendix A. Liters Conversion Chart

Unit Name	Value Equivalency to 1 Liter	Abbreviations
Kiloliter	1000	kL
Hectoliter	100	hL
Dekaliter	10	daL
Liter	1	L
Deciliter	0.1	dL
Centiliter	0.01	cL
Milliliter	0.001	mL

Appendix B

Unit Name	Value Equivalency to 1 Liter	Value Equivalency to 1 Milliliter	Value equivalency to 1 Hectoliter
Kiloliter	1000		
Hectoliter	100		
Dekaliter	10		
Liter	1		
Deciliter	0.1		
Centiliter	0.01		
Milliliter	0.001		

Appendix C

Unit Name	Value Equivalency to 1 Meter	Value Equivalency to 1 Centimeter	Value equivalency to 1 Kilometer
Kilometer	1000		
Hectometer	100		
Dekameter	10		
Meter	1		
Decimeter	0.1		
Centimeter	0.01		
Millimeter	0.001		

Unit Name	Object best measured in this unit
Kilometer	
Hectometer	
Dekameter	
Meter	
Decimeter	
Centimeter	
Millimeter	

Appendix D

Unit Name	Value Equivalency to 1 Meter	Value Equivalency to 1 Centimeter	Value equivalency to 1 Kilometer
Kilometer	1000	100000	1
Hectometer	100	10000	0.1
Dekameter	10	1000	0.01
Meter	1	100	0.001
Decimeter	0.1	10	0.0001
Centimeter	0.01	1	0.00001
Millimeter	0.001	0.1	0.000001

Unit Name	Object best measured in this unit
Kilometer	Length of a river
Hectometer	The length of a school building
Dekameter	The width of someone’s front yard
Meter	The height of a house
Decimeter	The length of someone’s leg
Centimeter	The width of someone’s hand
Millimeter	The width of someone’s fingernail

Citation

Tudor, M., Gundlach, M., & Osborne, M. (2023). Deciphering Deworming Metric Measurements. In A. Bicer (Ed.), *Creativity-Directed Mathematical Tasks for 5th Grade Common Core Classrooms* (pp. 405-414). ISTES Organization.

Task 30 - Shopping Abroad

Aylin S. Carey, Fay Quiroz, Traci Jackson

Mathematical Content Standard

CCSS.Math.Content.5.MD.A.1

Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real-world problems.

Supporting Standard

CCSS.MATH.CONTENT.5.MD.C.3

Recognize volume as an attribute of solid figures and understand concepts of volume measurement.

Mathematical Practice Standards

CCSS.MATH.PRACTICE.MP4

Model with mathematics.

CCSS.MATH.PRACTICE.MP5

Use appropriate tools strategically.

CCSS.MATH.PRACTICE.MP6

Attend to precision.

CCSS.MATH.PRACTICE.MP7

Look for and make use of structure.

Vocabulary

Convert, conversion, metric, meters, centimeters, decimeters, kilometers, inches, feet, yards, miles, grams, liters, calories.

Materials

6 sided dice (one for each group), prize (candy, juice, and nuts), one copy of Appendix A and B for each student, measuring cups and pitchers (optional), and base 10 blocks (optional).

Lesson Objective

In this lesson, students will describe how units within the different-sized standard measurement system (metric units and customary units) can be converted such as multiplying to find the number of smaller units in a larger unit or dividing to find the number of larger units in a smaller unit. The most important objective of this lesson is to have students understand the relationship between the units being converted and apply the relationship to real-life situations. Students will explore conversions in a creative way, through independent problem-solving skills and making generalizations. Students will be encouraged to use two-column tables to show relationships between units and make conversions to support calculation accuracy. This lesson will be completed in two-class time.

Engagement

While students have already learned in fourth grade to convert the relative size of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. and to express measurements in a larger unit in terms of a smaller unit, they have not made the connection between conceptual and procedural understanding or practiced thoroughly the use of the relationship between units to make conversions. To access students' background knowledge, ask students: "What do they know about measurement within the United States compared to around the world?" To help them understand that there are different types of measurement being used around the world, begin the lesson with a video that introduces the existence of a non-traditional measurement tool, called SMOOT (https://www.youtube.com./watch?v=bC_9IHOdd5c). If needed, teachers should pause the video to check for student understanding. For example, teachers may pause at 1:10 and ask students a series of questions: "Can anyone help clarify what is happening?," "What is a SMOOT? Is it a real unit?," and "What is the point of this task? What might happen next?" At the end of the video the same types of questions may be asked.

After students watch the video, create a discussion around the idea of different measurement units and tools. Ask students how they would measure a bridge if there were no units or tools given, what tools they would use, what they would call their unit, how they would describe it, and finally how they would check the accuracy of their unit. Then introduce the topic of traveling abroad with this question: "Have you ever traveled outside of the United States?" Wait for a response. If there are students who traveled outside of the country, ask the following question: "What measurement units have you noticed?" If no students traveled internationally, perhaps teachers can share their own travel stories or the stories of others with visual aids such as Figure 1, showcasing different weight units (i.e. kg and g) and currency (i.e. Turkish Lira).

Figure 1. Fruit Display Market in Izmir, Turkey

Explore-Explain Part I

(30 minutes) Tell students about the first store they walk into is a football store, but not just

American football, also European football! Ask students if they know the difference between American football and European football. Hopefully, students will recognize the similarities between European football and American soccer, and eventually realize that both sports are the same but with different names. They may also share information about the strategies and rules of the game. To build students' background about these two types of games, have a short conversation about the differences between American football and European football and how each game is played. The teacher can use this link to either watch as background information for themselves, or to show the students How to Play American Football. For example, the game begins with one team's kicker kicking the football to the other team's receivers. The receiving team then gets 4 chances to move the ball 10 yards with the goal of reaching the located at the opposite end of the field. The result of reaching the endzone is called a touchdown, awarding the team with six points. After scoring the touchdown, the scoring team may attempt one more play from a designated spot on the field. This team may choose to kick the ball between two upright bars for one point or may choose to run the ball or throw the ball forward into the endzone for two points. If at any point the team is unable to gain 10 yards within the four-down limit, they may choose to kick the ball in between the upright bars to be awarded three points. The game ends at the end of four 15-minute time periods, known as quarters, with the team scoring more points winning the game. Then, have a short conversation about how European football is played, and/or teachers can watch this short video on How to Play European Football. An example conversation may include the following information. European football is known to Americans as soccer. The game begins with one team kicking the ball off in the middle of the field to try to move the ball towards the opposing team's side of the field to score a goal by kicking it in the net, which is worth 1 point. The opposite team tries to stop the ball or gain possession of the ball, preventing allowed goals while simultaneously attempting to score goals while in possession. The only one who can touch the ball with their hands is the goalie unless the ball goes out of bounds (short for boundaries), which then a player can use their hands to pass the ball to their teammate. Play continues for two periods of 45 minutes separated by a resting "halftime," with the most periods at the end of time declared the winner.

Back to the store scenario. Remind students they have walked into a football store where half of the store is American football, and the other half is European football. When they walk into the store, the sales associate gives them a *Punt, Pass, and Kick Card* (see Appendix A). The associate says they will have to punt, pass and kick with an American football and

European football, and take their completed card to the checkout counter. The challenge at each station is to not only punt, pass or kick the specified footballs, but to convert their punt, pass or kick to the correct unit. When students have visited all the stations on both sides of the store, they take their cards to the checkout counter to collect their prize! For this activity, place students in groups of two-three people and give them each a six-sided dice and a *Punt, Pass, and Kick Card*. When students roll the dice, this will be the distance the ball will travel for either the punt, pass and kick (see figure 2). For example, if students begin on the American Football side of the store and roll a 2 for the punt, students would write 2 in the *Dice Roll* column and 38 in the *Distance and Units* column. In the *New Distance and Units* column, students convert 38 inches to yards by dividing 38 inches 36 = 1.056 yards. Similarly, for the European Football side of the store, students would write 2 in the *Dice Roll* column and 38 in the *Distance and Units* column. In the *New Distance and Units* column, students convert 38 kilometers to meters by multiplying 38 km 1000= 38,000 meters (see Figure 3).

Dice Roll	**Distance**
1	50
2	38
3	90
4	117
5	63
6	45

Figure 2. The Distance the Ball Traveled with a Dice Roll

Punt, Pass, and Kick Card

American Football	**Dice Roll**	**Distance and Units**	**New Distance and Units**
Punt	**2**	**38 inches**	**1.056 yards**
Show your work	**38 inches 36 = 1.056 yards**		

Punt, Pass, and Kick Card

European Football	Dice Roll	Distance and Units	New Distance and Units
Goalie Kick (Punt)	2	38 kilometers	38,000 meters
Show your work	38 km 1000= 38,000 meters		

Figure 3. Examples of Rolling a 2 on the Dice on the Punt, Pass and Kick Card

Explain to students the American football side will be in the U.S. Customary units (e.g., inches, feet and yards) and the European football side will be in Metric units (e.g., millimeters, meters and kilometers). Students should be familiar with both units of length from fourth grade.

Students should also have an understanding that when converting from bigger to smaller units one will multiply, and when converting from smaller to larger units one will divide. However, teachers may choose to explain further or have a more in-depth conversation with students and show Figure 4 as a reminder.

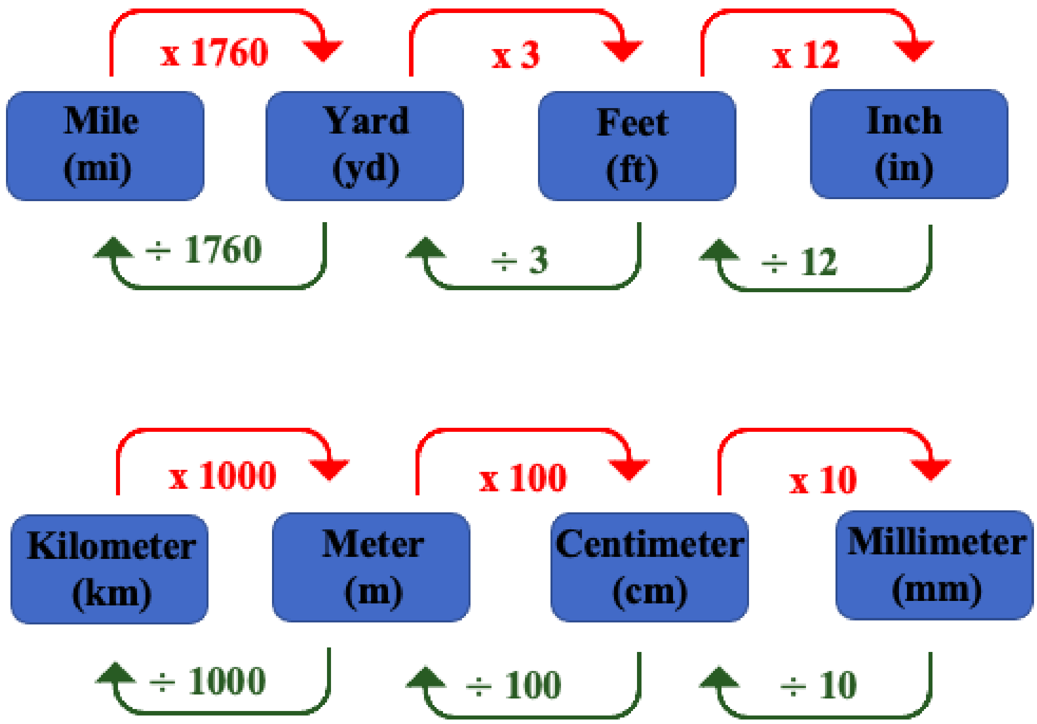


Figure 4. The U.S. Customary and Metric Unit Conversion Steps

For example, there are 12 inches in 1 foot and 3 feet in one yard for the U.S. customary units. An example of this students can think about is that there are more inches in a foot; therefore, to multiply by 12 (because there are 12 inches in 1 foot) when converting from inches to feet, like 6 feet equals 72 inches. Whereas converting from a smaller unit to a larger unit, like 6 feet to a larger unit like yards, one would divide by 3 (because there are 3 feet in 1 yard), and 6 feet would equal 2 yards. For metric the difference is the conversions are in increments of 10. Therefore, 1 meter is equal to 1,000 millimeters, and the decimal can be moved to the right three times because there are 1,000 times more millimeters in 1 meter.

The opposite is also true, the decimal can be moved to the left three times because in 1 meter there is equal to 0.001 kilometers. If the teacher feels like any students are struggling to understand this they may allow students to use the hallway or large enough space to measure the distance they punt, pass or kick to help understand the conversions conceptually. Tell students they can begin on either side of the store, but they must visit both sides of the store before taking their card to the checkout counter (this would be the teacher) to receive a prize (a suggested prize is a piece of candy). The prize choice can be changed by the teacher.

Explore-Explain Part II

(20 minutes) After visiting all of the stations on both sides of the football store, students are ready to cool down with some fresh juice. But first, they need to familiarize themselves with the international measurement for liquid. The international liquid measurement system includes milliliter (mL), centiliter (cL), liter (L), and kiloliter (kL), while the U.S. customary units include fluid ounce (fl oz.), cup, pint, quart, and liquid gallon.

It is important to note that the United Kingdom imperial units share the same name as the U.S. customary units; however, the number of ounces in pints, quarts and gallons are larger in the imperial system since they use different volumes measurements. The size of one fluid ounce is also different as the imperial system of one fluid ounce has 28.41 mL metric equivalency while the U.S. system of one fluid ounce measures 29.57 mL. It would be ideal for students to compare and contrast liquid measuring tools. For this, teachers should either show commonly used measuring cups and pitchers or the pictures of these tools with units on them (see Figure 5).

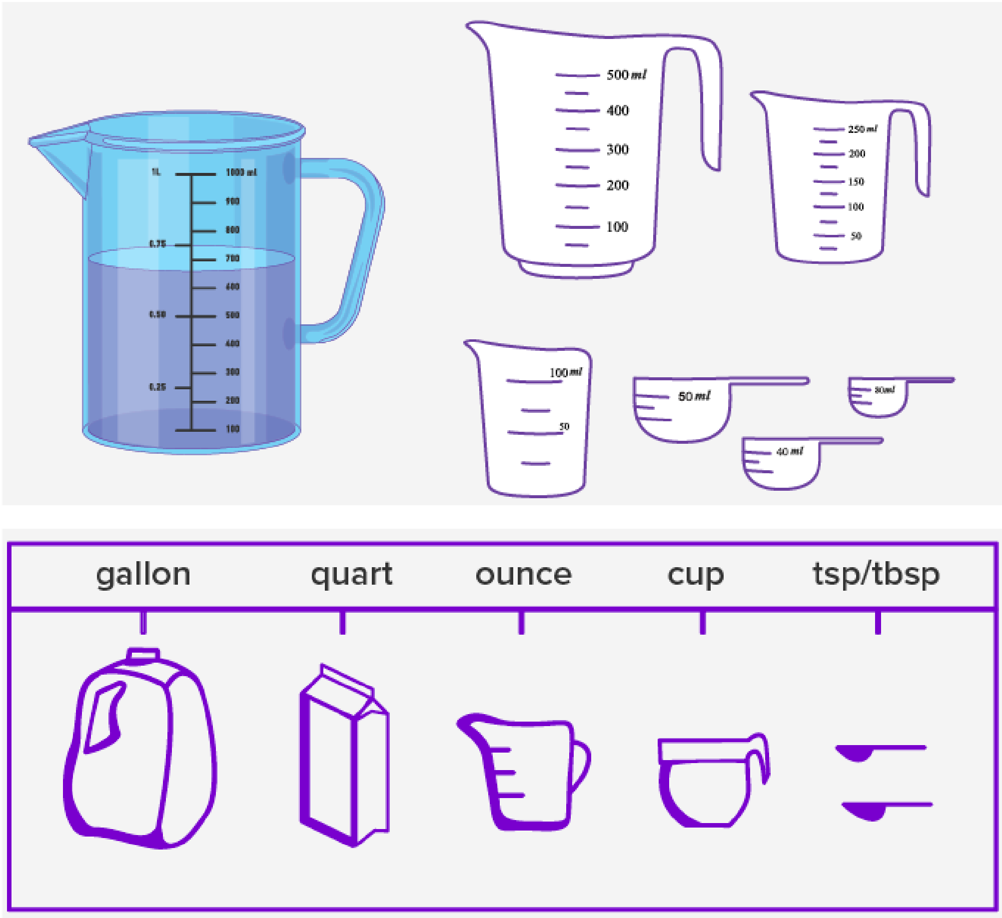


Figure 5. Examples of Measuring Tools in Metric and the U.S. Customary Units

The students should also be familiarized with the unit conversion. For this, show the conversion steps of the metric and the U.S. customary unit charts next to each other and ask students what they notice (see Figure 6). Students must be able to explain the relationship between liters and milliliters and use the relationship between units to make conversions as part of the learning targets. For example, similar to the length measurements, students should be able to generalize the international liquid measurements that each unit is 10 times smaller we move from liter to milliliter. Considering that there are hectoliter and decaliter between kiloliter and liter, the previous statement is also true when we move from kiloliter to liter. Similarly, when we move from liter to kiloliter or milliliter to liter, each unit is 10 times greater. Using Figure 6, students can provide examples to explain their generalization such as $1\ kL\ = 10x10x10\ =\ 10^3\ = 1000\ L$. After their generalizations, ask students what they notice between the metric and the U.S. customary unit charts. If students did not explain their thought process with conversions, ask students to write equalities for the U.S. customary units since they already did conversions with metric units. Students should provide examples to explain their generalization such as $2\ cups = 1\ pint$, $4\ cups = 2\ pints = 1\ quart$, $16\ cups = 1\ gallon$, or $1\ gallon = 4\ quarts = 8\ pints = 16\ cups$.

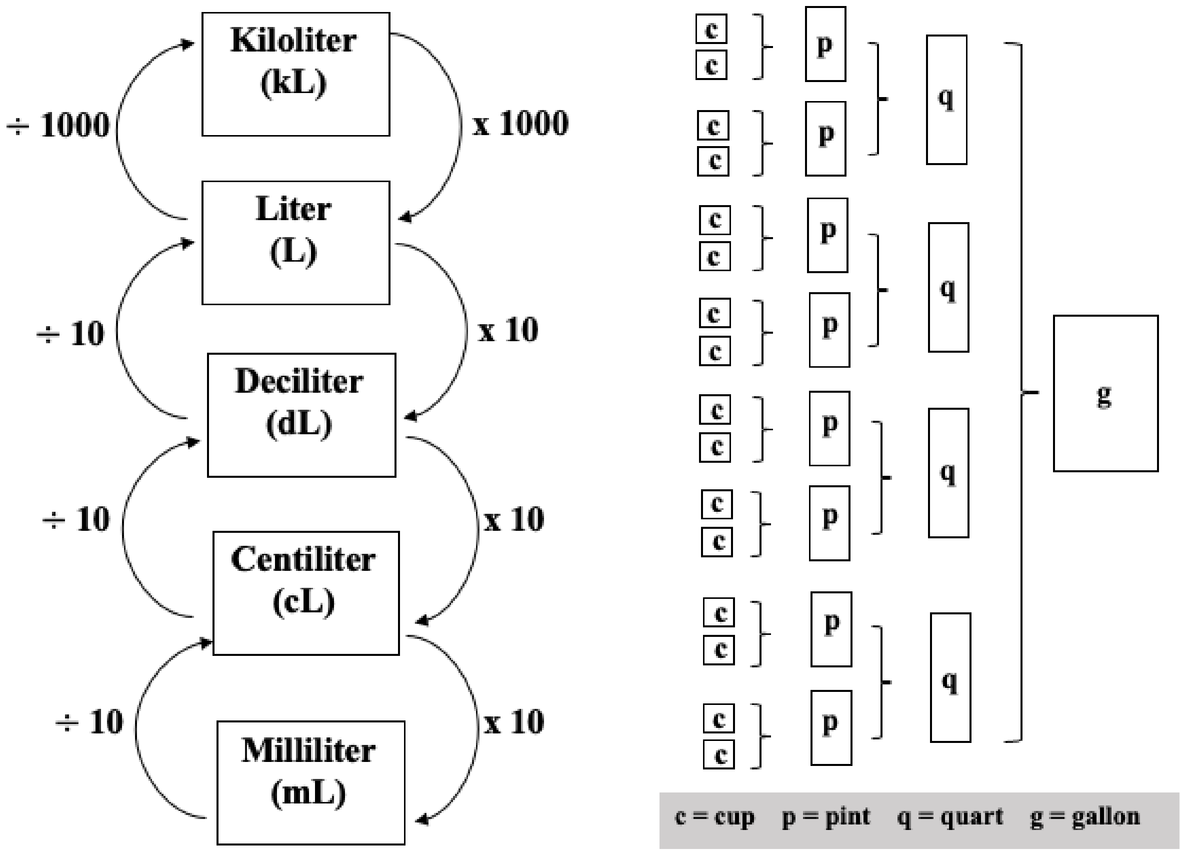


Figure 6. Metric and the U.S. Customary Unit Conversion Steps

After students explain their generalization, introduce the Juice Café they would visit after the football store. If students would like to purchase mixed fruit juice, the café serves them in one size cup. Only one flavor of fruit juice comes in a bottle. Since students are familiar with the U.S. customary units and most drinks come in small (12 oz.), medium (16 oz.), and large (20 oz.) size cups in the U.S., teachers should mention that both cups and bottles have a capacity of 350 mL in the Juice Café. Ask students to predict whether 350 mL would be close to small, medium, or large size U.S. cups. After students make their prediction, provide the conversion: 1 fluid ounce = 29.57 milliliters, and have them calculate to check their prediction. Students can use a calculator to divide 350 mL by 29.57 mL to notice the answer 11.836 oz. close to 12 oz. and therefore, the cup size is equivalent to a small U.S. cup size and both cups and bottles have a capacity of 12 oz. At the Juice Café, a challenge awaits students. The café only has 24 bottles left, and there are 30 students. Assuming that each student drinks about the U.S. size 1 cup (16 ounces = 2 cups). The task is to figure out if there will be enough drinks for everybody. Students can get together with their group members and solve this problem. Once they finish solving the problem, they visit with the café owner (this would be the teacher) to receive a prize (a suggested prize is juice). The prize choice can be changed by the teacher. Students are expected to do the conversion correctly: Figure out how many ounces in one cup (8 ounces = 1 cup) and calculate the total amount of 30 students would drink (since each student drinks one cup and one cup equals 8 ounces: $30 \times 8 = 240$ oz.). After this calculation, figure out how much juice the café has: Since one bottle has a capacity of 350 mL and there are 24 bottles, $24 \times 350 = 8400$ mL. Students should have the freedom to work with ounces since they earlier figured that 350 mL is about 12 oz, to build flexibility with estimation and go back and forth between the two different units. Students can either divide 8400 mL by 29.57 mL ($8400 \div 29.57 = 284.072$ oz.) since 1 fluid ounce equals 29.57 mL or multiply 24 bottles by 12 oz. ($24 \times 12 = 288$ oz.) since one bottle has a capacity of close to 12 oz. Students may also figure out the conversion of 240 oz (since 1 fluid ounce equals 29.57 mL, $240 \times 29.57 = 7096.8$ mL.) Students should be able to conclude that the café will have enough juice for all students since 240 oz is smaller than 284.072 oz. or 7096.8 mL is smaller than 8400 mL. All calculations and reasoning should be considered when assessing students' understanding.

Explore-Explain Part III

(20 minutes) The last store the students will visit is Candy & Nut Shop, but first, they need to familiarize themselves with the international measurement for weight. At the shop, candies and nuts are sold in "kilo" (see Figure 7). The international weight measurement system includes gram (g), kilogram (kg), tonne (tonne), and megagram (Mg), while the U.S. customary units include ounce (oz), pound (lb), hundredweight (cwt), and ton (ton). Metric weight units are based on the weight of respective metric volumes of water (i.e., 1 liter of water = 1 kg). The students should also be familiarized with the unit conversion. Before showing the conversion steps of the metric unit chart (see Figure 8), ask students to predict the similarities and differences of the units after they have explored the length and liquid measurement systems. By now, students should be able to generalize the weight measurement also includes numbers in powers of 10. By observing the previous charts and based on their

previous knowledge from fourth grade, students should be able to provide an example as 1 kilogram = 1000 grams. To convert kilogram to grams, students would multiply by 1000.

Figure 7. Walnuts on Display at the Candy & Nut Shop in Izmir, Turkey

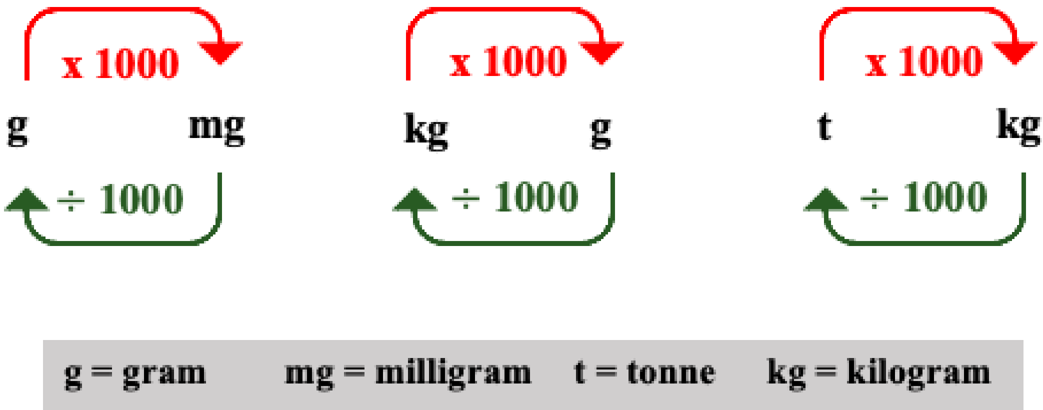


Figure 8. Metric Unit Conversion Steps

After students explain their generalization, come back to the Candy & Nut shop story. Since this is the last place they would visit, the goal is to purchase some candies and nuts to bring back with them to the U.S. However, they have to follow the travel restrictions with the weight limit. Each traveler is restricted to one suitcase with a maximum weight limit of 50 pounds. Assuming that their suitcase already weighs between 35 to 40 pounds, students need to figure out how much candies and nuts they can buy. Since they already spend some of their money in other places, students have to be very strategic with their food choices and spending at this last shop. Remind students that 1 kilogram = 2.20462 pounds. When calculating, allow students to use 1 kg = 2.2 lbs. The learning target is that students are able to use the relationship between units to make conversions. For example, if the shop has packaged goods with the measurement of a gram, students should be able to figure out the conversion between gram and pound (if 1 kg = 1000 g = 2.2 lbs, then 1 g = $\frac{2.2}{1000}$ = 0.0022 lbs.) Provide students with a price list (see Figure 9) to help their decision. Students can get together with their group members; however, each student should have their own list of items.

Roasted and Salted Peanuts	1 package – 142 g	110 L
Kadayif with Walnuts	1 kg	146 L
Sekerpare with Hazelnuts	1 kg	146 L
Raw Hazelnuts	1 package – 142 g	100 L
Mixed Turkish Delight	1 kg	146 L
Mixed Nuts	1 package – 142 g	88 L
Walnuts	1 kg	95 L

Figure 9. Produce, Amount, and Price List of the Candy & Nut Shop.

Students also need to know the currency rate (i.e., 1 U.S. Dollar = 14.82 Turkish Lira). Teachers can check the currency rate on the day they give the lesson if they prefer. Since each student would create their own list of items, teachers should assign the suitcase weight and amount of money students can spend. For example, student A has a suitcase that weighs 39 pounds and 256 Turkish Lira to spend, student B has a suitcase weighs 41 pounds and 320 Turkish Lira to spend, and student C has a suitcase weighs 45 pounds and 140 Turkish Lira to spend. Teachers can use the random number generator website to assign these numbers, using 35 for minimum and 40 for maximum for the suitcase weight and 100 for minimum and 500 for maximum for the spending allowance. Figure 10 is an example of what type of list students may create. For example, student A should be able to calculate the amount of candies and nuts they can buy by subtracting 39 pounds from 50 pounds ($50 - 39 = 11$ lbs) and converting 11 lbs to kilogram ($\frac{11}{2.2}$ = 5 kg). Based on their list, listen for a conclusion such as the student A's suitcase would have some room after the shopping but the student would only be left with 22 Turkish Lira, not leaving enough to buy for more candies or nuts.

Student A	**Student B**	**Student C**
Current suitcase weight: 39 lbs Room Capacity: 11 lbs (5 kg) Allowance: 256 L	Current suitcase weight: 41 lbs Room Capacity: 9 lbs (4.1 kg) Allowance: 320 L	Current suitcase weight: 45 lbs Room Capacity: 5 lbs (2.3 kg) Allowance: 140 L
Shopping List: 1 package mixed nuts 1 kg mixed Turkish delight Total weight: 1.142 kg Total amount: 234 L	Shopping List: 1 package mixed nuts 1 kg walnuts 1 package roasted and salted peanuts Total weight: 1.284 kg Total amount: 293 L	Shopping List: 1 package roasted and salted peanuts Total weight: 0.142 kg Total amount: 110 L

Figure 10. Student Example of a Possible Shopping List

Extend

Students can now explore how the units of length, area and volume change as the side lengths of a cube increase or decrease. Students will also explore how the metric system units are connected and add energy and temperature to their measurements. Tell students that at one of the stores they found a tiny cube charm. Show students a picture or actual centimeter cube (see Figure 11). Ask what the side length would be in millimeters. Ask what the area of the base would be in centimeters, then millimeters. Look for students saying 10 millimeters. Show them a picture of the centimeter cube divided into millimeters to address that area scales in two directions, creating 100 square millimeters. Ask students how many millimeter cubes they think would fit in a centimeter cube. Optional: Give students base ten blocks to explore this. Discuss how students are seeing this (in layers of 10 or 100 and then multiplying).

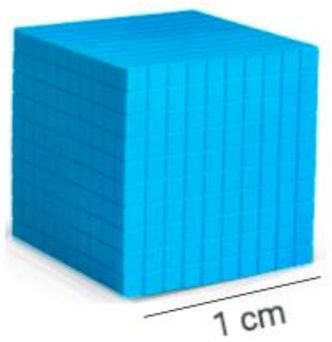


Figure 11. Visual of Centimeter Cube and Centimeter Cube Divided into Millimeter 100 Cubes

Next, hand out the printable (see Appendix B) and have students work in groups to create their own metric cube dimension and convert the side length, area of the base, and volume to either smaller or larger units. Once the group has their cube and conversions, have them provide their cube measurements to another group and ask them to convert. To close, ask students how their cubes compare to the centimeter cube length, area of the base, and volume. They may notice that the side from 1 cm to their cube increases by a scale factor, while the area increases by that scale factor squared, and the volume increases by the scale factor cubed. See Figure 12 for possible responses.

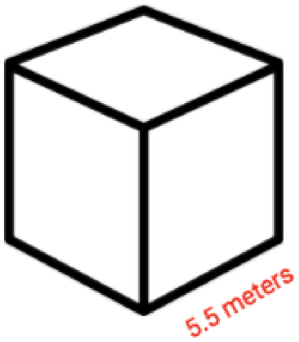
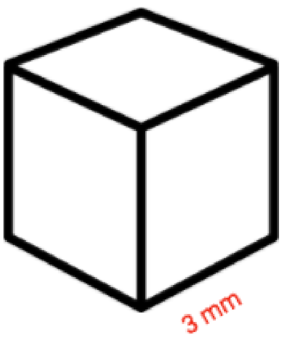
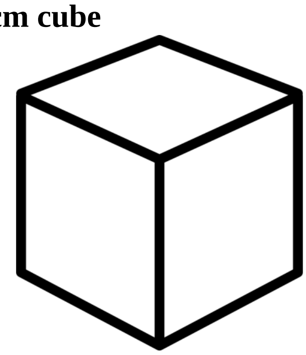
Our Cube 	**Side length** **5.5 m** **Side length with different unit** **550 cm**	**Area of the base** **30.25 square meters** **Area of the base with different unit** **302,500 square cm**	**Volume** **166.375 cubic meters** **Volume with a different unit** **166,375,000 cubic cm**
Neighbor's Cube 	**Side length** **3 mm** **Side length with different unit** **.3 cm**	**Area of the base** **9 square mm** **Area of the base with different unit** **.09 cm**	**Volume** **27 cubic mm** **Volume with a different unit** **.027 cubic cm**
1 cm cube 	**Side length** ***1cm*** **Side length with different unit** ***10mm***	**Area of the base** ***1 square cm*** **Area of the base with different unit** ***100 square mm***	**Volume** ***1 cubic cm*** **Volume with a different unit** ***1000 cubic mm***

Figure 12. Sample Student Responses for creating their own Measurements for the Cube

Ask students how much water they think would fit into an empty cm cube. What might that amount of water weigh? Ask students what they know about the word calorie. Students will likely mention food, some with more calories than others. Let students know that scientists use the word calorie to measure any kind of energy. The word “calorie” we use for food is actually a kilocalorie. Ask how many calories would be in a 1 kilocalorie. A calorie (not the one we use for food) can be defined as the amount of energy it takes to raise the temperature of 1 gram of water 1 degree centigrade (temperature).

Here the teacher may need to have a conversation about Celsius and Fahrenheit temperature. Tell students that 1 gram is actually the weight of a cubic centimeter amount of water. Ask how many millimeter cubes that would be to connect them back to the previous activity. Tell students that while shopping they bought a 2-liter bottle of water to make tea and ask how many calories it would take to heat the water from room temperature (give them 20 degrees Celsius, or allow them to look this up) to boiling (100 degrees Celsius.) Ask students to create their own questions about the 2-liter bottle such as “How much would the water weigh” or “How many cubic centimeters of water would fill the bottle?”

Evaluate

Teachers should first look for students to identify the two different measurement systems, standard (also known as U.S. customary) and metric, and the relative sizes of measurement within one system. Once students understand this relationship, they should be able to use the relationship to convert from bigger to smaller units by multiplication and from smaller to larger units by division. Students should generalize that this relationship is true for standard and metric systems. Additionally, when students are converting in the metric system, the generalization teachers can look for is that the decimal will move to the right for every 10 that is multiplied, or to the left for every 10 that is divided.

As students work through the extended problems, look for students using the length scale also for the area and volume. Use visual models to help students see the connection. Have students connect different ways to change their measurements. For example, in Figure 12, for the Our Cube column, they may multiply 550cm x 550 cm to arrive at 302,500 square cm or multiply 30.25 square meters by 10000 to arrive at 302,500 square cm. Be sure to point out that both side lengths are multiplied by the conversion so the 100 cm in 1 meter becomes

100x100 square centimeters in 1 square meter. Have students uncover how scaling the side impacts area and volume differently.

Reference

SplashLearn. (n.d.). *Liquid measurement chart: Definition with examples* [StudyPad]. SplashLearn.com. https://www.splashlearn.com/math-vocabulary/measurements/liquid-measurement-chart

Appendix A

Punt, Pass, and Kick Card

American Football	Dice Roll	Distance and Units	New Distance and Units
Punt		______________ ________ **inches**	________ **yards**
Show your work			
Pass		______________ ________ **feet**	________ **yards**
Show your work			
Kick		______________ ________ **yards**	________ **inches**
Show your work			

If you're finished with this side, go check the other side! If BOTH are COMPLETED, go to the checkout counter!

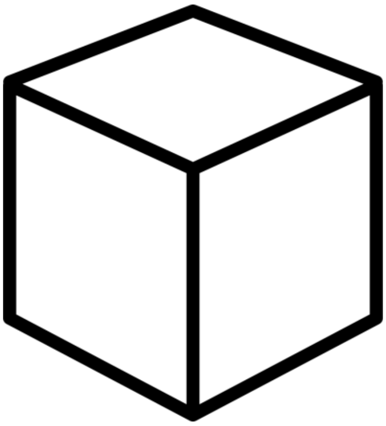
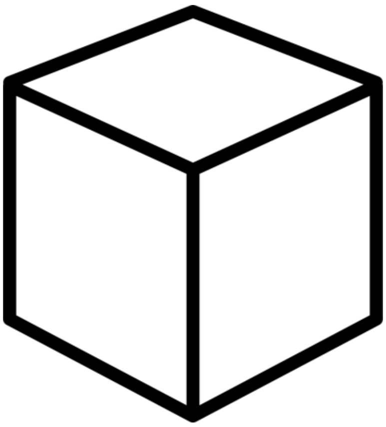
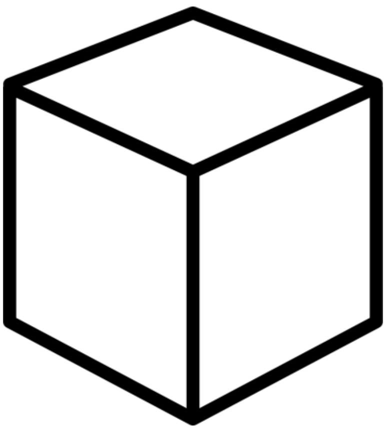
Punt, Pass, and Kick Card

European Football	**Dice Roll**	**Distance and Units**	**New Distance and Units**
Goalie Kick (Punt)		____________________ ________kilometers	____________meters
Show your work			
Throw-In (Pass)		____________________ ________millimeters	____________meters
Show your work			
Penalty Kick		____________________ ___________meters	__________kilometers
Show your work			

If you're finished with this side, go check the other side! If BOTH are COMPLETED, go to the checkout counter!

Appendix B

Measurement Creating Activity

Our Cube	Side length	Area of the base	Volume
Our Cube 	**Side length** ________________ **Side length with different unit** ________________	**Area of the base** ________________ **Area of the base with different unit** ________________	**Volume** ________________ **Volume with a different unit** ________________
Neighbor's Cube 	**Side length** ________________ **Side length with different unit** ________________	**Area of the base** ________________ **Area of the base with different unit** ________________	**Volume** ________________ **Volume with a different unit** ________________
 **1 cm cube**	**Side length** *1cm* **Side length with different unit** *10mm*	**Area of the base** *1 square cm* **Area of the base with different unit** *100 square mm*	**Volume** *1 cubic cm* **Volume with a different unit** *1000 cubic mm*

Citation

Carey, A. S., Quiroz, F., & Jackson, T. (2023). Shopping Abroad. In A. Bicer (Ed.), *Creativity-Directed Mathematical Tasks for 5th Grade Common Core Classrooms* (pp. 415-436). ISTES Organization.

Task 31 - How Long?

Jennifer Kellner, Amy Kassel, Chuck Butler

Mathematical Content Standards

CCSS.MATH.CONTENT.5.MD.A.1

Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real-world problems.

Mathematical Practice Standards

CCSS.MATH.PRACTICE.MP4

Model with mathematics.

CCSS.MATH.PRACTICE.MP5

Use appropriate tools strategically.

CCSS.MATH.PRACTICE.MP7

Look for and make use of structure.

Vocabulary

Metric system, standard system, nonadjacent measurements

Materials

Rulers with both metric and standard measurements, Meter/Yardsticks with both metric and standard measurements, measuring tape

Lesson Objective

Students will apply their understanding of taking measurements to convert within a given measurement system. They will use this conversion to solve real-world problems. This lesson will promote creative thinking by asking students to visually represent each conversion and includes multiple representations.

Engagement

(20 minutes) Begin the lesson by having the students get in groups of 2-3. The teacher will discuss the difference between the metric and US Standard systems of measurement. The following YouTube video can be shown to help with the difference Metric System Explained Simply (We Grow People, 2021).

Each group will select one of the systems. Each group will measure 4 to 5 objects (such as books, students, doors, desks) in the classroom using only one measurement in their chosen system (Appendix A). For example, if students choose the customary system, they can choose inches, feet, or yards. If the metric system is selected, students may choose centimeters, millimeters, or meters. The teacher should suggest the measurement of many different objects so not everyone is measuring the same.

Once the initial measurements are taken, have the groups switch with another group. Using the same 4-5 items from the group they switched with, have the students measure the same items using a different measurement from the same system. For example, if the first group used feet, the second group will measure the same 4-5 items in something like inches or yards.

Explore

(30 minutes x2) Part 1: Teachers will use the data students collected during the engagement phase of the lesson for students to explore how to convert between measurements within a given system.

Ask students to create class data by putting their data for their measurements in the appropriate table (either US standard measurement or metric measurement) (Appendix B). After students have added their data to the class table, do a Notice and Wonder. Teachers may ask students, “What do you notice and what do you wonder about the data?” Some sample student responses may include:

- Students may notice that when they measured an object in feet and inches there were always more inches than feet, or similarly for yards and feet.
- Students may notice that an object’s length measured in feet was 3 times the length measured in yards.
- Students may notice that when they measured an object in millimeters and centimeters there were always more millimeters than centimeters, or similarly for centimeters and meters.
- Students may notice that an object’s length measured in centimeters was 100 times the length measured in meters.
- Students may wonder how the measurements for yards and inches or feet and inches or meters and millimeters are related.
- Students may wonder if they can measure the data with other units.
- Students if how to relate the measurements between the systems.
- Students may wonder why there are two measurement systems.

Teachers may use the other notice and wonderings to help students explore the relationships between measurements in each system. Ask the class which system they wish to explore first.

Since both systems will need to be explored, responses for each system will be included within this lesson plan. However, teachers may choose to explore each system during different lessons.

Within the one system, ask students to do a Quick Write to write down everything they know about measuring length in the US Standard/Metric system. Sample student responses may include:

- We measure things in inches, feet, yards, and miles.
- I know there are 12 inches in a foot or 3 feet in a yard.
- We measure things in millimeters, centimeters, meters, and kilometers.
- I know there are 10 millimeters in a centimeter or 100 centimeters in a meter.

Teachers ask the class to share what they wrote about measuring length. Teachers may record the information on a table on the classroom display as students are talking.

Length	
1 foot	12 inches
1 yard	3 feet or 36 inches
1 mile	5280 feet or 1760 yards

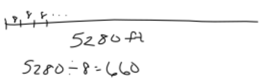
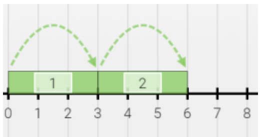
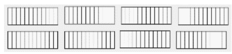
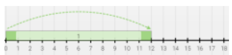
Length			
Prefix	Unit of measurement	Meaning	Abbreviation
kilo	kilometer	one thousand (1000)	k
hecto	hectometer	one hundred (100)	h
deka	dekameter	ten (10)	da
	meter	one (1)	
deci	decimeter	one-tenth (1/10)	d
centi	centimeter	one-hundredth (1/100)	c
milli	millimeter	One-thousandth (1/1000)	m

Part 2: Students will now explore converting between units (within one system). Teachers may pair up students to explore the conversions.

Ask each pair of students to select two objects and one of the corresponding measurements from the class data list.

Ask that students select different units of measures for each of their objects. Ask students to complete the conversions to the other units within the system by showing a model to represent their thinking (see Appendix C).

As students are working, teachers should observe their solution paths looking for a variety of solution paths for students to share. Sample student responses:

Object	Miles	Yards	Feet	Inches
Door	Student explanation: There are 5280 feet in a mile, so if I divide 5280 into 8 feet sections, there are 660 sections. Each doorway is 1/660 mile tall.	Student explanation: There are 3 feet in each yard, so in 8 feet there are $2\frac{2}{3}$ yards.	8 feet	Student explanation: There are 12 inches in each foot, so there are 96 inches in 8 feet.
Model				
Desk	Student explanation: I divided each time by the conversion factor.	Student explanation: Each yard has 3 feet, so $1\frac{1}{2}$ feet is the same as $\frac{1}{2}$ yard.	Student explanation: There are 12 inches in each foot, so in 18 inches there are $1\frac{6}{12} = 1\frac{1}{2}$ feet.	18 inches
Model	$18\ in \div 12\ in/ft = 1.5\ ft$ $1.5\ ft \div 3\ ft/yd = 0.5\ yds$ $0.5\ yds \div 1760\frac{yds}{mi} = \frac{1}{3520}\ mi$			

Object	kilometer	hectometer	dekameter	meter	decimeter	centimeter	millimeter
Height of student	Student explanation: Since there is 1/1000 km in each meter, 1.5 m is equivalent to 0.15 km	Student explanation: Since there is 1/100 hm in each meter, 1.5 m is equivalent to 0.15 hm	Student explanation: Since there is 1/10 dkm in each meter, 1.5 m is equivalent to 0.15 dkm	1.5 m	Student explanation: Since there are 10 dm in each meter, 1.5 m is equivalent to 15 dm	Student explanation: Since there are 100 cm in each meter, that means there are 50 cm in .5 m, so 1.5 m is equivalent to 150 cm	Student explanation: Since there are 1000 mm in each meter, 1.5 m is equivalent to 1500 cm
Model	$1.5 \times \frac{1}{1000} = 0.0015$	$1.5 \times \frac{1}{100} = 0.015$	$1.5 \times \frac{1}{10} = 0.15$			$1\ m = 100\ cm$ and $.5\ m = 50\ cm$ $100 + 50 = 150\ cm$	$1.5 \times 1000 = 1500$

Images from The Math Learning Center (The Math Learning Center, 2022).

Part 3: Students will complete How Long? Activity (Appendix C).

In this activity, students will explore converting between nonadjacent measurements through problems in context.

For example, converting from miles to feet directly or centimeters to kilometers directly. Ask students to get back in their original groups of 3-4 to complete this activity.

As students are working, the teacher may wish to monitor student thinking by probing them to look for generalizations that might make their work easier. Sample student explanations:

World Record	Original Measurement	New Measurement
Longest Monster Truck (Las Vegas, NV)	9.75 m	___________cm
Explanation	Since there are 100 cm in 1 m, $9.75 \times 100 = 975$ cm	
Tallest Waterfall (Angel Falls, Venezuela)	3212 feet	___________ miles
Explanation	Since there are $3 \times 1760 = 5280$ miles in a foot, then $3212 \div 5280 = 0.608$ miles	
Shortest spy (Richebourg, France)	58 cm	___________ dkm
Explanation	To convert from cm to dkm, you would divide by 10 for each step, $\div$ 10 to convert to dm, $\div$ 10 to then convert to m, $\div$ 10 to then convert to dkm, or $\div$ 1000 to convert directly from cm to dkm, 58 cm $\div 1000 = 0.058$ dkm	

Records from the Guinness World Records (Guinness World Records, 2022).

Explain

(30 minutes x2) Part 1: During the Notice and Wonder, teachers may be prepared to prompt students to see if they recognize how any of the units change, such as feet measurements are always 3 times the yard measurements or centimeter measurements are always 1/10 the millimeter measurements.

If students wonder how to relate the measurements between the systems, teachers will explain that while sometimes this needs to be done, the standard for 5th grade is asking to convert within one system not between systems.

If students wonder why there are two measurement systems, teachers will explain why the United States uses two measurement systems. See https://www.britannica.com/story/why-doesnt-the-us-use-the-metric-system for a brief history of why the U.S. does not use the metric system (Britannica, n.d.).

During the Quick Write debrief, teachers may be prepared to elicit students' background knowledge on measurement lengths by asking questions such as

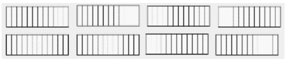
- What are some ways you measure lengths or distances?
- When you ride in a car, how do you measure how far you travel?
- When you look at a ruler, how do you know how many millimeters are in a centimeter?
- When you look at a yardstick, where do you see other measurements?
- How many centimeters are there on this meter stick?

As teachers are processing the information from the quick write, teachers may need to supply some conversions values as necessary. When generating information for the metric system, teachers may find it helpful to build students' knowledge around the prefix system in the metric system since it is used for measuring length, volume, and mass.

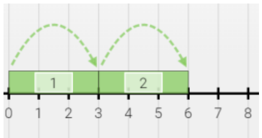
Part 2: As students are working with a partner to develop models for converting between the various measurements, teachers should be observing the various models to have students present to the class.

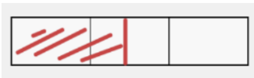
As students present their models, teachers may wish to ask students what similarities they noticed between converting. For example,

- when converting from a larger unit to a small unit, students may observe that the models represent multiplying, such as in these models. Example 1: Each foot has 12 inches. Since there are 8 feet, then there are 96 inches. Example 2: Each meter has 10 decimeters. Since there are 1.5 m, then there is 15 dm.

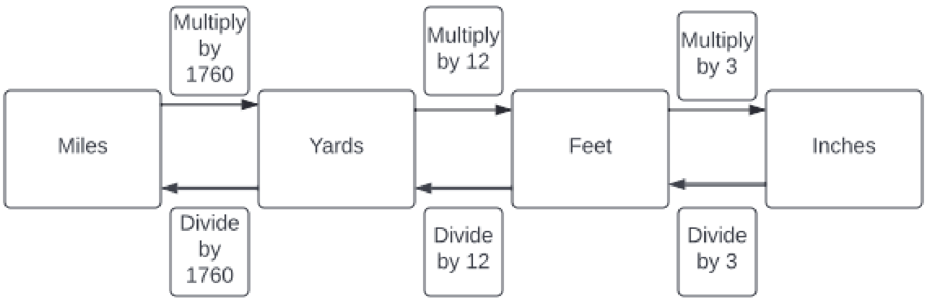


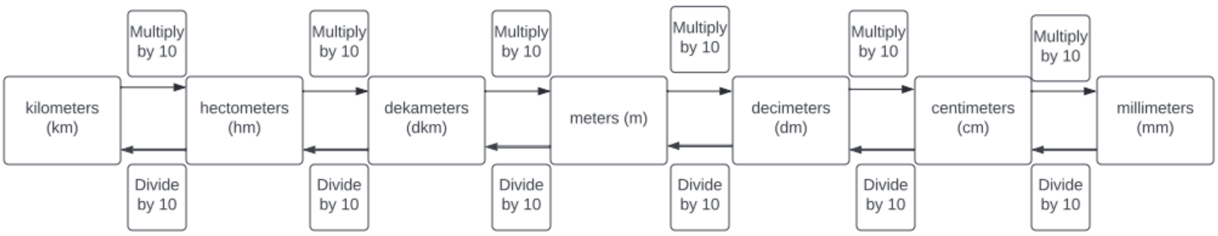
- when converting from a smaller unit to a larger unit, students may observe that the models represent division, such as in these models: There are 3 feet in each yard, how many groups of 3 are represented in 8 feet? or There are 2 groups of 1.5 feet in 3 yards.





As students recognize the patterns of multiplication and division, teachers may ask students to draw a visual representation of these patterns. Sample student responses may be:





Part 3: As students are working on the How Long? activity, teachers should monitor and observe student discourse looking to see that students have generalized how to convert between units with more than one step. Students should generalize that converting within the metric system can be done by multiplying and dividing by multiples of 10. If students need additional support, teachers may wish to review mental multiplication and division of multiples of 10. In the US Standard system, students may generalize the conversion between

inches and yards requires multiplying/dividing by 36 and the conversion between feet and miles requires multiplying/dividing by 5280.

Extend

The teacher may choose to assign one extension to the whole class or assign different extensions to different groups based on interest, ability, and readiness.

Which One is Better – At the end of part 1 or part 2, the students can decide which measurement within their chosen system is best to use for this specific item. For example, after measuring the book using inches and feet, the student could justify why they think one measurement is more appropriate than the other.

Disguise – Students convert measurements from the Guinness World Book of Records to a large or small unit. Once complete, students exchange the measurement with another group and then convert them again. For example, the tallest waterfall in the world would measure 38,544 inches.

222 – A person was born at 2:22 am on February 22, 2022. How old were they 222, 222, 222 seconds later? What time was it?

Oldest Person Ever – The oldest person whose age has been independently verified is 122 years and 64 days. Convert their age to seconds, minutes, or hours.

Ain Dubai – The world's largest Ferris wheel, the Ain Dubai (Dubai Eye), travels approximately 2,576 feet in 38 minutes. The student can pose unique mathematical questions and then answer them. Students could draw a visual model of the situation. For example, if you ride the Ain Dubai 10 times, how far did you travel? How long did it take? How far would you travel in 1 minute?

Evaluate

- In part 1, formative assessment for creativity and content occurs as students determine which objects to measure and what unit of measure to use within each system. While the students might not measure the items precisely, which could limit their ability to see the pattern, students can still analyze the data to get an estimate of the relationship between the two units of measurement.

- In part 2, formative assessment for creativity and content occurs as students decide on the object to convert, convert to two other units, and then model their thinking visually for each conversion. If a student struggles to model their thinking using a representation, then the teacher could encourage them to start with measurements that could be represented to scale on the paper, like inches and centimeters.
- In part 3, formative assessment for content occurs as students are converting to non-adjacent units. If students struggle to do this, the teacher could encourage the student to convert to a measurement that makes sense to them. For example, a student might struggle to convert from miles to inches directly but could convert miles to yards, yards to feet, and then feet to inches. If students complete the conversion with scaffolded questions, then formative assessment for content occurs as students try to write a rule for converting directly from miles to inches.
- In Which One is Better, formative assessment for content and creativity occurs as students decide in what contexts could specific units of measurement be most useful. The teacher should listen for justifications that are unique to the class and show understanding of the context. For example, measuring a book in centimeters makes sense but students might report the total length of books in a different measurement if they were stacking the books in large piles.
- In Disguise, formative assessment for content occurs as students convert the length of their object (and another groups' object) to a different measurement.
- In 222 and Oldest Person Ever, formative assessment for content occurs as students convert a real-world situation involving the measurement system of the time. If a student struggles to convert, then the teacher can ask scaffolded questions like "How many seconds in a minute?" or simpler cases like "If a person is 1 day old, how old are they in seconds, minutes, or hours?"
- In Ain Dubai, formative assessment for creativity and content occurs as students ask their mathematical questions and then answer their questions using a model. The teacher can look for trends in questions as well as unique mathematical questions that would allow students to accomplish the goal of the lesson.

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Appendix A

Which measurement system did you choose:

Metric or US Standard (Circle One)

1st Measurement Used ________ **2nd Measurement Used** ________	**1st Group Members**	**2nd Group Members**
What was the object?	**What is the length?**	**What is the length?**
1.		
2.		
3.		
4.		
5.		

Appendix B

US Standard System Measurements

Item Measured	Yards	Feet	Inches

Metric System Measurements

Item Measured	Meters	Centimeters	Millimeters

Appendix C

US Standard System

Object	Miles	Yards	Feet	Inches
Model				
Model				

Metric System

Object	kilometer	hectometer	dekameter	meter	decimeter	centimeter	millimeter
Model							
Model							

Appendix D

How Long?

Find the length of these Guinness Book of World Records. Explain your method.

World Record	Original Measurement	New Measurement
Longest Monster Truck (Las Vegas, NV)	9.75 m	__________cm
Explanation		
Tallest Waterfall (Angel Falls, Venezuela)	3212 feet	__________ miles
Explanation		
Largest Model Train Set (Miniatur Wunderland)	15.715 km	__________ m
Explanation		
Highest Mountain (Mt. Everest, Nepal)	9676.25 yds	__________ inches
Explanation		

Smallest Toad (Bufo Taitanus Beiranus, Africa)	24 mm	___________ dm
Explanation		
Shortest spy (Richebourg, France)	58 cm	___________ dkm
Explanation		
Shortest coastline (Monaco)	3.5 miles	___________feet
Explanation		
Shortest Street (Ebenezer Place, UK)	2.05 m	___________ hm
Explanation		

Citation

Kellner, J., Kassel, A., & Butler, C. (2023). How Long? In A. Bicer (Ed.), *Creativity-Directed Mathematical Tasks for 5th Grade Common Core Classrooms* (pp. 437-456). ISTES Organization.

Task 32 - Welcome to the Neighborhood!

Geoff Krall, Helen Aleksani

Mathematical Content Standards

Focus Content Standard

CCSS.MATH.CONTENT.5.MD.A.1

Convert among different-sized standard measurement units within a given measurement system, and use these conversions in solving multi-step, real-world problems.

Supporting Content Standard(s)

CCSS.MATH.CONTENT.5.NBT.A.1

Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.

CCSS.MATH.CONTENT.5.NF.B.3

Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

Mathematical Practice Standards

CCSS.MATH.PRACTICE.MP2

Reason abstractly and quantitatively.

CCSS.MATH.PRACTICE.MP5

Use appropriate tools strategically.

CCSS.MATH.PRACTICE.MP6

Attend to precision.

Lesson Objective

Students will create a tour of their or their school's neighborhood. The tour will include a map with a legend and proper unit conversions. Students will showcase their creativity as they design the walking tour, including visuals, charts, and an audio recording. The lesson takes a problem-based approach by activating curiosity with a solicitation from the principal of the school.

Engagement

(20 minutes) Start the lesson with a letter from the principal:

> Dear Students of __________ Elementary School,
>
> Hello students! I'm excited to tell you about an opportunity for your class! I constantly have parents asking me what they can do to support the local businesses in the area. Sometimes parents drop off their kids and want to see what's around. Other times they like to show up to the neighborhood early and do some shopping. I'd like you to help them out!
>
> I would like you to create a brochure for parents that might like to hang out near our school. Your brochure should feature the following:
>
> - A map of the school and surrounding area, highlighting at least four local businesses or attractions.
> - A 2-3 sentence description of each local business.
> - A chart showing the distances between each of the attractions as well as our school. The chart will be provided for you.
> - For each distance, please tell me the distance in miles, feet, kilometers, and meters and decide which might be the best to use for our brochure.
>
> I am excited about the wonderful products you will bring to our community!
>
> Sincerely,
> Your Principal

Alternatively, you may wish to ask the principal to come and share this information with the class directly. Using a letter or an in-person solicitation can make the task more engaging and exciting for students.

Begin the lesson by asking students how much they know about the neighborhood their school is in. Ask students to discuss businesses that are within walking distance of the school:

- Are there any restaurants?
- Are there any retail stores?
- Are there any other local businesses?
- Are there any other attractions (parks, trails, etc.)?

Write each business and attraction on the board as students are discussing them. Once you have a list of 5-10 places, ask students to estimate how far away each of the places are from the school and from each other.

Split students into groups of 3-4 and ask them to decide which four places they would like to include in their final product.

Explore

(20 minutes) Students will now research the actual walking distances between the school and each of the four places they have chosen. Students may wish to use google maps to find the distances.

Or, if given the proper permissions and supervision, they may actually go on a walk and measure.

Hand out the recording sheet (Appendix A). Have students find the distances first in miles and in kilometers before launching into the associated lesson (in the Explain section).

Figure 1 shows an example of a school neighborhood with surrounding business and attractions. (source: Google Maps).

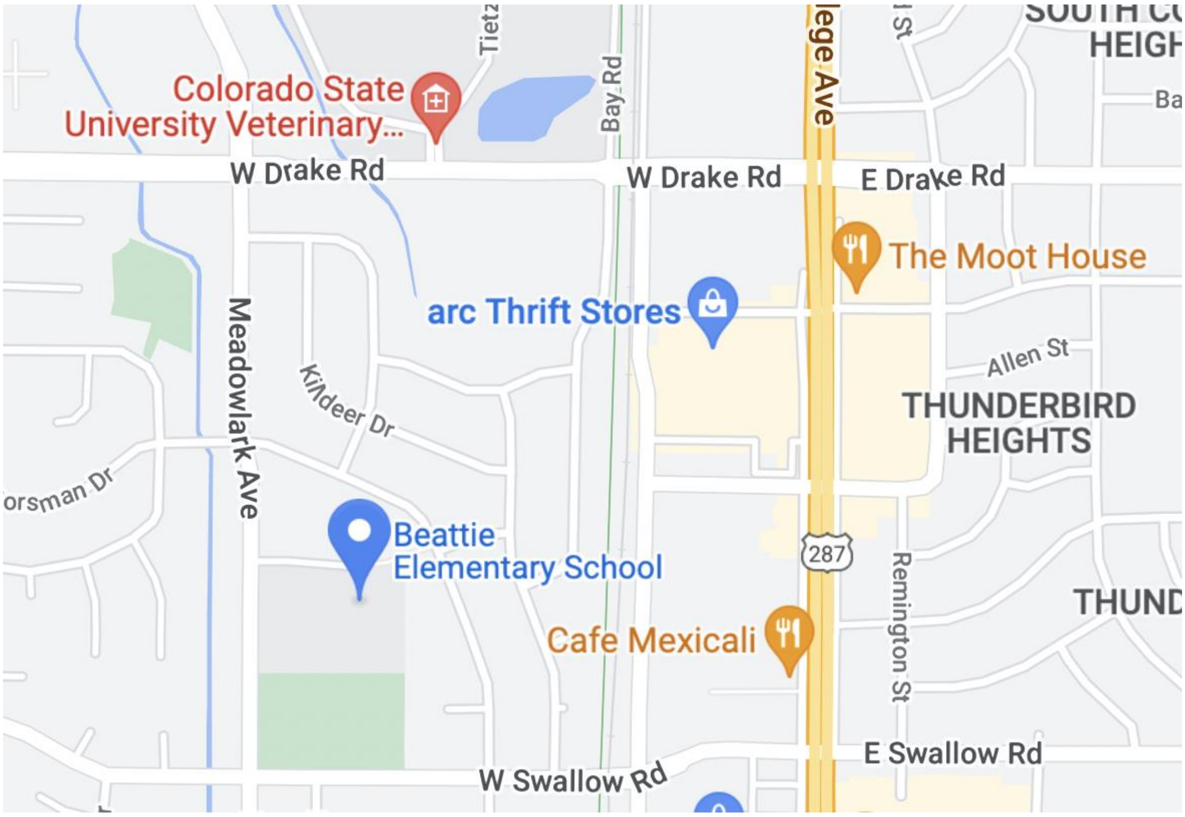


Figure 1. Example of a Neighborhood around an Elementary School

Explain

(20 minutes) Host a whole class lesson on converting units within standard unit types (for example: kilometers to meters to centimeters.

Explain to students that it is important to use a unit that helps the reader understand the distance or measure best. Use the following examples:

- Which distance is farther: 67,000 centimeters or 3 kilometers?
- How long will it take me to walk 67,000 inches? Will it take a long time or a short time?
- What if I told you to drive 30000 feet? Would that make sense?

Tell students that we are going to have a quick lesson on how to convert units within a measurement system. This means we are going to convert units of like system. For example, miles to feet to inches; centimeters to meters to kilometers; grams to kilograms, etc.

Demonstrate the following distance examples:

Convert 6000 meters to kilometers.

$6000\ meters \times \frac{1\ kilometer}{1000\ meters} = \frac{6000}{1000}\ kilometers$= 6 kilometers

Convert 4 kilometers to meters. Then convert that into millimeters.

$4\ kilometers \times \frac{1000\ meters}{1\ kilometers} = 4 \times 1000\ meters$= 4000 meters

Convert 500 meters into kilometers

$$\mathbf{500\ meters} \times \frac{1\ kilometer}{1000\ meters} = \frac{\mathbf{500}}{\mathbf{1000}}\mathbf{kilometers} = \mathbf{0.5\ kilometers}$$

Be sure to use proper units and unit notation as that will help keep track. Students may be used to crossing out the units that cancel one another out.

Then ask students to do the following to problems. During this time, monitor for proper units and unit notation.

Convert 7000 meters to kilometers.

Convert 600 meters to kilometers.

Convert 10 kilometers into meters.

Explain that metric units are nice and neat, but converting in standard units is more challenging.

Demonstrate the following distance examples.

Convert 6 feet into inches.

$6\ feet \times \frac{12\ inches}{1\ foot} = \ 6 \times 12\ inches$= 72 inches

Convert 48 inches into feet.

$48\ inches \times \frac{1\ foot}{12\ inches} = \frac{48}{12}\ feet\ = \ 4$ feet

Convert 3 miles into feet.

$3\ miles \times \frac{5280\ feet}{1\ mile} = 3\ \times 5280\ feet\ = \ 15840$ feet

Then ask students to do the following to problems. During this time, monitor for proper units and unit notation (see also, the Evaluate section).

Convert 108 inches into feet.

Convert 10560 feet into miles.

Once students demonstrate the capability to convert between units, return to the problem-based activity.

Extend

(40 minutes) Have students continue creating their walking tour. They should finish filling out their recording sheet.

Once students complete their recording sheet, students will begin creating an authentic artifact to showcase the attractions near their school. Options for final products will depend on the school's resources and the teacher's ability to show how to use various media software. Students may also choose to record audio, offering visitors an audio tour of the neighborhood. This could also occur as an accommodation for students for whom recording audio might be more suitable than typing or writing lengthy descriptions of businesses.

The time frame for this section could vary significantly based on the type of media product students produce.

Students will showcase their creativity in the design of the final product. Appendix B shows an example of a possible flyer for a final product. The flyer was made using Canva's free interface at canva.com.

Evaluate

Students may vary significantly in their understanding of unit conversations. It's possible you may wish to host the instructional section (the Explain section) to a small group instead of the whole group. After you demonstrate the examples, have students solve some conversion problems on their own. Monitor them for the correct use of units and an understanding of how to use units to make appropriate conversion.

Allow students a reference sheet for the unit conversions such as the one in Appendix C.

Some students may be able to do the calculations quickly, but make sure on their recording sheets they are including the unit conversions. Students that are demonstrating the material well can help other students, ensuring they are writing out the unit conversions.

The following might be some common mistakes students make:

- Multiplying by a unit conversion when they should divide (or vice versa).
- Not using the appropriate unit conversion.
- Losing track of the units they are converting from and the units they are converting to

In all of these cases, make sure students are writing out the full unit conversions as we did in the Explain section. If students need more practice, consider using the additional conversion problems in Appendix C.

In order to help students think carefully and deeply about each conversion problem, ask students to identify ahead of time whether the final number should be smaller or larger than the original numbers. For example, if you are converting feet to miles, the number should go down. If you are converting meters into centimeters, the number should go up.

Finally, you may wish to evaluate students even further by having them share their final products and ask different groups to do the conversions on another group's poster. For example have another group calculate four unit conversions of the poster in Appendix B. The groups can then check their answers based on their recording sheet (Appendix A).

Appendix A

Names of group members

The address of our school:

Our four locations with 2-3 sentences describing what each location is:

Location 1:

Location 2:

Location 3:

Location 4:

	Distance from school (miles)	Distance from school (feet)	Distance from school (meters)	Distance from school (kilometers)
Location 1				
Location 2				
Location 3				
Location 4				

Which unit makes the most sense to use in our brochure and why?

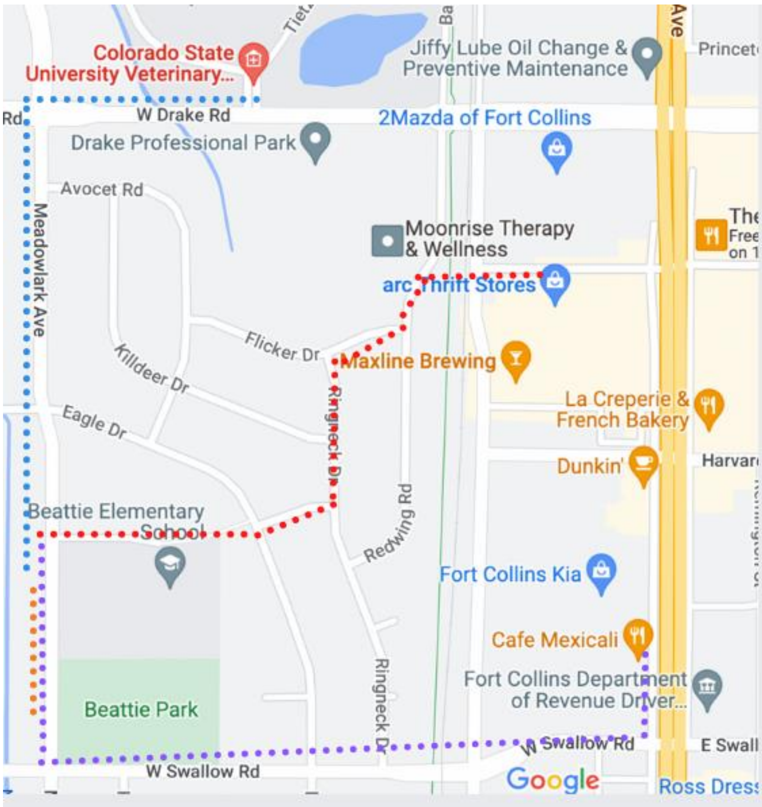
Appendix B

Beattie Neighborhood

Enjoy your time in our neighborhood!

What's in the area? How far is it from Beattie?

What's in the area?	How far is it from Beattie?
Beattie Park Beattie park is a great place to bring your younger kids. It has a slide, a merry-go-round, and a playscape. We like to go there after school and play freeze tag.	**400 feet**
Arc Thrift Store The Arc Thrift Store has a lot of used clothes and other items. The items are very cheap! You can also buy Halloween costumes there!	**0.7 miles**
Cafe Mexicali Cafe Mexicali is a Mexican restaurant. They have delicious food like tacos and chimichangas. Sometimes we have our school fundraisers there!	**1.2 miles**
CSU Vet Hospital This is where students at Colorado State University learn to be veterinarians. The building has a lobby where you can look at pictures of the animals they are studying.	**3000 feet**



Path to CSU Vet Hospital

Path to Beattie Park

Path to Cafe Mexicali

Path to Arc Thrift Store

This brochure was created by

Jess Lynn
Richie Stenson
Soleil Gomez

Appendix C

Unit conversion reference sheet

1 kilometer = 1000 meters $\frac{1\ kilometer}{1000\ meters}$ $\frac{1000\ meters}{1\ kilometer}$

1 meter = 100 centimeters $\frac{1\ meter}{100\ centimeters}$ $\frac{100\ centimeters}{1\ meter}$

1 foot = 12 inches $\frac{1\ foot}{12\ inches}$ $\frac{12\ inches}{1\ foot}$

1 mile = 5280 feet $\frac{1\ mile}{5280\ feet}$ $\frac{5280\ feet}{1\ mile}$

Appendix D

Convert 800 meters to centimeters.

Convert 800 meters to kilometers.

Convert 8500 meters into kilometers.

Convert 24 inches into feet.

Convert 15840 feet into miles.

Citation

Krall, G. & Aleksani, H. (2023). Welcome to the Neighborhood! In A. Bicer (Ed.), *Creativity-Directed Mathematical Tasks for 5th Grade Common Core Classrooms* (pp. 457-470). ISTES Organization.

SECTION 9 - REPRESENT AND INTERPRET DATA

Task 33 - Water Works

Michael Gundlach, Michelle Tudor, Melena Osborne

Content Standards

CCSS.MATH.CONTENT.5.MD.B.2

Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally*.

Practice Standards

CCSS.MATH.PRACTICE.MP1

Make sense of problems and persevere in solving them.

CCSS.MATH.PRACTICE.MP3

Construct viable arguments and critique the reasoning of others.

CCSS.MATH.PRACTICE.MP5

Use appropriate tools strategically.

Lesson Objective

Students will use the data collected and displayed on a line plot to determine equal amounts of water that can be distributed among the class. Students' creativity will be expressed through the ways they choose to use the information to solve the problem. For example, what kinds of models or operations do they use?

Vocabulary

Line plot, mean (average), data

Materials List

Water bottles (students will bring), sticky notes

Engage

Ask students to share their favorite type of treat. If a common favorite treat emerges, use that for the following number talk, but if not, any type of treat will do. The example herein will use "cookies."

There are four friends that together have 12 cookies. If they decide to share them equally, how many does each friend get? What if there are 18 cookies? Let students share different ideas of how the cookies could be shared. Some common methods could be dividing each cookie into four pieces, so each person gets 18 fourths, giving people four cookies each and then half of another cookie, or throwing away two cookies so each person gets four. After each group has concluded that each person gets four cookies and then a little bit more, but not a fifth cookie, have a group explain why the answer is 4 ½ cookies. If no group has come up with the ½, demonstrate to the class how it can be found by dividing each of the last two cookies in half and giving each person half a cookie.

Explore

Before class starts give each student a sticky note with either 0, ⅛, ¼, ½, or 1 written on it. Multiple students will have the same amounts. Before moving on, create a table with the different amounts on the sticky notes and ask students what value they have on their sticky notes. Tally these amounts in the table for the students to visually see. This will represent the part of a liter of water they brought to field day in their water bottle. Tell them they will be participating in the field day, and you want everyone to have the same amount of water before the games begin. Place students in small groups and ask them to share the amount of water that is on their sticky note. Ask the group to come up with any ideas on how they could evenly distribute the water. Allow students time to share ideas with the whole group. Ask questions to lead the group to understand that we have to know the total amount of water in order to divide it equally. Ask students, "Would it be easier if they could see how much water each person has all at one time? How could we do this?" If no one comes up with making a graph, suggest it as a possibility and ask students if they want to try it.

				x		x		
	x			x		x		
x	x			x	x	x		
x	x	x		x	x	x		x
0	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$	1

Figure 1. Sample

Explain

Have a line plot made with the fractional parts of the liters of water on the sticky notes the students have. Ask students to place their sticky note in the correct place on the line plot. Alternatively, you could have students tell you where to place a tally mark on the line plot representing their sticky note. Point out to the students that the line plot and the table above in the Explore part represent the same data. Ask students to work with their small group to calculate the total amount of water. Circulate to answer questions about how to do this. Students may use different operations or models to do this. After giving the groups some time to do this, ask each group to explain the method they chose to solve the problem. If there is a disagreement about the total amount, discuss how to determine which answer is correct. All groups should agree on the total amount. Next, ask groups to figure out how to distribute the water equally among the class (this amount will be the mean). Again, circulate to help with any questions about how to do this. Groups should show the work and be able to explain it. After a few minutes of work time, ask for groups to share their answers. Again, the answer should be the same, so discuss any differences and how to determine which answer is correct.

Extend

Have students return to their line plots and put a line one their line plot marking the location of the mean, as seen in the figure below.

				x		x		
	x			x		x		
x	x			x	x	x		
x	x	x		x	x	x		x
0	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$	1

Figure 2 Sample Table with Line

Have students try to determine any patterns they see with how the line representing the mean relates to the data points. If students are unsure how to proceed in finding this pattern, have them explore the following questions:

- How far is each data point from the mean? Make sure to note on what side of the mean each data point is.
- Are there any visual patterns?
- Try finding a pattern with the line plot and the mean using two, three, and four data points first, and then expand to using the whole line plot.

The goal of this activity is to help students understand that the mean is the "balance point" of the set of numbers. This idea may come up in some of their justifications for equal distribution, with people with more water giving that water to people with less water.

Evaluate

As a formative evaluation, monitor conversations about methods for totaling and dividing water to make sure students are on the right course. As a summative evaluation, collect group line plots, group totals, and group averages as well as supporting work. When grading summative evaluations, look for justifications that lend themselves well to general strategies for finding an average, especially as students likely have not mastered the fraction division skills necessary to perfectly find an average by hand.

Citation

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Task 34 - The Lemonade Stand

Fay Quiroz, Traci Jackson, Aylin S. Carey

Mathematical Content Standard

CCSS.Math.Content.5.MD.B.2

Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots.

For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.

Supporting Standard

CCSS.Math.Content.5.MD.A.1

Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.

Mathematical Practice Standards

CCSS.MATH.PRACTICE.MP4

Model with mathematics.

CCSS.MATH.PRACTICE.MP5

Use appropriate tools strategically.

CCSS.MATH.PRACTICE.MP6

Attend to precision.

CCSS.MATH.PRACTICE.MP7

Look for and make use of structure.

Vocabulary

Data, data collection, line plots, x-axis, y-axis, fractions, gallons, ounces

Materials

Lined paper, one copy of appendix B and C for each student, a half-gallon jug of lemonade (optional)

Lesson Objective

Students will investigate line plots by gathering and interpreting data from around the school to create a line plot using fractions (1/8, 1/4, and 1/2). Students will use creative thinking to decide what are the most valuable features to build a successful lemonade stand business. Based on these features, students will create questions, and give a survey to potential customers. They will use the data they collect to create a line plot, which will be interpreted through a series of questions to help students evaluate their findings to design a successful lemonade stand business. Students will complete this lesson in two to three class times.

Engagement

(30 minutes) Begin this lesson as a whole class by asking the students to think about ways they could make some extra money either on the weekends or in the summer (depending on when this lesson is taught and what seems most relevant). Sample answers could include dog walking, mowing lawns or selling lemonade. If needed, tell students their last year's class designed a lemonade stand business, and that this year’s class will also get to design one. Brainstorm with the class about what an owner would need to consider when building a successful lemonade stand. Expected student answers could include:

- making enough money

- amounts of lemonade
- size of cups
- location
- types of lemonade
- cost per cup
- expected number of people that will visit the stand
- powder lemonade vs. pre-made lemonade
- Table
- cookies or brownies.

Tell students that for this project they will work in groups of three. All three students will be equal owners in their lemonade stand business. First, students will pick a name for their lemonade stand company. Then ask groups to discuss, based on the list created by the class, which are the most important characteristics for our lemonade stand to be successful? Students should prioritize their list with the *top three features* in order from most important to least important.

After groups have created their lists, bring the class back together and let each group take turns sharing their name and the top three features. For example, the class may choose the top three features as: 1) the total amount of lemonade to sell, 2) cost per cup, and 3) how many people they think will visit their lemonade stand in a week. As these are just suggestions of what students could propose as the most important traits, the discussion should be guided to help lead students to two out of three features that should include fractional amounts. Teachers should use the first two features mentioned: total amount of lemonade to sell and 2) cost to sell include fractional amounts of $\frac{1}{2}$ and $\frac{1}{4}$, teachers can use a third feature of whichever their class feels most strongly about.

Build on these three characteristics by posing questions about how to collect the data. The discussion should center on questions such as: "Who are you going to ask to collect *good* data?" Students may decide to just ask their classmates, or they might decide they should ask other students around the school, since they will probably be the ones buying the lemonade. Other questions in the discussion could include: "How many people would you ask? Is there a range of people the class can agree on? Is two people enough? What would be enough data to have a reasonable sample size?" Teachers can discuss the idea of a sample size and if you only asked two people versus asking 20 people. Students should notice asking two people is not enough, but 100 people might be too many. Perhaps a range of 25-40 people might be an amount the class could agree on when their groups are going to collect the data. Lastly, the class should make a plan for how their groups should collect the data. Hopefully, students will suggest turning each feature into a question, collecting the data in a table, and offering choices for each question. See examples of sample data and line plots in the explore/explain section.

Explore/Explain (Amount of Lemonade)

(30 minutes) Now that students have created a lemonade stand business and have created a plan to collect data based on the important features that will make their lemonade stand successful, students can begin to collect the data and create a line plot matching the data. Teachers should manage this next part in steps to guide students through one feature at a time.

For example, in the first feature, teachers should provide more guidance in helping students create a table, and decide where, and how much data to collect. After the data is collected, teachers should remind students about their background knowledge from 4th grade on how to create a line plot using the data they collected. The second feature may have less teacher support, and by the third feature, groups can create plans for collecting the data and only check in with the teacher for approval.

The first data collection question will be for the total amount of lemonade needed for the first week of the lemonade stand. Students may choose to have two or more types of lemonade. Lead a short discussion on how much lemonade they might need based on the number of people they think they will serve so they can make reasonable estimates. It may help to bring plastic cups and talk about how large the cups are. Alternatively, share the images from Appendix A. After students decide which cup they might use, ask how many gallons they may need based on the cup size. Let students know that the containers of lemonade are in half-gallons, so when they survey people about how much lemonade they will need, they will need to say to the nearest half-gallon. If students choose two or more types, have students ask two survey questions, one for each type of lemonade. Students should make a plan on how to collect their data and then go around the school to collect data (see Table 1). When students return, teachers may have a short whole class discussion to remind students of the features of a line plot: title of the graph, x-axis label, y-axis label, numbers need to be an equal distance apart and one dot represents a person (see Figure 1).

Table 1. Example of How Students Use a Table and Sample Data Collection for the Amount of Lemonade

Business:		
Amounts of Lemonade	**Number of People Surveyed**	**Total**
3 ½ gallons	𝍷𝍷𝍷	3
4 gallons		
4 ½ gallons	𝍷𝍷	2
5 gallons	𝍸	5
5 ½ gallons	𝍸𝍸𝍷𝍷	12
6 gallons	𝍸𝍸𝍷𝍷𝍷	13
6 ½ gallons	𝍷𝍷𝍷	3
7 gallons	𝍸	5
7 ½ gallons	𝍷	1

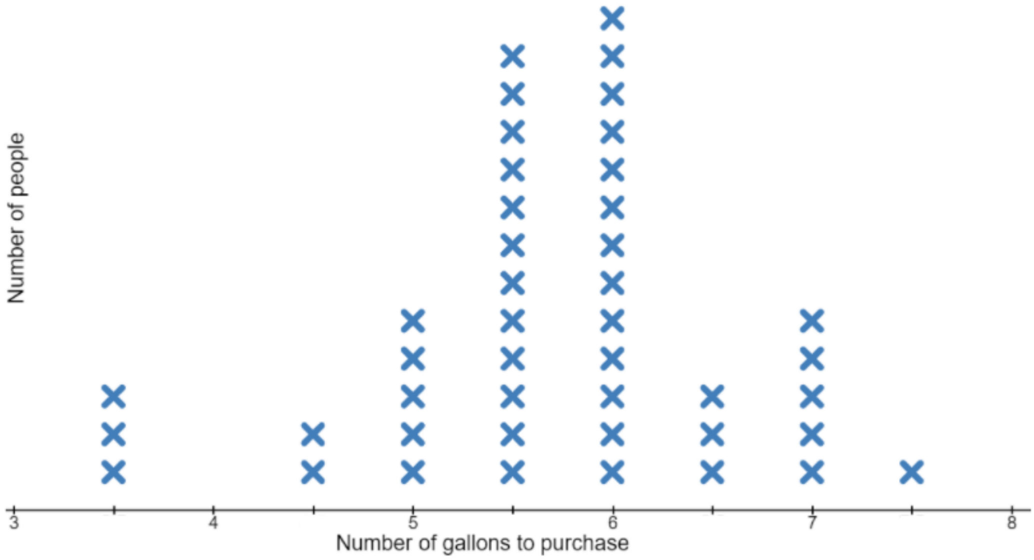


Figure 1. Student Example Line Plot for the Amount of Lemonade

Explore/Explain (Cost to Sell Lemonade)

(20 minutes) The second data collection question will be: “How much should each cup of lemonade cost?” Teachers can help students brainstorm amounts of money to the nearest fourth and remind students they need to graph the fraction equivalent on their line plot: 0.25 ($\frac{1}{4}$), 0.50 ($\frac{2}{4}$), 0.75 ($\frac{3}{4}$), 1.00 (1), 1.25 ($1\frac{1}{4}$), 1.50 ($1\frac{2}{4}$), 1.75 ($1\frac{3}{4}$), 2.00 (2). An example of the data table that students collected is below (see Table 2).

Table 2. Example of How Students Use a Table and Sample Data Collection for the Amount of Money to sell a Cup of Lemonade

Business:		
Amounts of Money to Sell Lemonade	**Number of People Surveyed**	**Total**
$0.25 (1/4)	XXXXXXXXXXXXXXX	15
$0.50 (2/4)	XXXXXXXXXXXX	12
$0.75 (3/4)	XXXXX	5

Business:		
Amounts of Money to Sell Lemonade	**Number of People Surveyed**	**Total**
$1.00 (1)	XXXXXXXXXXXXXX	14
$1.25 (1 ¼)		0
$1.50 (1 2/4)	XX	2
$1.75 (1 ¾)		0
$2.00 (2)	X	1

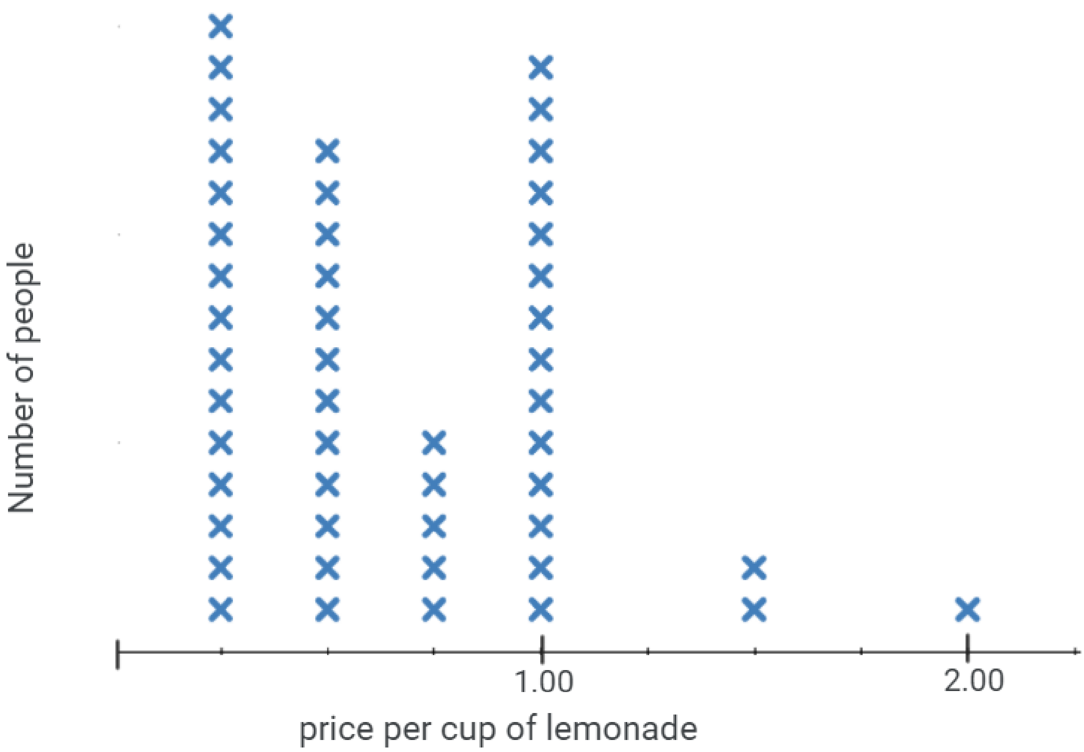


Figure 2. Student Example Line Plot for the Price per Cup of Lemonade

Explore/Explain (Cup Size)

(20 minutes) The third data collection question will be: “What cup size should we use?” For this activity, teachers suggest students to use the cup sizes of 9, 8, 12, 16, and 20 ounces (see Appendix A) and offer help to those with conversion of one ounce to a cup size (1 U.S. fluid

ounce = $\frac{1}{8}$ U.S. cup). When graphing on a number line, observe students' flexibility in conversion (ounce to cup) and ability in writing fractions in a decimal form. For example, a student can reason their calculation such as "If 1 ounce is $\frac{1}{8}$ cup, then 8 ounces is $8 \times \frac{1}{8} = 1$ cup." Another student may calculate as $12 \times \frac{1}{8} = \frac{12}{8} = \frac{12 \div 4}{8 \div 4} = \frac{3}{2} = 1\frac{1}{2}$ and should be able to reason 1 and $\frac{1}{2}$ as $1 + 0.5 = 1.5$.

Table 3. Example of How Students Use a Table and Sample Data Collection for the Size of Cups

Business:		
Cup Size	**Number of People Surveyed**	**Total**
9 oz. (1 1/8)	● ● ● ● ●	5
8 oz. (1)	● ● ●	3
12 oz. (1 ½)	● ● ● ● ● ● ● ● ●	9
16 oz. (2)	● ● ● ● ● ●	6
20 oz. (2 ½)	● ●	2

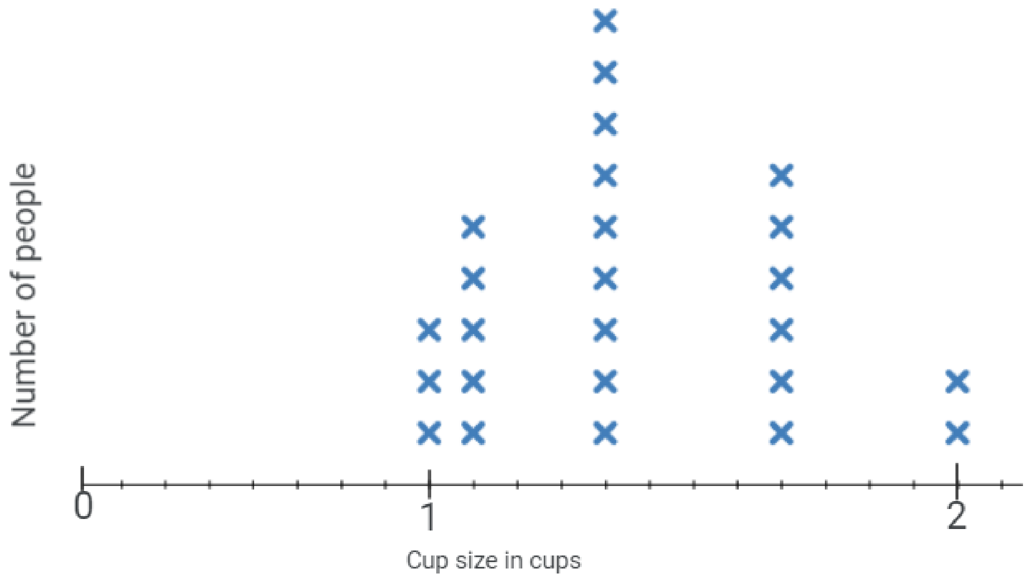


Figure 3. Student Example Line Plot for Cup Size

Extend

(30 minutes) Now that students have collected their own data for each question and created a line plot for each set of data, they can answer questions about their data to understand how to better design their lemonade stand. Tell students you will share with them data that was collected last year, at the end of the first week of selling lemonade.

Each day the lemonade stand traveled to two different locations, one in the morning and another in the afternoon. The distance is the number of miles from the school to the lemonade stand location. The data shows the number of people at the different locations. Analyze the data from the first week of last year to design your own plan for your first week.

Show students the line plot that was created from the first week last year, which is the distance the lemonade stand is from the school (see Figure 4). Students set up in a morning location and afternoon location.

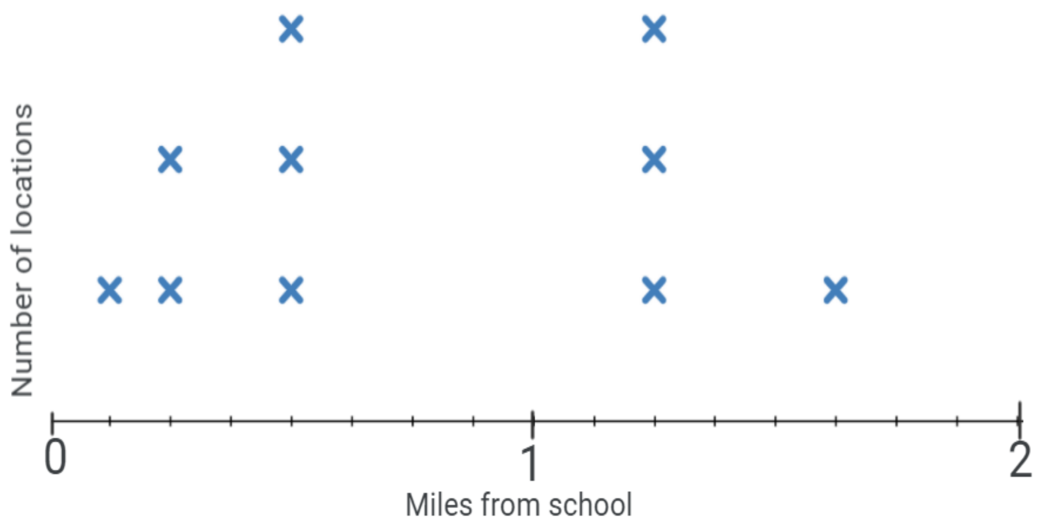


Figure 4. Line Plot for Locations the First Week from Last Year

Give each student a copy of Appendix B with a list of questions to help students analyze the data from the line plot. As a whole class and using the line plot in Figure 4, ask each question, discuss the answers, and how these questions might influence the design of a successful lemonade stand. Teachers should use this opportunity to clear up any misunderstandings about the data on the line plot.

In the next part, business teams should refer to the line plots they created from the top three features list in the explore-explain. As a group, students should answer the following questions for each line plot, discuss the answers, and decide how these questions might influence the design of a successful lemonade stand (see Appendix C). Teachers should rotate between groups to guide conversations about how to answer the questions regarding the data, but more specifically how the data will impact the success of their lemonade stand business.

Evaluate

Teachers should look for students who understand how to make a chart to collect the data, and how to collect the data. Furthermore, students should be able to create a line plot that includes a title, x-axis with a label, y-axis with a label, but more specifically if either axis is created with numbers, they should be spaced at an equal distance apart. For example, on the amount of lemonade used the spacing is $\frac{1}{2}$, apart so each number from zero should read 0, $\frac{1}{2}$, 1, $1\frac{1}{2}$, 2, $2\frac{1}{2}$, etc. As students create their line plots, listen in to how they are making sense of the data. Ask questions like "What do the 3 marks above 4.5 mean?" Students should understand the dots above the number represent a person who was asked in the survey they took. Moreover, teachers should look for students who can understand and interpret questions about the data on the various line plots. If students struggle with adding, subtracting, multiplying, and dividing the fractions, teachers can use fraction manipulatives or encourage students to draw fraction bars to solve the problems.

Use the third explore and explain collecting data for cup size should be used as an opportunity to evaluate students' overall understanding of collecting data, organizing the data into a line plot and answering questions about the data.

Appendix A

Retrieved from https://www.banksmarket.com.

Retrieved from https://www.banksmarket.com.

Appendix B

Business Name & Owners:		
Location Questions	**Answer**	**How does this help our business design?**
1) Which location was most popular? Which location was least popular?		
2) What location was the furthest away from the school?		
3) How many times further is the one with the greatest distance than the one with the least distance?		
4) What's the total miles traveled by the 3 longest distances?		
5) If we combined all the miles that people traveled to the lemonade stands, and readjusted the miles, so each person traveled the same distance. How many miles would each person travel?		

Discuss the findings with your group and make a plan for where to locate your lemonade stand the first week.	**<u>FINAL PLAN for LOCATION:</u>**

Appendix C

Business Name & Owners:		
Amount of Lemonade Questions	**Answer**	**How does this help our business design?**
1) Which amount was most popular? Which amount was least popular?		
2) What is the difference between the greatest amount and least amount of lemonade?		
3) How many times more is the most amount than the least amount of lemonade?		
4) What's the total price of the 3 least amounts of lemonade?		
5) If you took the amounts suggested and redistributed them so they were equal. How much would each amount be?		
Discuss the findings with your group and make a plan for how much lemonade you will bring the first week.	**FINAL PLAN for AMOUNT OF LEMONADE:**	

Business Name & Owners:		
Cost to Sell Questions	Answer	How does this help our business design?
1) Which price was most popular? Which price was least popular?		
2) What is the difference between the most expensive and least expensive cup of lemonade?		
3) How many times is the greatest amount of money than the least amount of money people will spend on a cup of lemonade?		
4) What's the total price of the 4 most expensive cups of lemonade?		
5) If we combined all the prices, and readjusted the prices, so each person paid the same amount. How much would one glass of lemonade cost?		
Discuss the findings with your group and make a plan for what price you will charge for your lemonade during the first week.	**FINAL PLAN for COST TO SELL the Lemonade:**	

Business Name & Owners:		
Cup Size Questions	Answer	How does this help our business design?
1) Which cup size was most popular? Which cup size was least popular?		
2) What is the difference between the biggest and smallest cup?		
3) Compare the most popular cup size to the most popular amount of lemonade. Make a decision on the number of cups you would need for the most popular amount of lemonade.		
4) Make a prediction of the amount of lemonade and the number of cups you would need.		
5) What adjustments do you need to make to the cost to sell that you would still make profit?		
Discuss the findings with your group and make a plan for what cup size to use for your lemonade stand during the first week.	**FINAL PLAN for Cup Size:**	

Citation

Quiroz, F., Jackson, T., & Carey, A. S. (2023). The Lemonade Stand. In A. Bicer (Ed.), *Creativity-Directed Mathematical Tasks for 5th Grade Common Core Classrooms* (pp. 477-494). ISTES Organization.

Task 35 - Plot the Distance

Amy Kassel, Chuck Butler, Jennifer Kellner

Mathematical Content Standards

CCSS.MATH.CONTENT.5.MD.B.2

Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.

Mathematical Practice Standards

CCSS.MATH.PRACTICE.MP4

Model with mathematics.

CCSS.MATH.PRACTICE.MP5

Use appropriate tools strategically.

CCSS.MATH.PRACTICE.MP7

Look for and make use of structure.

Supporting Standards

3-5-ETS1-1. Define a simple design problem reflecting a need or a want that includes specified criteria for success and constraints on materials, time, or cost.

3-5-ETS1-2. Generate and compare multiple possible solutions to a problem based on how well each is likely to meet the criteria and constraints of the problem.

3-5-ETS1-3. Plan and carry out fair tests in which variables are controlled and failure points are considered to identify aspects of a model or prototype that can be improved.

Vocabulary

Data set, preciseness, horizontal axis, x-axis, vertical axis, y-axis, line (dot) plot, difference, independent and dependent variables

Materials

Used crayons (teachers may wish to collect used crayons for this or have students look in their desks for used crayons), rulers (https://www.inchcalculator.com/printable-ruler/), sticky notes, items such as old CDs, straws, plastic bottles (16.9 oz), Legos, cardboard, glue, tape, wooden skewers (anything that would make a body, wheels, and axels of a car), bags of balloons, sample constructed balloon-powered cars

Lesson Objective

Students will apply their understanding of fractions and measurements to plot data in a line plot.

This lesson will promote creative thinking by having students create their own cars to use for data collection. They will also write their own statistical questions, conduct a survey of their classmates, and use creative methods to represent the data.

Engagement

(20-60 minutes) For engagement, the students will conduct a science experiment. The teacher will show the following YouTube video to the students – Making a Balloon Powered Car (https://www.youtube.com/watch?v=yGXg6uWMG24). According to Newton's third law of motion, "for every action, there is an equal and opposite reaction", the air coming out of the balloon pushes the car forward (Finio, 2022)

The teacher will decide if students can take multiple days to build their balloon-powered cars. Teachers may choose if students work individually, in partners, or groups of 3 to build their cars. If preferred, students can take class time to design their cars on the first day. The cars will use materials found around the students' houses or from the teacher in the school to create balloon-powered cars. They could bring supplies from home or select what the teacher has (recommended) and build them the next day. To avoid having to use sharp objects like knives, the teacher may precut items, so they are ready for the students to use. Students will use these cars to collect data on how far the car travels. This data will be used to conduct the math lesson. Figure 1 shows examples of students' cars.

Figure 3. Examples of Balloon-Powered Cars

Explore

(30 minutes) Part 1: After students plant their seeds but before the seeds sprout, students will practice measuring to the nearest 1/8 inch.

- Teachers will ask students to gather 5 used crayons and a ruler with inches.
- Teachers will then ask students to measure the crayons.
- Students may measure the different levels of preciseness (nearest $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}$ inch).
- Ask students to hold up one crayon and find a partner with a crayon approximately the same size.
- After students have paired up, ask students to share their measurements with each other.
- Ask students if their measurements were the same? Why or why not?
- After students have shared with one another, ask students if the class could agree on how precise they should measure the crayons.
- Teachers may wish to project a picture of a ruler (Figure 2) on the classroom display for students to reference during this discussion. Teachers may probe students to discuss which measurement would be the most accurate.
- After the class agrees on the preciseness of measurement (to the nearest 1/8 inch), ask

students to measure their crayons again and compare their results.

- Students will then write the length of each of their crayons on a sticky note.
- The class will then create a large-scale dot plot of their crayon length data using the sticky notes on big poster paper.
- Ask students what they want to title the graph. Sample student responses might be "Crayons," or "Crayon Length," or "Lengths of Used Crayons in our Classroom."
- Ask students to describe how they would create horizontal and vertical axes. Students may suggest that the axes represent the length and how many. Teachers may ask students which variable will go on which axis. Once it is determined that the horizontal axis will represent the lengths of the crayons, teachers may ask students how they want to label them. Students may give examples that are on their sticky notes. Teachers may ask students at which value the axis should, by which values the axis should count, and at which value the axis should end. Teachers may ask students how they should label the vertical axis. Students may recognize that the vertical axes should count how many. Ask students for the beginning and ending values for the vertical axis and label them appropriately.
- After the axes are created, ask students to place their sticky notes in the appropriate place. Teachers may help students place their sticky notes above other sticky notes with the same measurements (not overlapping).

Part 2: Students will answer questions about the dot plot (Appendix A). Students should use models to represent their solutions when appropriate.

Part 3: Students will collect data on how far their balloon-powered cars travel (Appendix B) to the nearest 1/12 foot. After collecting data for 20 trials, students will create a dot plot of their data. Then students will get into groups and share their dot plots. The students will then work as a group to answer questions about the data (Appendix C).

Explain

(30 minutes) Part 1: As students measure their crayons for the first time, they may ask what unit they should measure. Teachers should tell students to decide for themselves. This will generate a variance in how accurate the measurements are for the crayons, which will create

the need for the class to be precise in describing their measurements.

After students share their measurements with their partner, the teacher should ask students to show their crayons and tell their respective measurements. For example, two students may have measured a crayon with approximately the same length as $2\frac{1}{2}$ and $2\frac{3}{8}$. A teacher may ask the class which measurement is more precise. If students are struggling, teachers may project the crayon and ruler images (Figure 2) or actual rulers with crayons on the classroom display and ask students which image would provide the most accurate length of the crayon. If students are struggling to recognize the measurements on the ruler, teachers may wish to provide each student with the picture and label the rulers with the appropriate measurements.

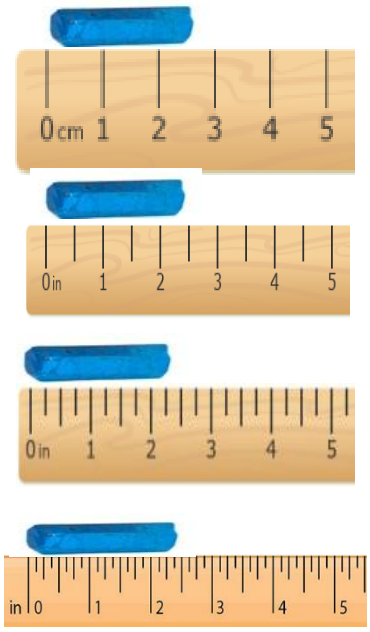
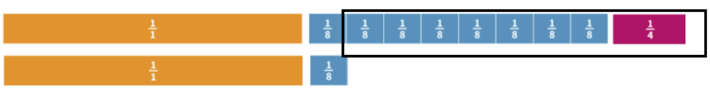


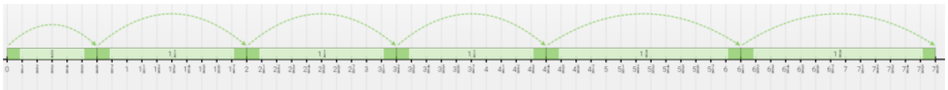
Figure 2. Different Rulers with Same Crayon

After the class agrees to measure their crayons to the nearest $\frac{1}{8}$ inch, teachers may wish to circulate the classroom as students are measuring their crayons for the second time. Teachers may look to see that students are correctly reading the ruler and understand the fractional measurements. For example, if a student is struggling to recognize how many $\frac{1}{8}$'s their crayon measures, then the teacher may ask the student how many parts are in each inch. Teachers may also ask students to label a paper ruler with $\frac{1}{8}, \frac{2}{8}, \frac{3}{8}, \frac{4}{8}, \frac{5}{8}, \frac{6}{8}, \frac{7}{8}$ to help students measure their crayons. Teachers may print paper rulers at https://www.inchcalculator.com/printable-ruler/. After students have labeled their ruler with $\frac{1}{8}$'s, teachers may discuss equivalent fractions and ask the students to label the ruler with the simplified equivalent fraction, $\frac{1}{8}, \frac{1}{4}, \frac{3}{8}, \frac{1}{2}, \frac{5}{8}, \frac{3}{4}, \frac{7}{8}$.

After students have completed their measurements and written the lengths of each of their 5 crayons on a sticky note, teachers may lead a whole-class discussion on how to construct a dot plot graph for this data. Teachers should prompt students to give an appropriate title by asking questions such as "What does our data represent?". Teachers should support students in thinking about independent and dependent variables when helping students decide which variable should be represented on the horizontal and vertical axes. In this case, length of the crayon would be the independent variable and the number of crayons would be the dependent variable. Teachers may also support students in deciding how to count on the horizontal axis by asking different students to share their lengths and then asking students how they could label the axis to represent all the possible lengths. Teachers may also support students in deciding how to label the vertical axis by having some students with the same measurements come up to put their sticky notes on the graph and ask students what each of those sticky notes represents. As students come up to put their sticky notes on the dot plot, teachers should be sure students place the sticky notes for the same measures above one another, not overlapping so that the count is shown accurately.

Part 2: As students are working on solving the problems, teachers may look to see that students are taking the appropriate data from the graph. For example, when the question asks for lengths, the students should be using data from the horizontal axis and when the question asks for how many, the students should be using data from the vertical axis. Teachers should also look for students' models and explanations to share during the conclusion of the lesson. Sample student responses might include:

1. What is the difference between the longest and the shortest crayon?	The longest crayon is $2\frac{1}{4}$ in and the shortest crayon is $1\frac{1}{8}$ in. The difference is $2\frac{1}{4} - 1\frac{1}{8} = 1\frac{1}{8}$ inches. Model: 

2. How many crayons are longer than $1\frac{1}{2}$ inches and shorter than 2 inches?	I counted how many crayons were at each measurement for $1\frac{5}{8}$, $1\frac{3}{4}$, $1\frac{7}{8}$. There were 6 crayons $1\frac{5}{8}$ in long, 5 crayons $1\frac{3}{4}$ in long, and 1 crayon $1\frac{7}{8}$ in long. So, there are 12 crayons longer than $1\frac{1}{2}$ in and shorter than 2 inches.
3. If we put all the crayons shorter than $1\frac{3}{4}$ inches together, how long would the new crayon be?	The shortest crayon is $\frac{3}{4}$ in. There is 1 crayon $\frac{3}{4}$ in long, 3 crayons $1\frac{1}{4}$ in long, and 2 crayons $1\frac{5}{8}$ in long. The length of a crayon putting all these together would be $\frac{3}{4}+1\frac{1}{4}+1\frac{1}{4}+1\frac{1}{4}+1\frac{5}{8}+1\frac{5}{8}=\frac{3}{4}+3\times1\frac{1}{4}+2\times1\frac{5}{8}=7\frac{3}{4}$ 

Models from The Math Learning Center (Math Learning Center, 2022) and Mathigon (Mathigon, 2022).

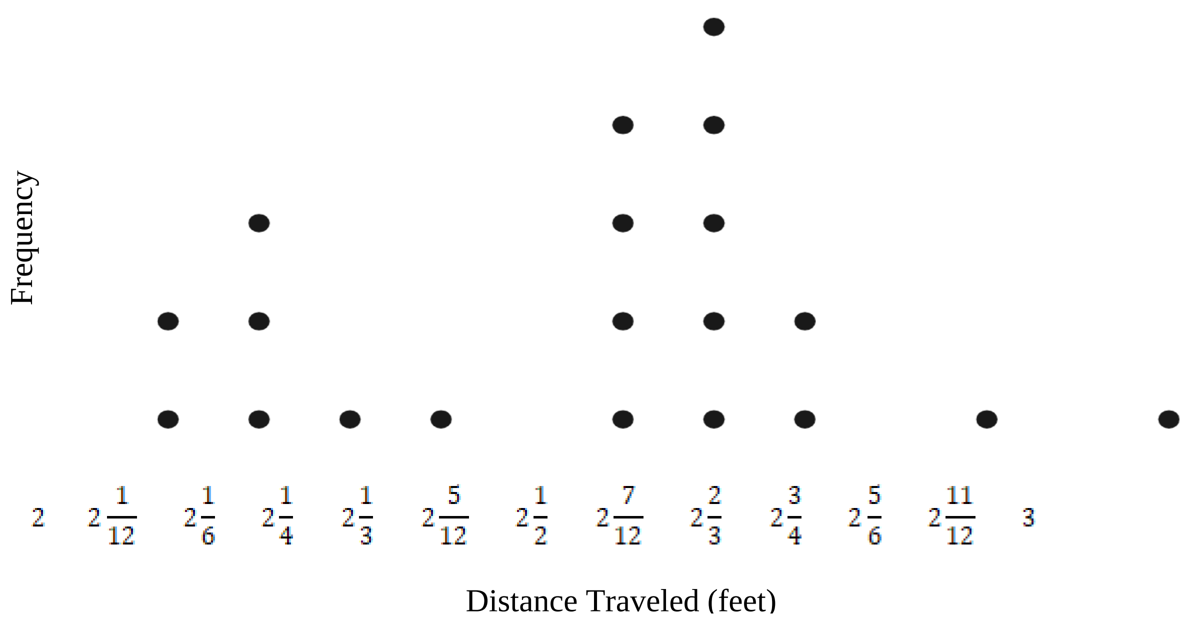
Part 3: Students will use the balloon-powered car they created in the engage part of the lesson to collect data. Students will collect data on the distance the car traveled to the nearest $\frac{1}{12}$ foot for 10-15 trials. Before the students begin measuring, teachers may have a discussion with the class to discuss how accurate they think the measurements for the distance traveled should be. Students may suggest the nearest $\frac{1}{8}$ inch since that was their experience in the prior problem.

Teachers may do a demonstration with their car and measure how far it traveled and ask students if they could accurately measure to the nearest $\frac{1}{8}$ inch. Students may say no. As a class, determine how accurate the measurements could reasonably be. Students may suggest to the nearest foot or nearest inch. Teachers may help students recognize that they could measure to the nearest part of a foot. Teachers may need to support students' thinking in developing $\frac{1}{12}$ of a foot by asking students how many inches are in one foot. Teachers may have students label each inch as $\frac{1}{12}, \frac{2}{12}, \frac{3}{12}, \frac{4}{12}, \frac{5}{12}, \frac{6}{12}, \frac{7}{12}, \frac{8}{12}, \frac{9}{12}, \frac{10}{12}, \frac{11}{12}$ initially. Then ask students to write each fraction in its simplified equivalent form, $\frac{1}{12}, \frac{1}{6}, \frac{1}{4}, \frac{1}{3}, \frac{5}{12}, \frac{1}{2}, \frac{7}{12}, \frac{2}{3}, \frac{3}{4}, \frac{5}{6}, \frac{11}{12}$.

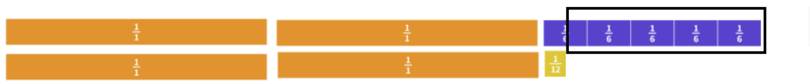
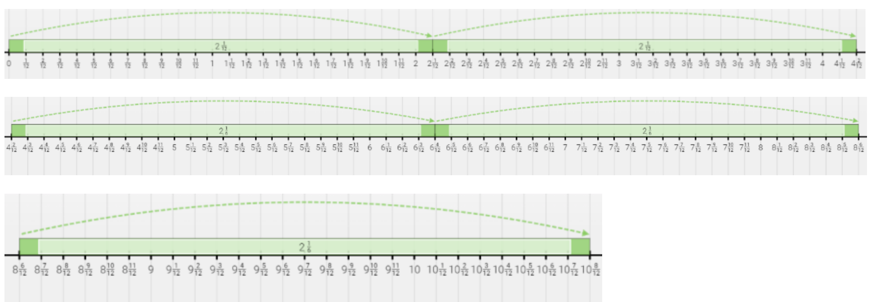
Teachers ask students to test how far the car traveled each time for 20 trials. After each trial, the students should measure and record the distance their car traveled to the nearest $\frac{1}{12}$ of a foot (Appendix B). If a student's car does not work, the teacher can have a couple of sample cars the student could use or have them use another student's car. This ensures the student can collect data.

After students have collected their data, they should create a dot plot of their data with the distance traveled on the horizontal axis and the frequency on the vertical axis. Teachers should ask students to title the graph and label the axes. Sample student responses may include:

Distance Traveled



After students have completed their dot plots, students will answer questions about their data. Teachers may place students in groups of $3-4$ to work together (Appendix C). Sample student responses may include:

4. How many times did the car travel more than $2\frac{1}{3}$ feet and less than $2\frac{7}{12}$ feet?	The frequency for $2\frac{1}{2}$ is 3. It is the only distance between $2\frac{1}{3}$ and $2\frac{7}{12}$ for which the car traveled.
5. What is the difference in the distance traveled between the car that traveled the least distance and the car that traveled the second greatest distance?	$2\frac{5}{6} - 2\frac{1}{12} = \frac{9}{12} = \frac{3}{4}$ 
6. If we measure put all the distances for the cars that traveled less than $2\frac{1}{4}$ feet together, what would be the total distance traveled?	The car traveled $2\frac{1}{12}$ feet two times and $2\frac{1}{6}$ feet three times. This can be modeled as an equation as $2\frac{1}{12} + 2\frac{1}{12} + 2\frac{1}{6} + 2\frac{1}{6} + 2\frac{1}{6} = 2 \times 2\frac{1}{12} + 3 \times 2\frac{1}{6} = 10\frac{2}{3}$ or as a number line model. 

Models from The Math Learning Center (Math Learning Center, 2022) and Mathigon (Mathigon, 2022).

Teachers may ask students to share their models with their groups or highlight specific models whole-class during consolidation.

Extend

The teacher may choose to assign one extension to the whole class or assign different extensions to different groups based on interest, ability, and readiness

How Likely – At the end of part 1 and part 2, the teacher can ask questions about the probability of an event happening. In part 1, the teacher could put all the crayons in a bag. If the students select one crayon, what is the probability that the crayon will be a certain length? For example, what is the probability that the crayon will be longer than $1\frac{1}{8}$ inches?

Data Representations – At the end of part 1 and part 2, students can represent data in a frequency table or histogram.

How Did You Do that? - Teacher could provide a box plot of the data and ask students to figure out how the teacher created the boxplot. Students could create steps for creating a box plot.

Write your own – Each group writes their own statistical question and then asks the rest of the class. Students can create a line plot in addition to other visual representations of the data.

Evaluate

- At the beginning of the lesson, formative assessment for content and creativity occurs as the class builds and measures the distance their car travels with precision. For example, a group might originally build their car to maximize the distance traveled.
- In parts 1- 3, formative assessment for content and creativity occurs as the class creates dot plot, students model their thinking using unique representations, and answers context-specific questions.
- In the extend task, How Likely, teachers can formatively assess for content as students find the probability of an event.

- In the extend task, Data Representation, formative assessment for content occurs as students represent their data using new representations.
- In the extend task, How Did You Do That, formative assessment for content occurs as students seek to understand the boxplot. If a student struggles to write steps for creating a box plot, the teacher can ask scaffolded questions like "What does this (point to a section of the box plot) mean?"
- In Write Your Own, formative assessment for content and creativity occurs as students write their own question, survey the class, and create a visual representation of the data. If a student struggles to write a statistical question, the teacher can encourage students to write a question that has a variety of answers.

References

Finio, B. (2022, March 3). *Balloon-Powered Car Challenge*. Retrieved from https://www.sciencebuddies.org/science-fair-projects/project-ideas/Phys_p099/physics/balloon-powered-car-challenge

Mathigon. (2022). *Polypad*. Retrieved from Mathigon: https://mathigon.org/polypad

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Appendix A

Using the dot plot that represents the class data for lengths of used crayons, answer the following questions. Draw a model or explain your thinking.

Question	Model
1. What is the difference between the longest and the shortest crayon?	
2. How many crayons are longer than $1\frac{1}{8}$ inches?	
3. How many crayons are longer than $1\frac{1}{2}$ inches and shorter than 2 inches?	
4. What is the difference between the shortest crayon length and the fifth shortest crayon length?	
5. If we put all the crayons shorter than $1\frac{3}{4}$ inches together, how long would the new crayon be?	
6. If we put all the crayons that are $2\frac{1}{4}$ inches long together, how long would the new crayon be?	
7. You want to select 2 groups of crayons so that the total length of all the crayons in those groups is the longest when you put all those crayons together. How long would the new crayon be?	

Appendix B

Data Collection Form

Trial	Distance Traveled
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Appendix C

Using your dot plot representing how far your car traveled, answer the following questions. Draw a model or explain your thinking.

Question	Model
1. How much further did the car that traveled the furthest distance go than the car that traveled the shortest distance?	
2. How many times did all the car travel more than $2\frac{7}{12}$ feet?	
3. How many times did the cars travel more than $2\frac{1}{3}$ feet and less than $2\frac{7}{12}$ feet?	
4. What is the difference in the distance traveled between the car that traveled the least distance and the car that traveled the second greatest distance?	
5. If we measure put all the distances for the cars that traveled less than $2\frac{1}{4}$ feet together, what would be the total distance traveled?	
6. How many times did the cars travel more than $2\frac{2}{3}$ feet and less than $2\frac{1}{4}$ feet?	
7. You want to select 3 groups of distances traveled so that the total distances for those cars would travel the greatest distance when you put all the distances together. What would be the distance?	

Citation

Kassel, A., Butler, C., & Kellner, J. (2023). Plot the Distance. In A. Bicer (Ed.), *Creativity-Directed Mathematical Tasks for 5th Grade Common Core Classrooms* (pp. 495-510). ISTES Organization.

Task 36 - Pencils, Make Me Rich!

Helen Aleksani, Geoff Krall

Mathematical Content Standards

Focus Content Standard

CCSS.MATH.CONTENT.5.MD.B.2:

Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.

Supporting Content Standards

CCSS.MATH.CONTENT.5.NBT.B.7

Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Mathematical Practice Standards

CCSS.MATH.PRACTICE.MP1

Make sense of problems and persevere in solving them.

CCSS.MATH.PRACTICE.MP3

Construct viable arguments and critique the reasoning of others.

CCSS.MATH.PRACTICE.MP5

Use appropriate tools strategically.

Vocabulary

Fraction Line plot, Line Plot, operation on fractions, coins, dime, nickel, penny, quarter, dollar, Fraction, data.

Material:

Paper, pencil, sticky notes, a bag of coins.

Lesson Objective

Students will be part of an experiment in collecting data and making a line plot to display a data set. In this task, students are building a business and will be asked to generate data on a line plot that includes whole numbers and fractions. They will be able to use coins to calculate the amount of money made from the business they are starting.

They will then compare their profit with other groups in class. They will then discuss some strategies to help improve the profitability of their business. Students' creative thinking will be fostered by enabling them to be involved in the process of starting a business and discussing its profit by plotting the data on a line plot with whole numbers and connecting it to the fraction line plot.

Students' creativity will be developed by enabling them to see the connection between the different steps of starting and developing a business using mathematical knowledge. Students' creativity will be also developed by allowing them to think of various strategies to help improve their business model and increase profit.

Engagement

(15 minutes) For this task, explain to students that we are going to use line plots to help us start a new business. Before explaining more about the business plan, start with explaining the concept to the line plots. A teacher draws a number line on the board and writes the numbers from 0 to 10+ under the line (see Figure 1).

Figure 1. Number Line to use for Line Plot labeled 0 through 10+ (10 or more items)

Explain to students that the number 0 represents 0 items, 1 represents 1 item, etc. Also, explain that 10+ means students with 10 or more items. Provide students with sticky notes and have them write the number of pencils they currently have on them. Then, model to students how to place their sticky notes on the number line (see Figure 2).

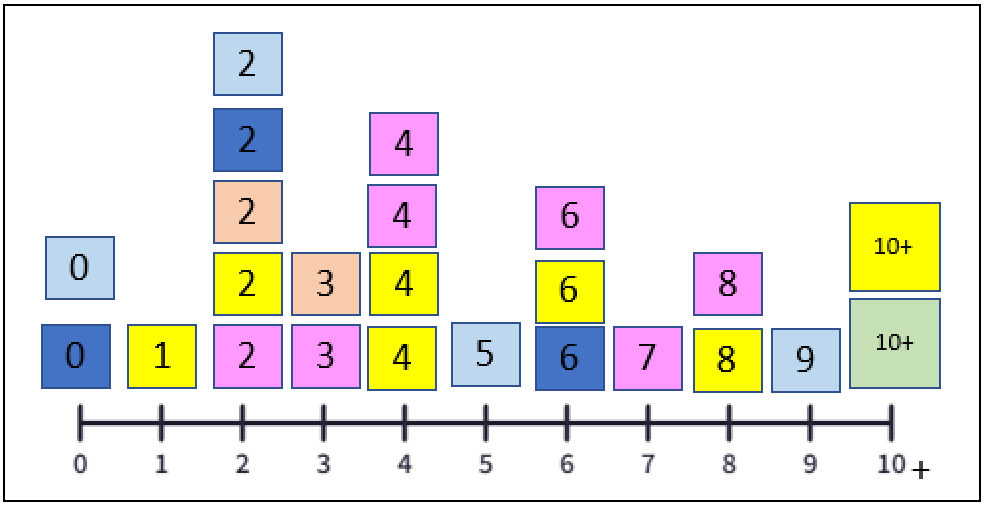

Figure 2. Shows Students how to place their Sticky Notes on the Line Plot

Then, help students understand how to read the line plot. For example, ask students the following questions:

- How many students participated in this activity?
- How many students have 6 pencils and more?
- How many students have less than 3 pencils?
- What is the highest number of pencils a student has?
- What is the least number of pencils a student has?

After students have the opportunity to discuss every question with a partner and share the answers with the class, show students that these sticky notes can be replaced with a circle, square, or the letter "X". see figure 3 below.

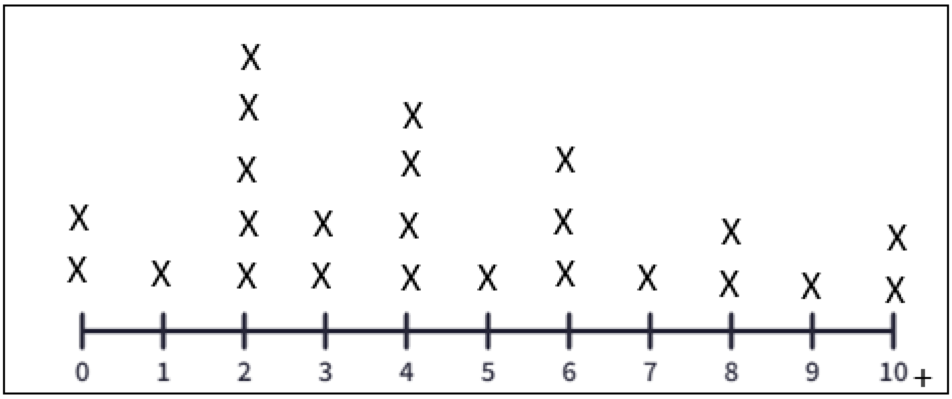

Figure 3. Line Plot with Letter "X"

Discuss with the class that there might be an opportunity here for them to start a business since many students have few pencils and they might be interested in purchasing some during school. Clarify that students are going to use a line plot to help develop a business.

Explore

(15 minutes) Now, explain to students that we are going to start a pencil business. This business will sell students different pencils at the school. For the start, our customers can choose from the following set of pencils (see Figure 4).

Figure 4. Customers' List to choose From

Every pencil has a different price. Students will be asking six customers about their orders and recording their responses in the following table (see Table 1, Appendix A).

Table 1. Students Record their Customers' Responses on the Table

Customers	Types of pencil						
	Sponge Bob	Finding Nemo	Paw Patrol	Mickey mouse	Harry Potter	Incredibles	Minions
Customer A	X		X		X		X
Customer B		X		X	X		
Customer C		X			X		
Customer D	X	X		X			X
Customer E			X		X	X	
Customer F		X		X	X		X

Discuss the importance of using a table to help organize the data. Then, students can create their line plot (see Figure 5).

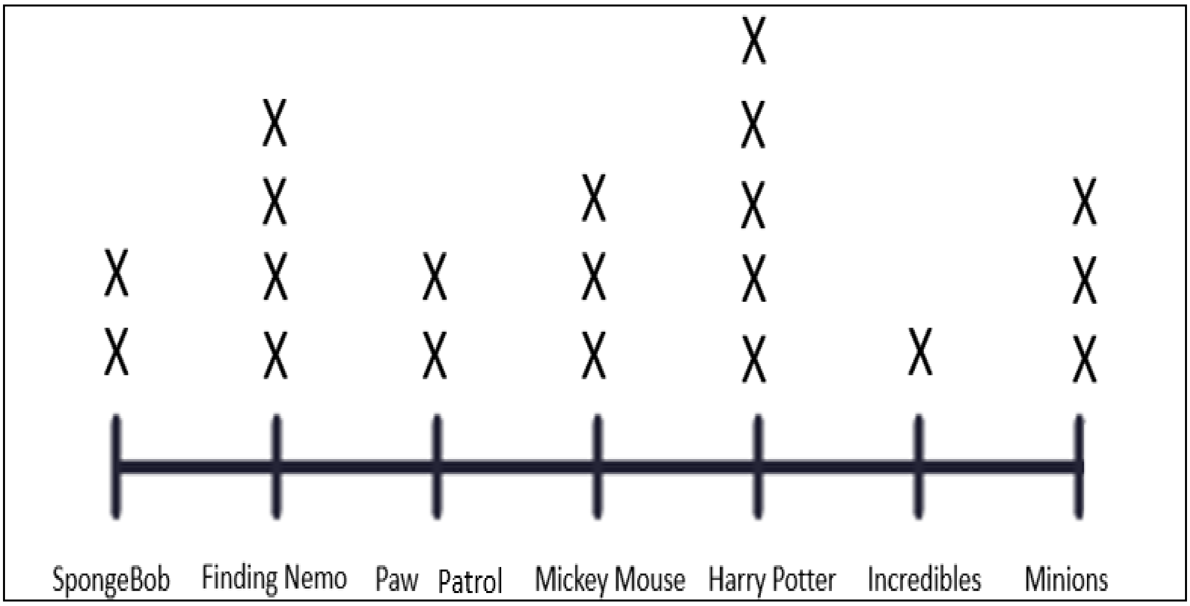

Figure 5. Line Plot of Customers' Order marked with "X"

Explain

(20 minutes) After students complete the table and the line plot, have them transfer their data from the line plot into a table such as the one below (Appendix B). For this, students can use https://mathigon.org/polypad to transfer their data into a table so they can use it to graph a pie chart. At this point, students are not generating the pie chart by hand.

Instead, they are using https://mathigon.org/polypad to see the connection between the size of the sector and the fraction representing it. Teachers can model an example on https://mathigon.org/polypad to help students learn how to complete a table and how to use the arrow to display the results on a pie chart.

Table 2. Using a Table to represent the Data

Pencil Type	Amount
Sponge Bob	2
Finding Nemo	4
Paw Patrol	2
Mickey Mouse	3
Incredibles	1
Minions	3
Harry Potter	5

Students can use the table to calculate the total number of pencils being ordered. For example, in this scenario, 20 pencils have been ordered as 2+4+2+3+1+3+5=20. They can then use the same website to create the pie chart. Help students also add the fraction of type of the pencil to the total number of pencils to the pie chart to help them see the connection between the size of each sector in the pie and the fraction that represents it. They can see that the bigger the fraction the larger the sector of the pie chart is. (see Figure 6)

x(Types of pencils)	y (total per type)
SpongeBob	2
Finding Nemo	4
Paw Patrol	2
Mickey Mouse	3
Incredibles	1
Minions	3
Harry Potter	5

Total number of pencils ordered=20

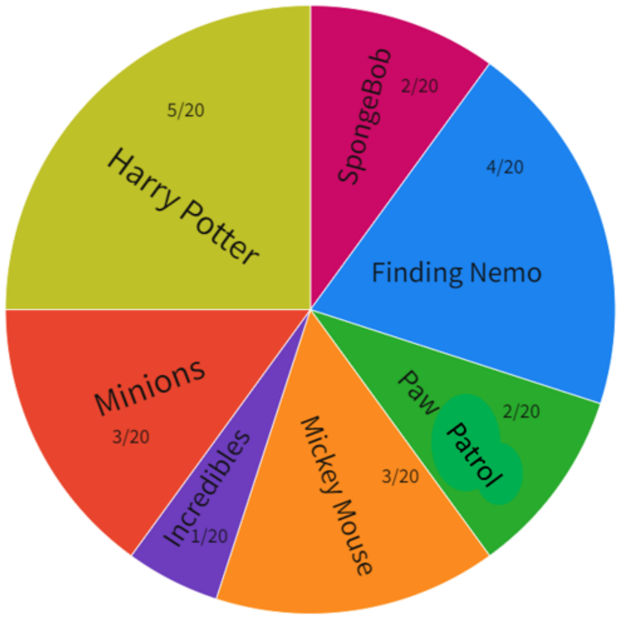

Figure 6. Using the Pie Chart and Table to represent the Data

Extend

(20 minutes) Since students were allowed to practice building line plots and charts, they can now use the line plot to represent data that include fractions. They can now create a line plot and a table to represent the money received from the orders they have taken (Figure 7, Appendix C). They can sort the money into coins and dollar bills.

x(Types of pencils)	y (total per type)	Money received
SpongeBob	2	0.15x2=0.30$
Finding Nemo	4	0.20x4=0.80$
Paw Patrol	2	0.30x2=0.60$
Mickey Mouse	3	0.45x3=1.35$
Incredibles	1	0.85x1=0.85$
Minions	3	1.00x3=1.00$
Harry Potter	5	0.55x5=2.75$

Figure 7. Using the Table to represent the Money received from the Orders

Students then can sort their money into pennies, nickels, dimes, quarters and a dollar bill and place them on a line plot (Figure 8). Students then can switch the coins into “x” to represent each type of coin received (Figure 9).

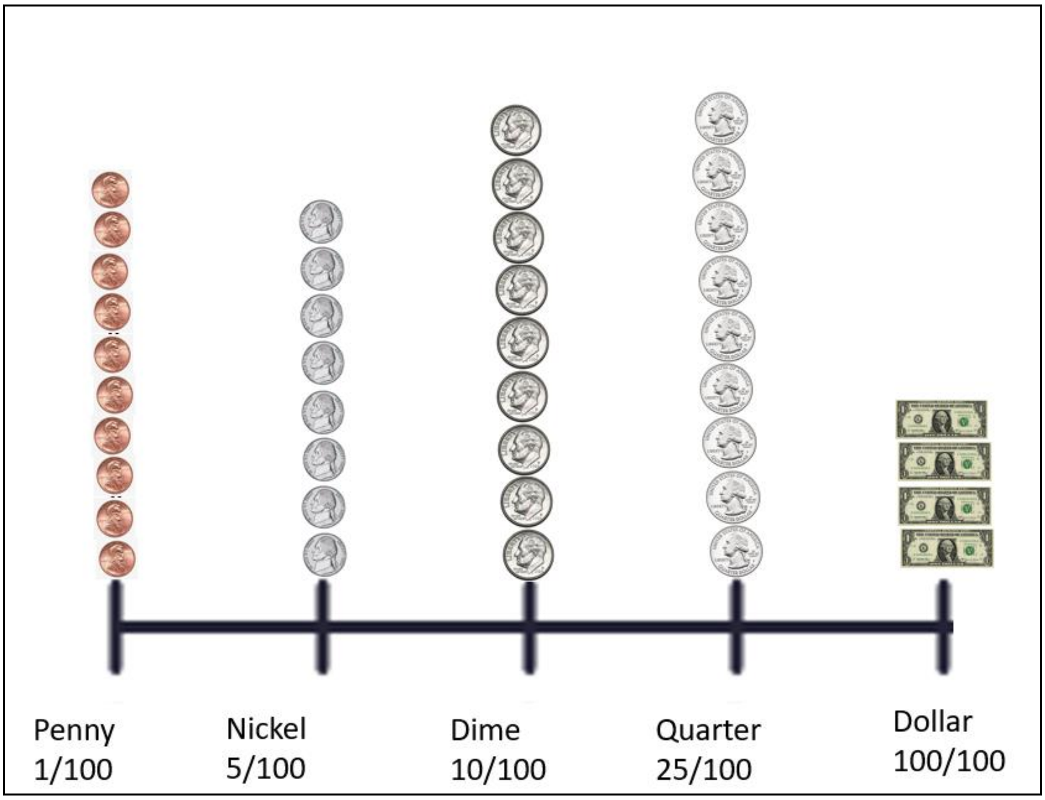

Figure 8. Sorting the Money received into Coins and Dollar Bills on a Line Plot

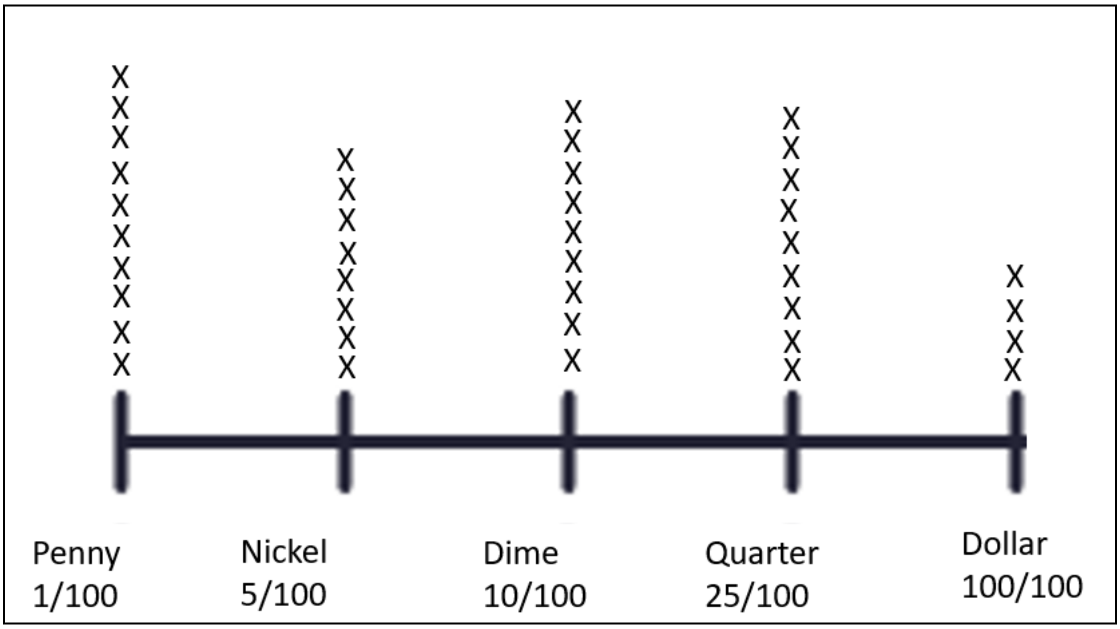

Figure 9. Replacing the Coins with “X”

After showing their data set with a line plot, ask the following questions:

- Which pencil-type was the most popular?
- Which pencil-type was the least favorite?
- Which pencil-type was the second most popular?
- How many different types of coins are presented in the Figure 9 line plot?
- How much money was received after collecting all the orders?
- Of the five categories listed in the Figure 9 line plot (penny, nickel, dime,...) which category received the most coins or dollar bills?
- Of the five categories listed in the Figure 9 line plot (penny, nickel, dime,...) which category received the least coins or dollar bills?

After students finished responding to the questions above, have them answer the following questions:

> Based on the information collected and the responses to the questions above, what would you conclude for your business? Based on your customers' responses, what changes will you make to your list of pencils?

Have students create a poster that demonstrates the connection between the table, the line plot, the pie chart and the scenario. Students should use words, pictures, and diagrams to fully explain their business. Students can use their creativity to showcase their graphics as well as the strategies they use to represent the data.

This activity promotes students' critical thinking and encourages them to think about strategies to improve their business. Based on the data received, they can carefully examine the type of pencils they are selling and replace the least favorite with pencils similar to the ones that their customers favor so they can increase the profit.

Evaluate

During the activity, teachers need to listen to students' descriptions of their data and line plots and look for their understanding of line plots representing the frequency of their items. Also, look for students' ability to convert tables into pie charts on https://mathigon.org/polypad and their interpretation of the line plot as well as the pie chart. Remember, students are not creating the pie chart by hand in this activity. Students need to understand the concept of the area of a section in the pie chart related to the fraction representing that sector. For example, the area of the sector representing 1/20 must be smaller than the area of the sector representing 5/20. Teachers should also look for those students who struggle to convert the value of the coins into fractions so they can provide instruction to help them understand the concept. For example, they can start by asking how many pennies are in a dollar. Students answer 100. Then, the teacher can clarify that the value of a penny compared to the dollar is 1/100. The teacher then can move to the next coin and ask the same question about a nickel. If students continue to struggle, the teachers can use the following website to review the concept of area and Money: https://apps.mathlearningcenter.org/money-pieces/

Appendix A

Customers	Types of pencil						
	SpongeBob	Finding Nemo	Paw Patrol	Mickey mouse	Harry Potter	Incredibles	Minions
Customer A							
Customer B							
Customer C							
Customer D							
Customer E							
Customer F							

Appendix B

Pencil Type	Amount
Sponge Bob	
Finding Nemo	
Paw Patrol	
Mickey Mouse	
Incredibles	
Minions	
Harry Potter	

Appendix C

x(Types of pencils)	y (total per type)	Money received
SpongeBob		
Finding Nemo		
Paw Patrol		
Mickey Mouse		
Incredibles		
Minions		
Harry Potter		

Citation

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SECTION 10 - GEOMETRIC MEASUREMENT: UNDERSTAND CONCEPTS OF VOLUME AND RELATE VOLUME TO MULTIPLICATION AND TO ADDITION

Task 37 - Brick by Brick

Melena Osborne, Michael Gundlach, Michelle Tudor

Mathematics Content Standards

CCSS.MATH.CONTENT.5.MD.C.5

Relate volume to the operations of multiplication and addition and solve real-world and mathematical problems involving volume.

CCSS.MATH.CONTENT.5.MD.C.5.A

Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.

CCSS.MATH.CONTENT.5.MD.C.5.B

Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real-world and mathematical problems.

Mathematical Practice Standards

CCSS.MATH.PRACTICE.MP1

Make sense of problems and persevere in solving them.

CCSS.MATH.PRACTICE.MP6

Attend to precision

CCSS.MATH.PRACTICE.MP8

Look for and express regularity in repeated reasoning

Materials

Unit cubes, base ten blocks or Legos, grid paper, copies of Boeing article, copies of Appendix A, wrapping paper

Lesson Objective

Students will develop the volume formula of $V = l \times w \times h$ by attempting to build a box the same size as an unknown box. Creativity will be emphasized by having students build the box with little to no guidance, encouraging exploration. Student justifications of the size of the box will also encourage creativity.

Engage

Show the following video about the Boeing Factory in Everett, WA. It is the largest building by volume in the world. https://www.youtube.com/watch?v=V5Oah5CosAU
This article can also be given to students to help them see some of the actual dimensions of the building. https://science.howstuffworks.com/transport/flight/modern/boeings-everett-facility-is-largest-building-on-earth.htm

After students have watched the video and read the article, ask them why is the volume of the building so important? (They have to be able to build the planes inside). What measurements are important when designing the building? Students may think of length and width, but not height. Guide the discussion to make sure students recognize that designing a building also requires thinking about the height.

Explore

Give each group of students a box made out of base-ten blocks that has been wrapped in wrapping paper or newspaper. Tell students that their goal is to make a box that is the same size as the wrapped box and to justify why their box is the same size. (This may work better with Legos or snap-together cubes depending on the size of the box.) Students will likely justify the boxes are the same size by holding their box up to the side of the box of unknown size. After students have justified the boxes are the same size, have them determine how many blocks are in the box of unknown size without counting the blocks in their box. This will help students connect the idea of a volume measurement to the number of unit cubes that can fill a space of a certain size.

Explain

Have 2-3 groups present their methods for counting the number of blocks in their boxes. Select groups that emphasize breaking down the box into "layers" of rectangles of the same size. Students may not use the "layers" language, but look for students that recognize their boxes are made of rectangular layers of the same size. These layers may be stacked vertically or horizontally. After students have shared their solution methods, emphasize to the class language used by the presenting students that hints at multiplication. Using the dimensions of a box used by one of the groups, walk students through one of the groups' methods of splitting the box into layers, and use this demonstration to develop the formal volume formula of $V = l \times w \times h$. At this point, you could mention to students why volume has units cubed. For example, if the units are inches, then $inches \times inches \times inches = inches^3$.

Extend

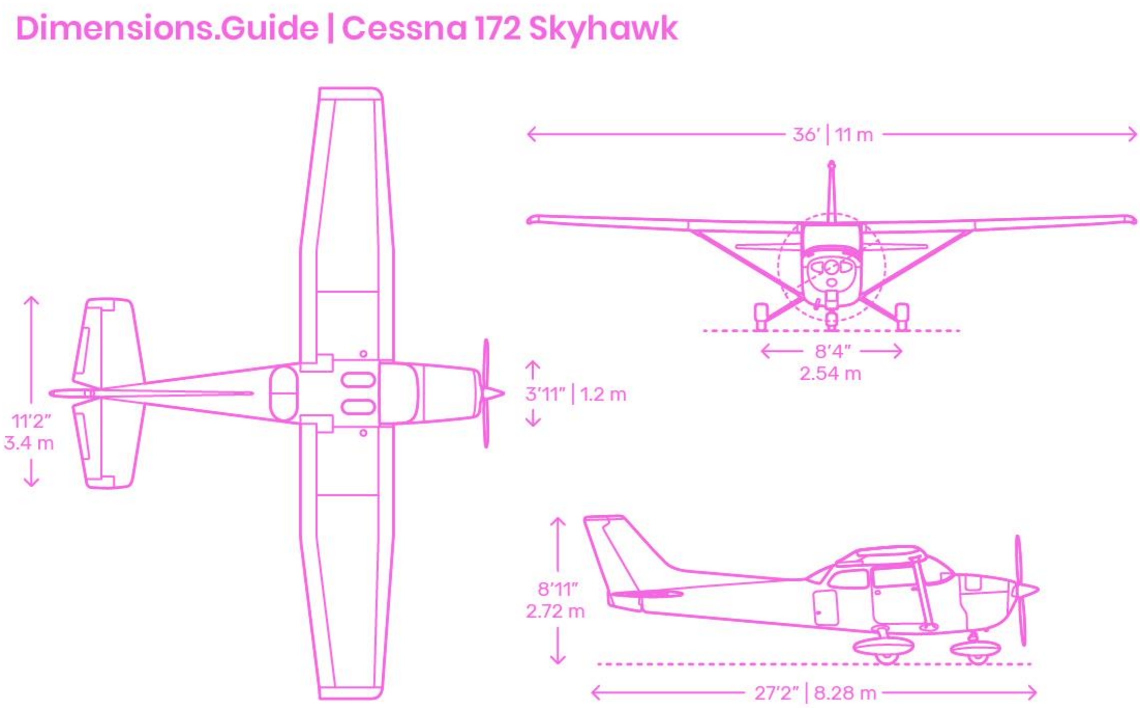
Give students the diagram with the dimensions of a plane in Appendix A. Have small groups of students work together to determine the volume of an airplane hangar where the plane could be stored. Remind students to think about getting the plane in and out of the building, people moving around the plane to work on it, etc. so they don't design a building that exactly matches the dimensions of the plane. Students should use base ten blocks, unit cubes or Legos to build a model (each block should represent 1 $ft.^3$ or the students could decide individual units) and then they should draw the 3-dimensinoal model on grid paper; alternatively, the students could use a computer program (such as GeoGebra) to build their 3-dimensional model and then print it. The drawing should show the length, width and height as well as the volume of the building. When groups have finished, have them come back together to share their designs and compare the size of the hangars they created.

Alternatively, this concept could be extended by assigning students a take home project. Have students create a 3-dimensional model of a plane (design and dimensions of their choice) and the necessary building to store the plane. The students should include a model ("blueprint") of the plane and building on grid paper (or use a computer program to build the models) along with measurements and units. This could be a group project or individual project.

Evaluate

Formative assessment occurs as group conversations are monitored. Look for students finding dimensions of boxes and recognizing the volume formula for a rectangular prism as an extension of the area formula for a rectangle. Encourage students to view the volume formula for a rectangular prism as a stacking of rectangles of height or width 1. Summative evaluation should come through collecting student work from the "extend" portion of the lesson. Students can practice finding the volumes of various rectangular prisms, either at home or at school, as a summative assessment of their understanding of the volume formula. Having students find the volume of at least 8 objects would likely be sufficient.

Appendix A



Citation

Osborne, M., Gundlach, M., & Tudor, M. (2023). Brick by Brick. In A. Bicer (Ed.), *Creativity-Directed Mathematical Tasks for 5th Grade Common Core Classrooms* (pp. 525-530). ISTES Organization.

Task 38 - Let's Build!

Traci Jackson, Aylin S. Carey, Fay Quiroz

Mathematical Content Standards:

CCSS.MATH.CONTENT.5.MD.C.3

Recognize volume as an attribute of solid figures and understand concepts of volume measurement.

CCSS.MATH.CONTENT.5.MD.C.3.A

A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume.

CCSS.MATH.CONTENT.5.MD.C.3.B

A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units.

CCSS.MATH.CONTENT.5.MD.C.4

Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.

CCSS.MATH.CONTENT.5.MD.C.5

Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.

CCSS.MATH.CONTENT.5.MD.C.5.A

Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.

CCSS.MATH.CONTENT.5.MD.C.5.B

Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.

Supporting Standards

CCSS.MATH.CONTENT.5.MD.C.5.C

Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems

CCSS.Math.Content.6.EE.A.1

Write and evaluate numerical expressions involving whole-number exponents.

CCSS.Math.Content.6.EE.A.2

Write, read, and evaluate expressions in which letters stand for numbers.

Mathematical Practice Standards

CCSS.MATH.PRACTICE.MP1

Make sense of problems and persevere in solving them.

CCSS.MATH.PRACTICE.MP3

Construct viable arguments and critique the reasoning of others.

CCSS.MATH.PRACTICE.MP7

Look for and make use of structure.

CCSS.MATH.PRACTICE.MP8

Look for and express regularity in repeated reasoning.

Vocabulary

Volume, rectangular prism, unit cube, Soma Cubes, base

Materials

Soma Cubes or snap cubes, grid paper (Appendix C), sugar cubes (optional), colored pens or markers

Lesson Objective

Students will investigate volume using cubes to explore different shapes. Students will represent the volume in different ways by grouping non-overlapping sections and writing expressions to represent groupings. Mathematical creativity will be encouraged having students discover and represent the volume of a building in multiple expressions.

Students will compare and contrast the models to justify multiple ways to represent the volume. Students will also engage creatively by reversing the task and different models with the same volume. This lesson is a 2-3 day lesson.

Engagement

(30 minutes) Show students the picture in Figure 1 and ask if they notice the location or can name some of the buildings? Students may name the location, New York City, and the famous building such as the Empire State Building.

Students should also be able to compare buildings and mention that some buildings are taller than the others, most of the buildings are rectangular prisms, some buildings have pointed tops, and other buildings that are shorter are also longer.

Figure 1. Famous Buildings in New York City

Show students one Soma Cube and explain to them this is one unit (see Figure 2), which represents volume or the amount something contains. Ask students to predict how many soma cubes (see Figure 3) or cubic units could be in the Empire State Building ? Have students share their answers with the whole class. Place students in groups of 2-3 students and give each group *many* soma cubes. Tell students they are going to have a race and will need to build two buildings from the picture (see Figure 1). One building needs to represent the Empire State Building , and the other building needs to represent another building in the picture to scale (Figure 1). Remind students about scale as they discussed earlier in the year when multiplying fractions, and when sharing with the class they need to defend their models with reasoning about their size in cubic units. Now, show students Figure 3 and ask what they notice and wonder about this building. Discuss how cubic structures are assembled together to create one building. Ask students to predict how they might find the volume or cubic unit of this building.

Figure 2. Unit Cube

Figure 3. Cubic Building

Figure 4. Soma Cube Structures Built From Link Cubes

Tell students that these cubes are arranged in specific structures and are known as Soma Cubes (Figure 4). The way these structures were put together was designed by a poet while listening to a science lecture. Have students construct the structures (Soma Cubes) out of link cubes (see Figure 4.)

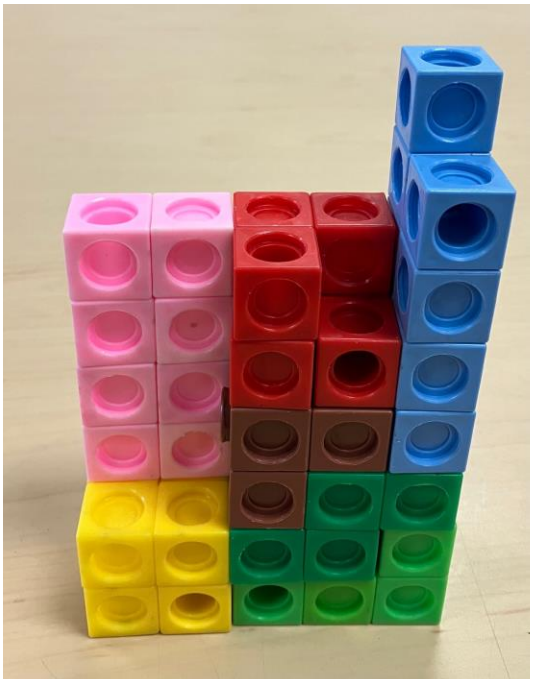
Alternatively, create these ahead of time for each group. Ask what they notice about the cubes. Take all responses but highlight what one cubic unit is.

Explore

Now using structures students built from the engagement, they all fit together to form a larger cube. Ask students how many total unit cubes there are in all the structures and how they counted. They may see the cubes with 6 structures each with 4 unit cubes and 1 structure composed of 3 unit cubes (brown in Figure 5) . As how they might write that as an expression and write it on the board. The expression could be $6 \times 4 + 3$. Check with other students that they understand what each number represents (6 structures made of 4 unit cubes each plus one structure made of 3 cubes.) They may see 7 structures with one missing 1 unit cube. This expression could be represented by $7 \times 4 - 1$. Students may also sort the structures by the number of unit cubes touching the table and then add the additional unit cubes that are not. The expression will vary depending on how the structures are laying on the table (in this case the green, pink and brown structures have 2 unit cubes touching the table and the other 4 colors have 3 cubes touching the table so the expression would be $(3 \times 2 + 4 \times 3 + 9)$. Ask for other ways to count the unit cubes and write the expressions that represent their counting. Let students know that the structures will stay in these formations throughout the lesson, remind them to not pull apart the cubes.

In groups of 2-4 ask students to try and create a large cube using all of the pieces without breaking them apart. This will be a challenge for some groups, so let students know in advance that all groups might not be able to solve this puzzle today (perhaps place the pieces at a back table for students to explore later if they are not able to finish creating the cube today.) After students have created a larger cube ask if they think there is another way to make the cube and challenge them to try again.

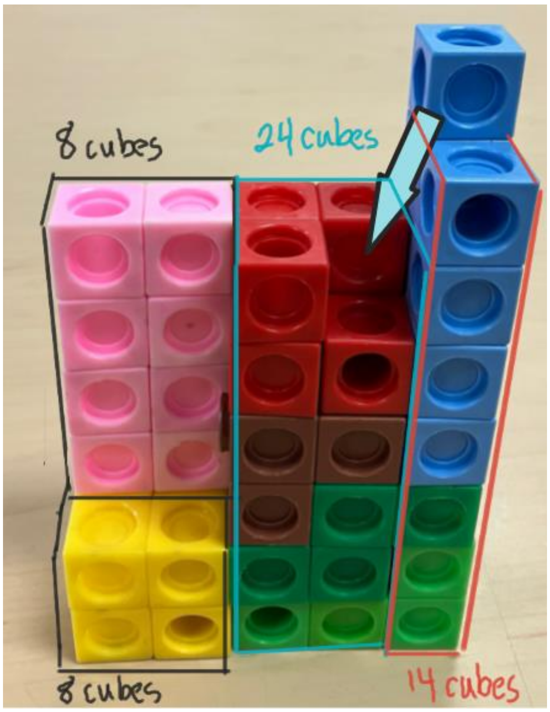
After several groups of students have created $3 \times 3 \times 3$ cubes bring the class together. Ask how many total unit cubes are in the larger created cube. Tell students the number of unit cubes in a figure is called the volume. Connect back to the expressions written on the board in pieces and ask how this compares to the individual structures added together. Ask students if there are other ways to count all the unit cubes now that they are all together in this larger cube. Listen for students to recognize that they can count in layers. Students may start with the top, side, or bottom layer of cubes and then multiply by 3 for each layer.



Example Expression and Explanation

Students divide the building by colors they may get an expression for the total unit cubes as

$8 + 8 + 8 + 8 + 8 + 6$



Example Expression and Explanation

They may calculate the number of cubes using the volumes of individual pieces.

$(2 \times 2 \times 2) + (2 \times 4) + (2 \times 2 \times 6) + (2 \times 7)$

Figure 5. Example Building with 2 Sets of Soma Cubes with Sample Expression

If students add layers of 9 cubes or rows of 3 unit cubes, ask if there is a way to represent this with multiplication both in the first layer to make a base (3×3) then to multiply by the total layers (3) by the area of one layer 9. Write $3 \times 3 \times 3$ on the board and check to see that students can use the large cube to describe what each part of the expression represents. Ask what would happen if you added another layer on top and ask what the new expression would be for the volume. If students say add 9 unit cubes to 27 unit cubes, ask if there is another way to calculate using multiplication. Have a 3×3 layer ready to add on the top of the student cube to help students visualize this to represent the expression $3 \times 3 \times 4$

Tell students they are now going to create different buildings with the same volume as the cube, 27 unit cubes. Have them rearrange the cubes to form another type of building of their choice then share an expression to represent the volume from the way they visualize it.

Next, have 2 groups of students combine their Soma Cubes and create a building (without pulling apart the Soma Cube structures, so students will have a total of 14 Soma Cube structures, with 54 total unit cubes.) On a printout (see Appendix A) instruct the two groups students write an expression that represents the volume (see Figure 5 for an example building). Have the groups leave their expressionrotate so that each pair of groups is at another building. Challenge them to see how the creators saw the figure by looking at their expression. Then have the new pair of groups try to find a different way of combining the parts and write a new expression that represents the volume. They are welcome to write more than one expression if they see it in multiple ways. Switch groups one more time then repeat the process.

Explain

Have student groups return to their original building. Ask them to look at the other two (or more) expressions and see if they can understand how the visiting groups saw the building. Next ask how students can check to see if the expressions are an accurate measure of volume (evaluating the expressions.) Have them evaluate the expressions and if they find an error, ask what that group might have been thinking.

Next students will apply their knowledge to rectangular prisms. Have students remove the pieces from the building back into 2 sets (not breaking apart the Soma Cube structures) and split back into groups of 2-4 students.

Tell students a rectangular prism is like a cube, but instead of a square base, it can be different sizes of rectangles. A cube is a special case of a rectangular prism where all sides are equal. Tell students we sometimes draw figures to represent three dimensional objects. Ask students to imagine these as sitting on their desk. Have students examine the two rectangular prisms (see Figure 6) and ask what is the same and what is different. Take all student responses. Keep taking responses until you have the first layer is the same or the area of the base is the same. Save this information for after students have played with making prisms.

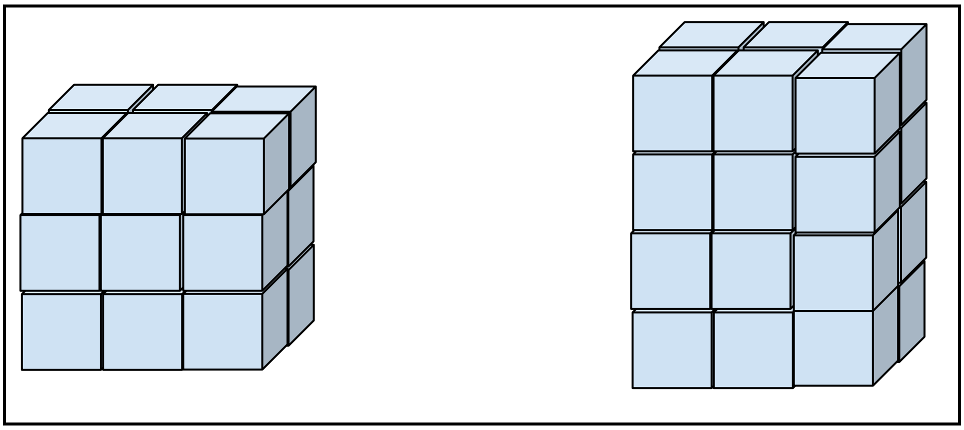
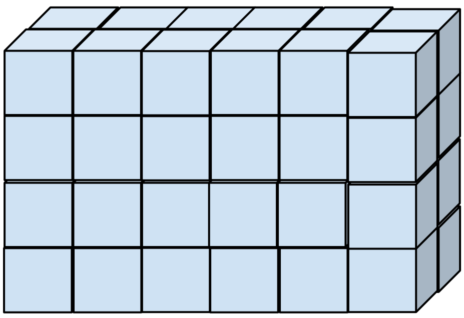


Figure 6. Two Rectangular Prisms to compare what is the Same and what is Different

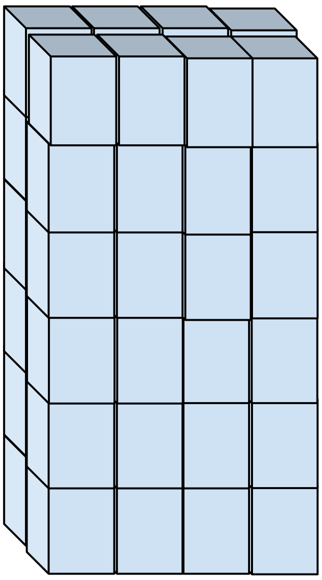
Next give students the printout (see Appendix B) to explore different challenges with the structures. Let students know that some might not be possible and if they find one that is not possible have them justify why this is not possible. For example, a large cube with 8 unit cubes would have to be 2 cubes long, wide, and high so would use 2 of the 4 unit cube structures. No two of the structures can be put together without changing the length, width or height to 3 cubes, so a cube of 8 unit cubes is impossible.

After students have explored trying to make some of the prisms, bring them together and ask what possibilities for a rectangular prisms length, width, and height could be a volume of 12 unit cubes (assuming you could have individual cubes.) Build the figures students suggest with unit cubes. Optionally, make ahead of time bases of 2x6, two 2x3 and three 2x2 (but don't let students see these) Ask students what the height of the 2x6 structure is. Ask students what expression they would use to calculate each of the representations of the structures that have a volume of 12 unit cubes. Here highlight finding the area of the base by multiplying the length and width, then multiplying by the height by seeing layers.

Return to the rectangular prisms that students noted what was the same and what was different and the response that the bases are equal. Ask how many cubes were added to the left rectangular prism to create the right rectangular prism. Ask students what they would multiply the base in the left figure to get the volume of the entire prism. Then, ask students what they would multiply the right figure to get the volume of the entire prism. Show students the first picture and have them visualize what this would look like in unit cubes on the desk. Ask students for an expression that would efficiently calculate the volume of prism (see Figure 7). Then show students the second rectangular prism.



First Prism



Second Prism

Figure 7. Rectangular Prisms

Ask what is the same and what is different. Highlight the rotation of the first picture to create the second. Ask if they could use the same expression. Ask students if they had to program a computer to calculate the volume what would they tell the computer to do?

Extend

After creating some models, have students think about painting their buildings. Would they paint one side, two sides, or all sides? Would they paint every side with one color or different colors? How would they calculate the amount of color needed to paint one side, two sides, or all sides? Have a short discussion as a whole class to address the previous questions. Then, show the example building in Figure 8 and have students think about this task: Suppose they take a 3x3x3 cube as seen in Figure 8 and completely dip into a pot of red paint. Remind the students that the cube is made of unit cubes and that each unit can come apart (Note: If teachers would like to demonstrate, sugar cubes can be great for this activity.)

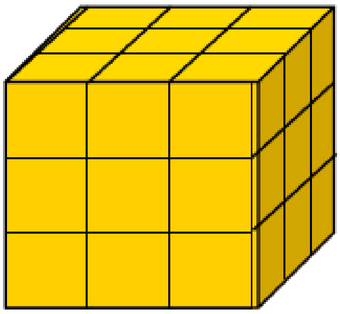


Figure 8. 3x3x3 Cube

After dipping the cube into paint, let it dry. After the paint dries, students would take the cube apart and examine the cube units. Ask students what patterns they notice. Encourage students to think visually: How many cubes have 0 sides painted? 1 side painted? 2 sides painted? 3 sides painted? Have students work in groups of three or four. To engage students in creative thinking and independent problem solving, have each student individually record the way they visualize the number of painted cube units. Provide grid papers and colored pens or markers. After students complete answering the questions and individually record their predictions, teachers can ask students to visit the other groups so that students can see how others identified their predictions and compare their findings. Bring the class together, briefly discuss each group’s findings, and ask students to think whether we can use patterns to find the answers to these questions for larger cubes such as 4x4x4 or even 50x50x50. Is there a way to find a general method, if calculations are repeated? Then introduce Table 1 to help students write general expressions. Fill out the table with students' predictions (See Appendix D for answers). For example, after calculating how many 0-, 1-, 2-, and 3-sides painted for a 2x2x2, 3x3x3, 4x4x4, and 5x5x5 cube, students are expected to describe their predictions of the n[th] structure as n^3 for the number of unit cubes, 8 cube units have 3-sides painted, $12(n-2)$ or $12n-24$ cube units will have 2-sides painted, $6(n-2)(n-2)$or $6(n-2)^2$ cube units will have 1-side painted, and finally for 0-side painted cube unit will have the general method: or , $(n-2)(n-2)(n-2)$or $6(n-2)^3$.

Table 1

Side Length of Cube	Number of Unit Cubes	3 Sides Painted	2 Sides Painted	1 Side Painted	0 Side Painted
2					
3					
4					
5					
n					

Evaluate

As students work with the Soma Cubes, circulate, ask questions, and check students' understanding of what a unit cube is and how it relates to volume. Ask them how large the larger cube will be ($3 \times 3 \times 3$) based on what they know about the pieces. As students get frustrated, let them know this is common for all problem solvers and encourage them to take a quick break and keep working. If students don't complete the $3 \times 3 \times 3$ cube task, assure them that they will have a chance to play with the cubes another time.

When students are working on their own buildings, listen for students breaking the prisms into smaller chunks and adding them together. Ask if any chunks have the same volume and if they could write an expression that would include multiplication. Ask students if they would ever use subtraction to calculate volume. This brings up the idea of negative space where students would fill the imaginary spaces to make an easy to calculate figure then subtract out the missing cubes.

When students are working to create prisms of a particular volume, ask students what possible side lengths the rectangular prisms have to create to that volume. They may forget to include the third measure. For example, they may say (3x4) or (6x2) for a volume of 12 unit cubes. Let them know that that would work if they could create a height of 1. Ask if enough of the Soma Cubes can lie flat to create this. If students are frustrated with one of the challenges, give them a suggestion to move to another (Appendix B). If they complete all the challenges, ask them to write one of their own.

When pulling students together for the explain portion of the lesson, be sure to talk about different orientations of seeing the same rectangular prism and how the expressions are connected by asking students how they see the multiplication. For example, figure 7can be represented by the equivalent expression $6 \times 2 \times 4 = 4 \times 2 \times 6 = 2 \times 4 \times 6$, all having the same volume of 48 unit cubes.

When asking students how a computer might calculate the volume of a prism, ask what information the computer would need to know. Listen for either the length, width and height and the area of the base and height. They may say the number of total cubes in the prism, ask if there is a way to use less information. Ask what letter they might store the information in for each measurement. After the relevant information as students what they tell the computer to do with that information to calculate the volume. How might they use letters to tell the computer to do that? Write on the board $l \times w \times h$ or $b \times h$. Teachers look for students who can generalize these letters as the formula for volume.

References

Soma Cubes https://americanhistory.si.edu/collections/search/object/nmah_1425322
Buildings https://www.britannica.com/list/13-iconic-buildings-to-visit-in-new-york-city
Cubic Building https://archziner.com/home/architecture/worldwide-brutalist-architecture-70-photos/?image_id=24961

Appendix A

Building Creators' Expressions:
1st Rotation Expressions:
2nd Rotation Expressions:

Building Creators' Expressions:
1st Rotation Expressions:
2nd Rotation Expressions:

Appendix B

Create a rectangular prism with a volume of 12 unit cubes.	Possible?	Expression
Create a rectangular prism with a volume of 16 unit cubes.	Possible?	Expression
Create a cube with a volume of 8 unit cubes.	Possible?	Expression
Create a rectangular prism with a volume of 20 unit cubes.	Possible?	Expression
Create a rectangular prism with a volume of 30 unit cubes.	Possible?	Expression

Create a rectangular prism with a volume of 12 unit cubes.	Possible?	Expression
Create a rectangular prism with a volume of 16 unit cubes.	Possible?	Expression
Create a cube with a volume of 8 unit cubes.	Possible?	Expression
Create a rectangular prism with a volume of 20 unit cubes.	Possible?	Expression
Create a rectangular prism with a volume of 30 unit cubes.	Possible?	Expression

Appendix C

Appendix D

Side Length of Cube	Number of Unit Cubes	3 Sides Painted	2 Sides Painted	1 Side Painted	0 Side Painted
2	8	8	0	0	0
3	27	8	12	6	1
4	64	8	24	24	8
5	125	8	36	54	27
n	n^3	8	$12(n-2)$	$6(n-2)^2$	$(n-2)^3$

Citation

Jackson, T., Carey, A. S., & Quiroz, F. (2023). Let's Build! In A. Bicer (Ed.), *Creativity-Directed Mathematical Tasks for 5th Grade Common Core Classrooms* (pp. 531-548). ISTES Organization.

Task 39 - T-Rex for Breakfast

Chuck Butler, Jennifer Kellner, Amy Kassel

Mathematical Content Standards

CCSS.MATH.CONTENT.5.MD.C.5

Relate volume to the operations of multiplication and addition and solve real-world and mathematical problems involving volume.

CCSS.MATH.CONTENT.5.MD.C.5.A

Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.

CCSS.MATH.CONTENT.5.MD.C.5.B

Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real-world and mathematical problems.

Mathematical Practice Standards

CCSS.MATH.PRACTICE.MP4

Model with mathematics.

CCSS.MATH.PRACTICE.MP5

Use appropriate tools strategically.

CCSS.MATH.PRACTICE.MP7

Look for and make use of structure.

Vocabulary

Rectangular prism, multiplicative relationship

Materials

Individual whiteboards and markers, cereal boxes of the same brand with different sizes (individual, regular, family size), multilink cubes

Lesson Objective

Students will apply their understanding of the area to find the volume of rectangular prisms. This lesson will promote creative thinking by having the students find multiple ways to represent the same volume of a rectangular prism as well as using multiple pathways to solve the same problem.

Engagement

(20 minutes) To get students engaged with volume, have them share what they believe would be the difference between a human brain and a T-Rex brain. More than likely, students will think the volume of the T-Rex brain would be larger than the human brain. Next, teachers will give them these comparisons: 100 cm^3 = volume of a T-Rex brain and 600 cm^3 = volume of a human brain (Prodigy Science, n.d.). This means the human brain is 6 times the volume of a T-Rex's brain. The class can discuss if they were correct with their assumptions.

Teachers will have students get into groups of two to four. Each group will get all three-size boxes of the same brand of cereal. Have students write on their whiteboards the estimated volume of multilink cubes for each size of cereal box. Next, have students use the cubes to cover the base of the smallest box. Ask them to write on their boards how many more rows it would take to “fill” the box with cubes. Students will then fill the small cereal box with cubes until it is filled. Count the number of cubes and compare it to the estimated guess. Discuss within the groups what the difference was between their guess and the actual volume.

Teachers will have students estimate what the volumes of the remaining two boxes will be based on their knowledge of the smallest box and write those values on their whiteboards. Students can repeat the process from the smallest box with each of the remaining sizes. If students are using different brands of cereal, the groups can share their results with the class.

Explore

(30 minutes) Part 1: Students will create as many different rectangular prisms as possible using 60 multilink cubes and record the area of the base and the height of the prism on the recording sheet (Appendix A). Teachers may choose to put students in groups of two or three to complete this task.

Part 2: Teachers will regroup students so that each group will consist of prisms that have the same base area, i.e., all students with a prism with a base of area 20 will group together, and so on. Ask students to explore how their prisms can have the same volume and same base area but look different. After students have had time to explore, teachers may hand out the recording sheet and ask students to record their observations (Appendix B).

Part 3: Students will create rectangular prisms given a set of information (Appendix C). Students may work in groups of two or three to complete the task.

Explain

(30 minutes) Part 1: Teachers may wish to remind students of important vocabulary as they facilitate the lesson. As students are creating their rectangular prisms, teachers may ask them to identify the height and base of the prism by reinforcing the vocabulary. If students are struggling to find the area of the base of the prisms, teachers may ask students how they would measure the bottom (or top) of the prism, and if necessary, prompt them to use the vocabulary of length and width.

Teachers should collect group data on a classroom display. As groups are completing the recording sheet, teachers may ask students to record their data on the classroom display and be prepared to share the rectangle prism that corresponds to the data they record and show the class how they calculated the area of the base and the height. Sample student responses may include (see Figure 1):

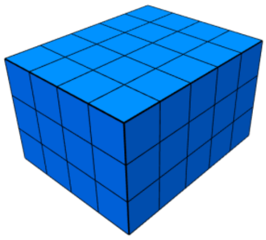
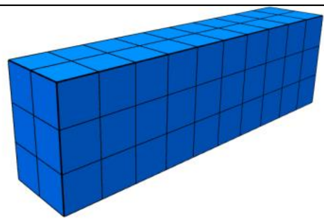
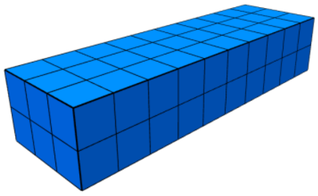
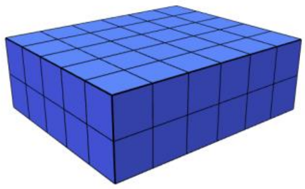
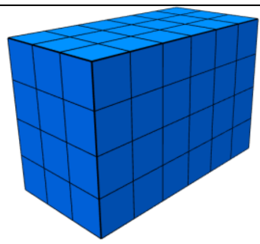
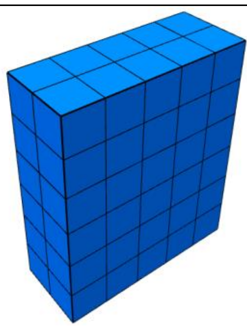
Area of the base	Height	Volume	Model
20	3	60 units^3	
20	3	60 units^3	
30	2	60 units^3	
30	2	60 units^3	
15	4	60 units^3	
10	6	60 units^3	

Figure 1. Images created from Toy Theater (Toy Theater, 2022).

After students have recorded and shared the class data, teachers may ask students to consider how the area of the base is related to the volume with their elbow partners. After students have discussed with their elbow partners, regroup students with another partner such as a face partner and share their observations from their previous partner. Then ask students to share their observations with the whole class. If students are struggling to see the relationship of $V = Area\ of\ Base \times h$, teachers may ask students how the numbers in each row are related, guiding them toward recognizing a multiplicative relationship.

Part 2: Teachers should collect group data on a classroom display. As groups are completing the recording sheet, teachers may ask students to record their data on the classroom display and be prepared to share the rectangle prism that corresponds to the data they record and show the class where they identified the length, width, and height of the prism (see Figure 2). Sample student responses may include:

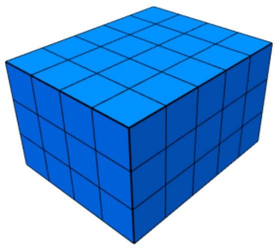
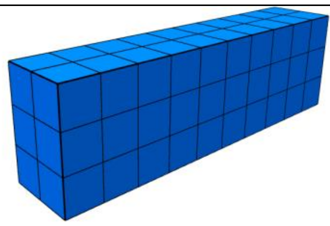
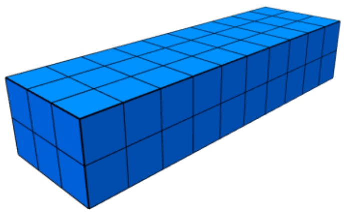
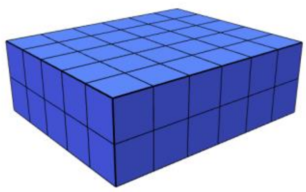
Area of the base		Height	Volume	Model
20		3	60 units^3	
Length	Width			
4	5			
20		3	60 units^3	
Length	Width			
2	10			
30		2	60 units^3	
Length	Width			
3	10			
30		2	60 units^3	
Length	Width			
5	6			

Figure 2. Images created from Toy Theater (Toy Theater, 2022).

Teachers may do a "What do you notice? What do you wonder?" protocol with students. Ask students to individually consider the data and write down 3 things that they notice and 3 things that they wonder about the data. Then ask students to share their observations and wonderings with a partner. After students have had time to share, teachers may ask students to share their observations and wonderings about the data. If students are struggling to observe the relationship $V = l \times w \times h$, teachers may ask how the numbers in the row are related, guiding them toward recognizing a multiplicative relationship.

Teachers may conclude this part of the lesson by asking students if there was a way to determine the volume of a rectangular prism without counting or knowing the number of cubes. Students will discover the formula $V = l \times w \times h$ for volume of rectangular prisms and be able to relate it back to or $V = Area\ of\ Base\ \times h$.

Part 3: As teachers facilitate the students solving the problems, teachers should ask students to create models and explain their thinking. As students are working on problems 3 and 4, teachers should challenge students to create multiple prisms that would satisfy the requirements. Sample student responses may include:

- The Paper Company needs to create boxes that will hold at least 24 cubic feet. What dimensions could the boxes be? $1 \times 1 \times 24, 1 \times 2 \times 12, 1 \times 3 \times 8, 1 \times 4 \times 6, 2 \times 3 \times 4, 2 \times 4 \times 3,$ and so on.

- You want to build a dog pen in your backyard. The pen must be at least 6 feet tall, so your dog does not jump out. You have read that your dog should have at least 288 cubic feet of space to play. What dimensions could you make the pen? $6 \times 8 \times 6, 4 \times 12 \times 6, 3 \times 16 \times 6, 2 \times 24 \times 6$, and so on. Teachers may ask students

  what considerations would have to be made in determining the final dimensions.

As students complete problem 5, teachers may need to support students in developing their understanding that volume is additive by decomposing the shape into the smaller rectangular prisms. Students may find it helpful to build the individual prisms to see the individual volumes and then compose them to find the total volume.

As students complete problem 6, teachers may need to support students by reminding them to create models of the boxes, so they can manipulate the boxes to fit into the smallest crate. If students need additional support, teachers may support students by helping them create an open box from centimeter grid paper printed on cardstock. Students may then place the models of the boxes inside the open crate to see how they fit. Students may create an open box by cutting out the corners equivalent to the size $h \times h$. For example, if students wanted to create a crate of dimensions $5 \times 7 \times 6$, students would cut on the dotted lines and fold up the remaining pieces.

As students complete the problem set, teachers may select various groups to present their solutions. Teachers may wish to select solutions that transition from least complex to most complex. To conclude the lesson, teachers may ask students to complete an exit ticket asking them how they would find the volume of a rectangular prism.

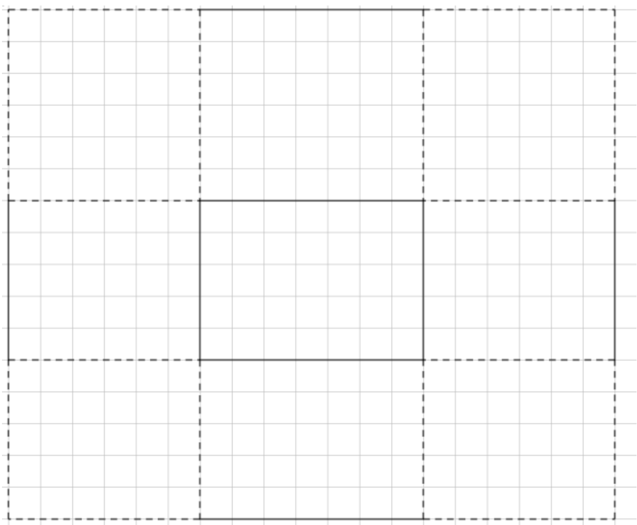


Figure 3.

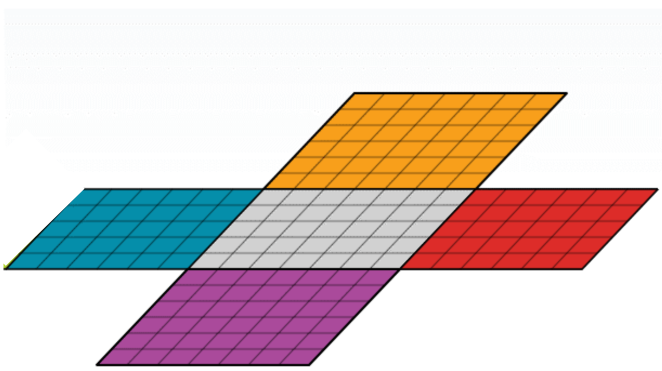


Figure 4.

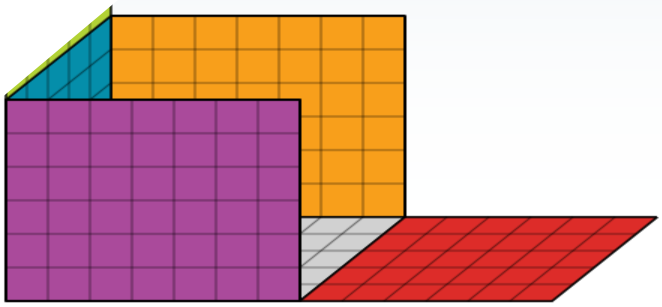


Figure 5.

Extend

Teachers may choose to assign one extension to the whole class or assign different extensions to different groups based on interest, ability, and readiness.

What If – At the end of part 1, teachers can ask students to imagine that the cubes represent a half unit. Estimate the volume of the shape and then use the cubes to determine the volume of the shape.

Missing Height– At the end of part 2, teachers ask students to find the volume of a rectangular shape with missing height. For example, you know the dimensions of the base are 5 cubes by 4 cubes. What could the volume be? Find at least three different volumes using different heights and then try to find an expression for all volumes of this shape.

Missing Area- At the end of part 2, the teacher asks students to find the volume of a rectangular shape with a missing base. For example, the height is 4 cubes. What could the volume be? Find at least three different volumes using different bases and then try to find an expression for all volumes of this shape.

Fractional Length – At the end of part 3, teachers can ask a student to create another solution to one of the examples using a fractional length. For example, for problem #4, the homeowner wants the height of the fence to equal $6\frac{1}{2}$ ft tall. What dimensions could you make the pen?

Evaluate

- In part 1, formative assessment for content and creativity occurs as the class builds different rectangular prisms with a volume of 60 units^3.
- In part 2, formative assessment for content occurs as students examine the relationship between the shape and volume of the object. For example, students will notice that a shape with the dimensions of 5 cubes by 4 cubes by 3 cubes would have the same volume as a shape with 2 cubes by 10 cubs by 3 cubes.
- In part 3, formative assessment for content and creativity occurs as students create models and solve problems that have several solution paths.
- In What If, formative assessment for content occurs as students find the volume of a shape with a fractional edge. If a student struggles to find the volume, the teacher can encourage them to find the volume of each layer of the rectangular prism.
- In Missing Height and Missing Area, formative assessment for content and creativity occurs as students imagine the height (or area of the base) of the rectangular prism and then find the volume. Students are also encouraged to generalize the formula with one unknown.
- In Fractional Length, formative assessment for content occurs as students are encouraged to use a fractional length in one of the problems from part 3.

References

Prodigy Science. (n.d.). *Fun Volume Facts*. Retrieved from Prodigy Science: http://prodigyscience.pbworks.com/w/page/18718713/Fun%20Volume%20Facts%21

Toy Theater. (2022). *Toy Teather Cube*. Retrieved from Toy Teather: https://toytheater.com/cube/

Appendix A

For each prism created, record the area of the base and the height.

Area of the base	Height	Volume
		60 units^3
		60 units^3
		60 units^3
		60 units^3
		60 units^3
		60 units^3
		60 units^3
		60 units^3
		60 units^3
		60 units^3
		60 units^3
		60 units^3
		60 units^3
		60 units^3
		60 units^3

Appendix B

For each prism created, record the area of the base and the height.

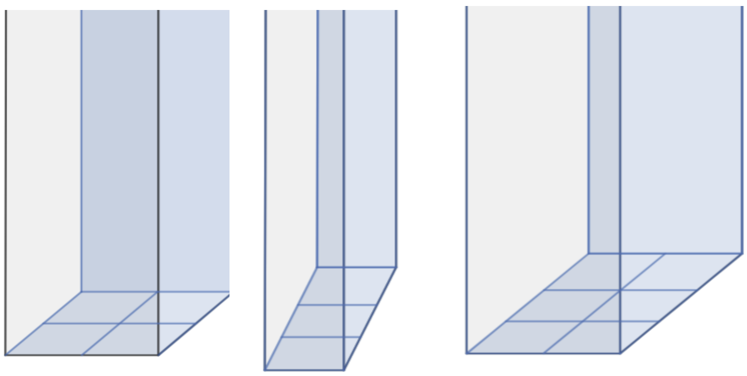
Area of the base		Height	Volume
Length	Width		60 units^3
Length	Width		60 units^3
Length	Width		60 units^3
Length	Width		60 units^3
Length	Width		60 units^3

Appendix C

Solve each problem. Create a model to show your thinking.

1. Create two rectangular prisms that have the same volume but different dimensions.

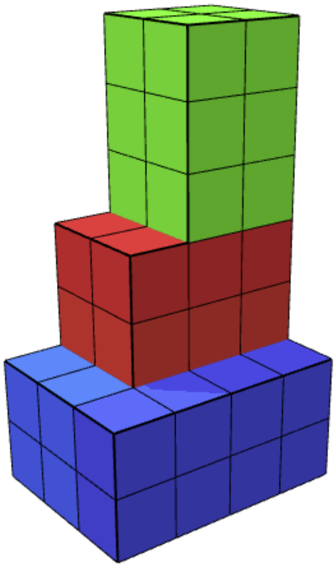
2. The prisms below have the same volume. Which prism has the greatest height? What could the heights of the prisms be?



3. The Paper Company needs to create boxes that will hold at least 24 cubic feet. What dimensions could the boxes be?

4. You want to build a dog pen in your backyard. The pen must be at least 6 feet tall so your dog does not jump out. You have read that your dog should have at least 288 cubic feet of space to play. What dimensions could you make the pen?

5. There is a stack of boxes of different sizes in the corner. What is the volume of the stack of boxes? Explain your thinking.



6. The Paper Company needs to ship 4 boxes of materials to a school. What is the smallest crate the Paper Company will need to ship the 4 boxes of materials? Justify your answer.

   Box 1: 4 feet long, 2 feet wide, and 3 feet high

   Box 2: 3 feet long, 5 feet wide, and 2 feet high

   Box 3: 2 feet long, 6 feet wide, and 1 foot high

   Box 4: 4 feet long, 5 feet wide, and 3 feet high

Citation

Butler, C., Kellner, J., & Kassel, A. (2023). T-Rex for Breakfast. In A. Bicer (Ed.), *Creativity-Directed Mathematical Tasks for 5th Grade Common Core Classrooms* (pp. 549-562). ISTES Organization.

Task 40 - A Fishy Situation

Geoff Krall, Helen Aleksani

Mathematical Content Standards

Focus Content Standard

CCSS.MATH.CONTENT.5.MD.C.5

Relate volume to the operations of multiplication and addition and solve real-world and mathematical problems involving volume.

CCSS.MATH.CONTENT.5.MD.C.5.B

Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real-world and mathematical problems.

Supporting Content Standard(s)

CCSS.MATH.CONTENT.5.MD.C.5.A

Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.

Mathematical Practice Standards

CCSS.MATH.PRACTICE.MP1

Make sense of problems and persevere in solving them.

CCSS.MATH.PRACTICE.MP3

Construct viable arguments and critique the reasoning of others.

CCSS.MATH.PRACTICE.MP7

Look for and make use of structure.

Lesson Objective

Students will develop their understanding of the volume of rectangular prisms by designing multiple fish tanks according to customer specifications. Students will apply the volume formulas of $V = l \times w \times h$ and $V = b \times h$ to ensure their fish tanks have the correct volume. The concept of volume will be further highlighted by a discussion of the amount of water contained within the fish tank. Students will demonstrate their mathematical creativity by crafting multiple different solutions to the customers' specifications of volume.

Engagement

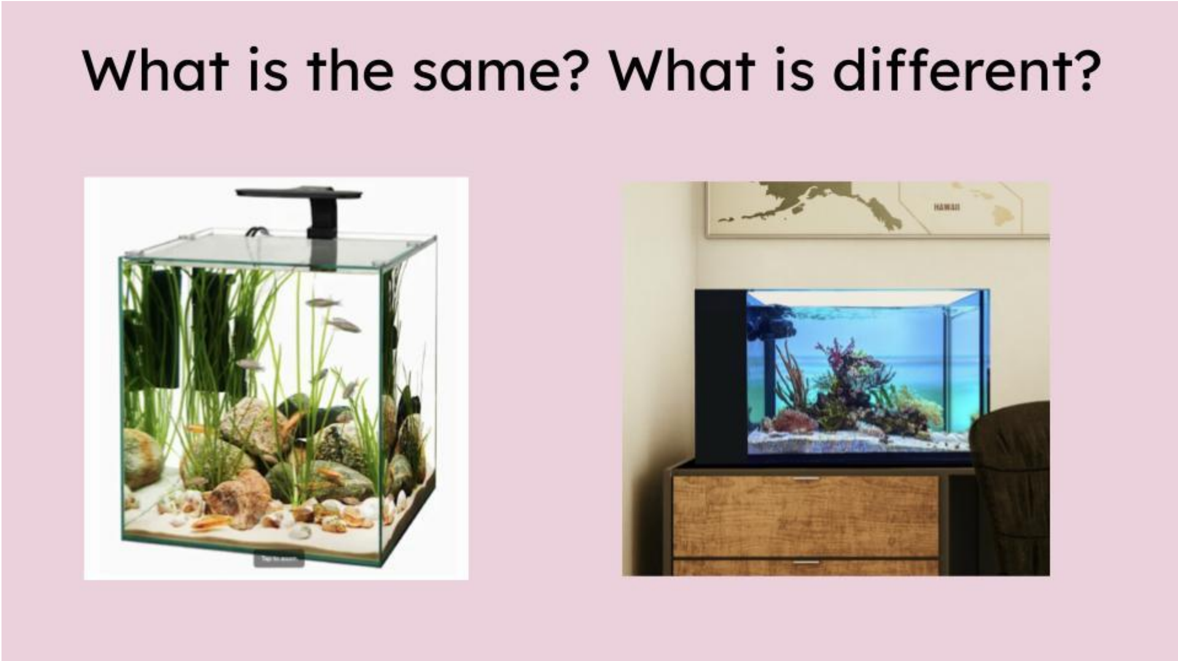
(10 minutes) Start the lesson by asking if anyone has a fish tank at home. Ask students to share what fish are in the fish tank, what size the fish tank is, where it is located, and other interesting details. Ask a student or two to draw a picture of the fish tank on the board. Some students might understand how to represent three-dimensional pictures using perspective. Students will receive practice on this concept during this lesson if they do not know how to represent three-dimensional prisms. Engage students in a conversation to see how much they understand about the volume of their fish tank:

- Do you know how much water it holds?
- How large is the fish tank?
- Can you estimate the lengths of each side? (Label the student's diagram if this is the case.)

This may be students' first experience with volume, so they may need additional explanation. As students if they can explain the difference between area and volume in their own words since they have learned about area before.

Explore

Display the following slide to students.



(Image Sources: https://www.petco.com/shop/en/petcostore/product/aqueon-frameless-cube-aquarium; https://www.saltwateraquarium.com/fusion-pro-2-14-gallon-aio-peninsula-aquarium-tank-only/?gclid=CjwKCAjwopWSBhB6EiwAjxmqDTEWVqE2SXfkb06wRVcmEKTSl6fBgUjBCrsOX1NO1DZDSsg1VJ3GJRoCubcQAvD_BwE)

Ask students to describe what is the same in these pictures and what is different. Here are some potential student responses.

Same:

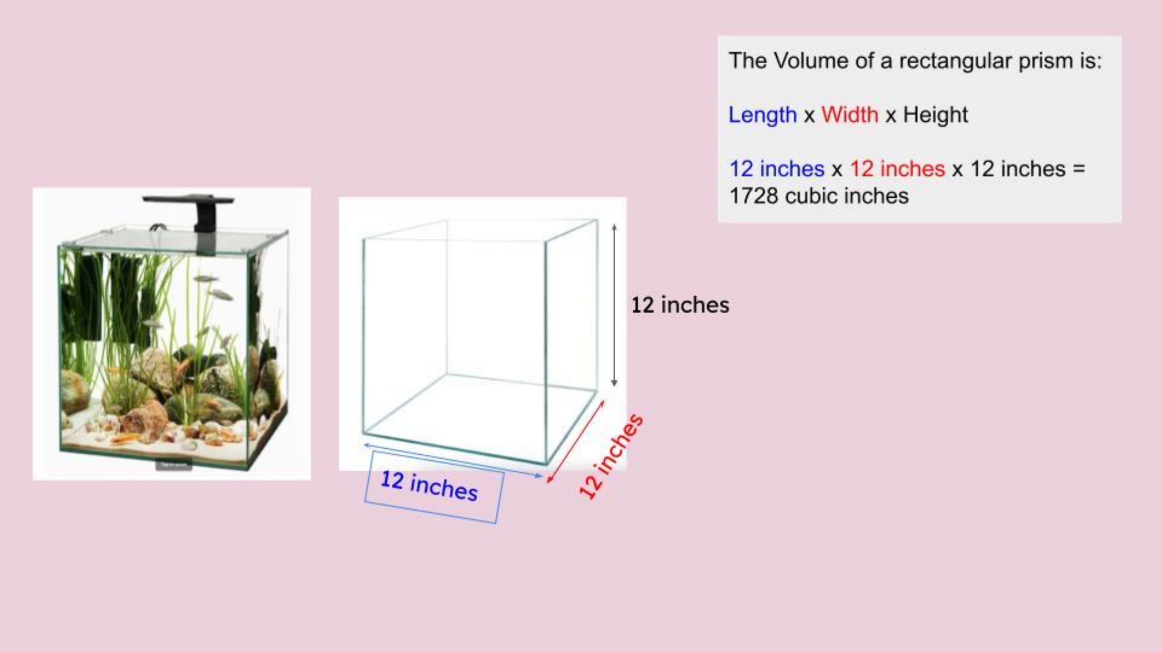
- They are fish tanks.
- They are rectangular prisms (students may use other vocabulary).
- They have rocks and plants in them.

Different:

- One is taller, the other is longer.
- One looks like it has a blue light in it.
- One is on a dresser.
- One has a light on top.

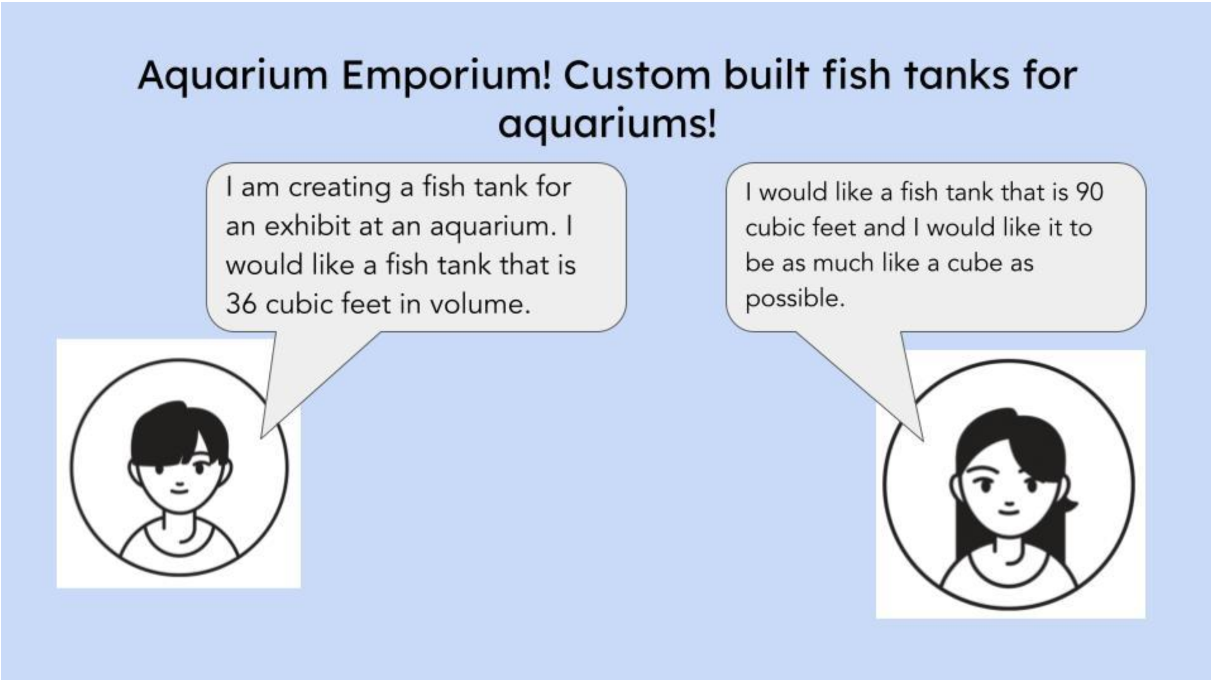
Ask students which one they think holds more water. Have students take a vote. Then reveal that actually each water tank holds the *same* amount of water. Explain that this is because they have the same volume. Even though they have different dimensions they have the same volume. Alternatively, if some students already know how to calculate the volume of a rectangular prism, you may wish to have them calculate the volume to discover the result of having the same volume.

Use the following slides to demonstrate how to find the volume of rectangular prisms using a formula.



Be sure to build on prior lessons concerning volume.

Tell students they are going to design two custom fish tanks for two different customers. Show students the following slide.



Customer A: “I am creating a fish tank for an exhibit at an aquarium. I would like a fish tank that is 36 cubic feet in volume”.

Customer B: “I would like a fish tank that is 90 cubic feet and I would like it to be as much like a cube as possible.”

Tell students we are going to focus on one customer at a time, starting with Customer A.

Explain

(20 minutes) Hand out isometric dot paper (Appendix A) and have students draw diagrams that might fit Customer A’s specifications (36 cubic feet). This may be the first time students have used isometric dot paper. Explain that isometric dot paper can be very helpful when drawing three-dimensional objects. Artistically creative students may be able to see how the dot paper can help with three-dimensional perspectives.

Tell students that one dot-to-dot length represents one foot, as shown on the scale.

Have students attempt to draw diagrams that represent 36 cubic feet. There are many possible dimension combinations that would yield 36 cubic feet using whole numbers (Table 1).

Table 1

Length	Width	Height	Volume
6	2	3	36
4	3	3	36
9	2	2	36
4	3	3	36

Challenge students to come up with multiple possible solutions and dimensions.

Once all students have completed at least one drawing showing 36 cubic feet, have them compare their drawings with each other, looking for similarities and differences.

Now we are going to address Customer B's request.

Challenge students to create a diagram of a rectangular prism with a volume of 90 cubic feet that's as close to a cube as possible, using only whole numbers. A cube, is a type of rectangular prism in which all sides are the same length. We will not be able to make a perfect cube with 90 cubic feet of volume using only whole numbers. But we will try to get as close as possible!

There are many possible dimensions that achieve a volume of 90 cubic feet. Here are some.

Once students have completed their drawings, select a few drawings to showcase and discuss with the entire class. Choose drawings with differing dimensions. First, verify that the drawings are, in fact, showing a prism with a volume of 90 cubic feet. Then ask students to decide which of the drawings are *most* like a cube. Have students come up with a justification for which drawing is the “cubest.”

Table 2

Length	Width	Height	Volume
10 feet	3 feet	3 feet	90 cubic feet
5 feet	9 feet	2 feet	90 cubic feet
5 feet	6 feet	3 feet	90 cubic feet
6 feet	5 feet	4 feet	90 cubic feet

Possible ideas for justification might include the following:

- The dimensions have the smallest gap from largest to smallest number.
- Two of the dimensions should be the exact same.
- The bottom side has to be a square.

Students may use their argumentation skills to make a case for any of the drawings you showcase. There are many possible solutions to the task; encourage students who demonstrate mathematical creativity by creating original or multiple solutions.

Extend

(15 minutes) Extend student’s understanding of volume, demonstrating that the formula $V = b \times h$ where b is the area of the base also is an acceptable method of finding the volume of a rectangular prism.

Have students calculate the volume of prisms in Appendix B. See if they notice a pattern. Students should first notice that all of the prisms have the same volume (70 cubic inches). They might also notice that the heights of each prism is the same (10 inches). Once all students have noticed those two features, ask students what’s interesting about the remaining dimensions (3” and 4”, 6” and 2”, and 12” and 1”).

Demonstrate that in each of these cases the area of the base is the same (12 square inches). Our formula for the volume of a rectangular prism can be simplified from $V = l \times w \times h$ to $V = b \times h$, where b is the area of the base (or bottom side) of the figure. This works because for a rectangular prism, the bottom side will be a rectangle. So the area of the base (b) is just l x w.

Evaluate

This may be students' first exposure to the concept of volume, so there are multiple opportunities to evaluate student understanding throughout the lesson.

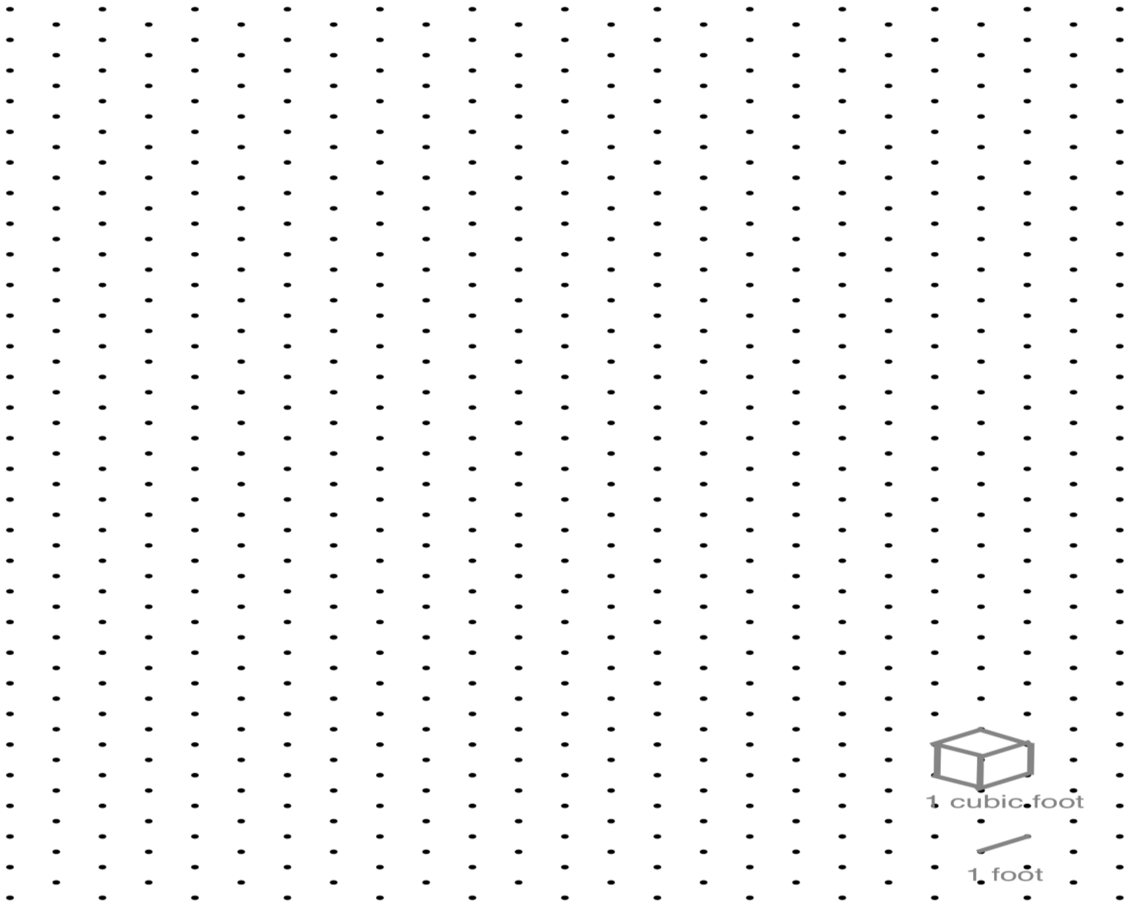
Students may get confused between the difference of area and volume. Students at this grade level are still relatively new to multi-dimensional units (i.e. square inches, cubic feet). Evaluate students' understanding of volume by asking for examples where volume is appropriate and area is appropriate. You may choose to host a small workshop with a card sort. See Appendix C for a set of cards to be sorted into area or volume.

During the Explain phase, check for student understanding by monitoring student drawings and calculations. Make sure that the work for Customer A yields 36 cubic feet. Have students write out their calculations (including units). Students may have difficulty finding unit combinations that yield 36 cubic feet. If that is the case, ask them to factor 36. Factors of 36 include 18, 12, 9, 6, 4, 3. Once students have the first factor, they may be able to find additional factors of the factor pair. For example, if a student factors 36 into 9 and 4, they may then choose to factor the 9 into 3 and 3 or 4 into 2 and 2. Use similar remediation strategies and evaluation techniques for Customer B.

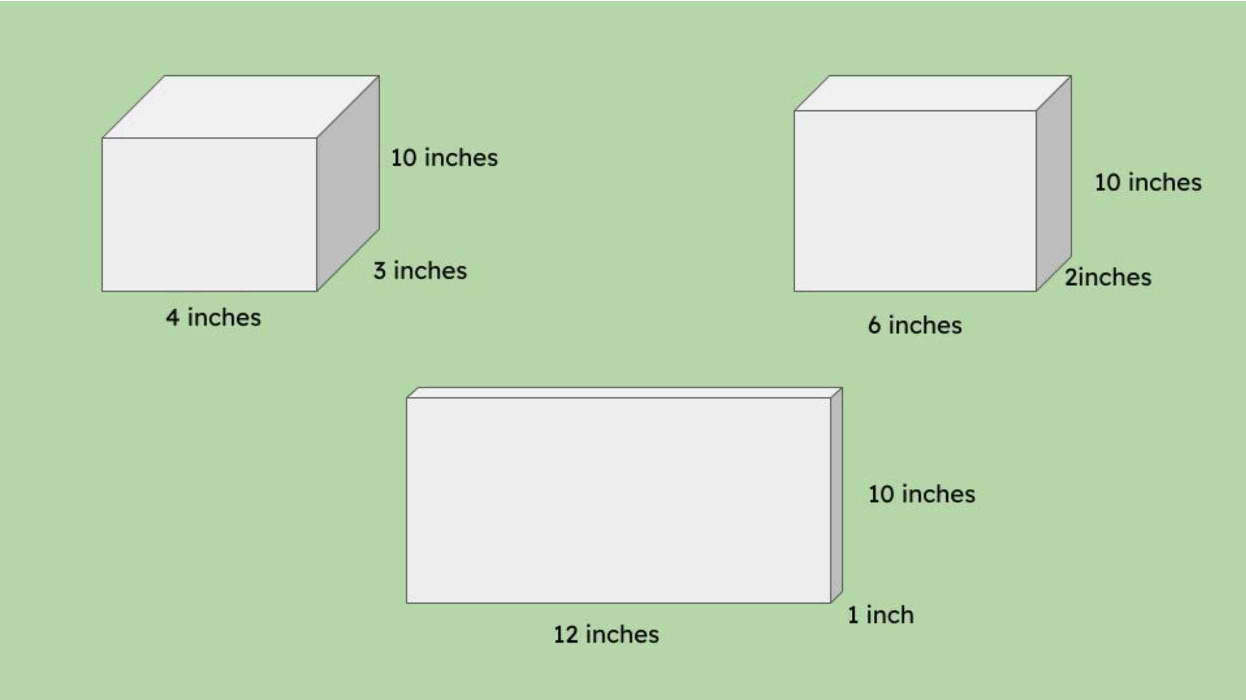
During the Extend phase, be aware that the vocabulary "area of the base" is easy to mix up with simply "base." Many students have heard volume expressed as "base times width times height." It is important to emphasize that *b* in our volume formula stands for the *area* of the base, not just the bottom length. Ask students what is the *area* of the base for each of our examples in Appendix B. Also ask for the individual dimensions to make sure students understand the difference.

Appendix A

Isometric dot paper



Appendix B


10 inches
3 inches
4 inches
10 inches
2inches
6 inches
10 inches
1 inch
12 inches

Appendix C

Area or Volume Cards

(Note: the first two columns are area concepts, the second two columns are volume concepts. Cut out the cards and mix up the cards before asking students to sort them).

Sort the following cards into either Area concepts or Volume concepts.

The amount of paint on a Wall	The size of a bedsheet	The amount of water in a cup	The amount of popcorn in a bag
The amount of carpet on a floor	The size of a football field	The amount of sand in a bucket	The amount of milk in a bowl
Square inches	Square meters	Cubic inches	Cubic Meters
ft^2	km^2	ft^3	km^3
Square miles	Write your own area concept:	Cubic Miles	Write your own volume concept:

Citation

Krall, G. & Aleksani, H. (2023). A Fishy Situation. In A. Bicer (Ed.), *Creativity-Directed Mathematical Tasks for 5th Grade Common Core Classrooms* (pp. 563-574). ISTES Organization.

SECTION 11 - GRAPH POINTS ON THE COORDINATE PLANE TO SOLVE REAL-WORLD AND MATHEMATICAL PROBLEMS

Task 41 - Plotting Directions

Melena Osborne, Michelle Tudor, Michael Gundlach

Mathematical Content Standards

Graph points on the coordinate plane to solve real-world and mathematical problems.

CCSS.MATH.CONTENT.5.G.A.1

Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates.

Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate).

CCSS.MATH.CONTENT.5.G.A.2

Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.

Mathematical Practice Standards

1. MP 5 Use appropriate tools strategically.
2. MP 7 Look for and make use of structure.
3. MP 8 Look for and express regularity in repeated reasoning.

Lesson Objective

Students will learn about the coordinate system including: the axes, origin, and quadrants. Students will be able to identify the coordinates of a given point and plot points in quadrant 1. Students' creativity will be fostered by challenging them to manipulate a number line (in some way) to show the directions North, South, East, and West. Students will then explore how to plot points on the coordinate plane to reach a specified point.

Vocabulary

Perpendicular lines, axes (x and y), coordinate system/plane, coordinates, origin, ordered pair(s), quadrants (I, II, III, IV), intersection

Materials List

Graph paper (or Appendix A), map of Denver Zoo (Appendix C)

Engagement

Tell students that you are going to be learning about writing mathematical directions today. Have students each write directions on how to get to their desk from the door to the classroom. Choose one or two students to be your "walkers." Select some students' directions to their desk and then have your walkers use those directions and see how well they can follow the directions. Choose some vaguely written directions and some specifically written directions to help students recognize the need for good, specific directions.

Explore

For the Explore, hand students a new copy of Appendix B. The students are going to use the Denver Zoo map (Appendix C) to practice plotting and identifying points in Quadrant 1 (you might want to pull up a clearer picture of the map online or print copies).

Have students answer questions (including plotting the ordered pairs) on Appendix B:

> Sample Questions: (1) From the entrance (starting point), you decide to go 1 block East and 1 block North. What animal are you visiting? What ordered pair does this represent on the coordinate system? (2) From the entrance (starting point), you decide to go 5 blocks East and 1 block North. What animal are you visiting? What ordered pair does this represent? (3) Go back to question (1). You decide that from this spot, you will go 4 more blocks East and 1 block North. What animal are you visiting? What ordered pair does this represent?

You can ask multiple questions like this. Additionally, you could have students pick animals to visit from quadrant 1 and identify the ordered pair, then plot the ordered pair on their coordinate system (see Appendix B).

Explain

Now, hand students a copy of Appendix B (copy of coordinate system/plane) so that they can label the coordinate system as you go over it. Explain to students what the coordinate system is (a pair of perpendicular lines called axes). You may have to remind the students of what perpendicular lines are (remember, perpendicular lines are two lines that intersect to create a 90 degree angle); have them give you some real-life examples of perpendicular lines (where walls meet the floor, cross-walks, tops of power poles, etc.).

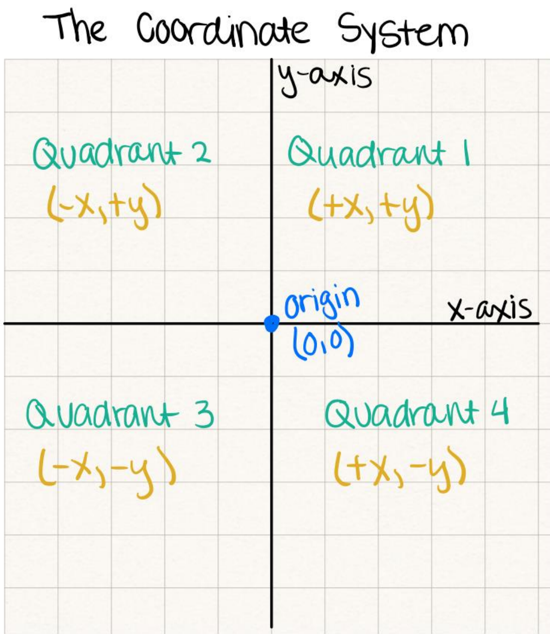
Once you have done this, explain to students that the horizontal axis is called the x-axis and the vertical axis is called the y-axis and have them label this; you could also go ahead and mention that the axes can represent different things (variables) depending on the type of problem. Once you have discussed the axes, you can move on to the origin. The origin is the intersection point of the two axes, and it represents the ordered pair (0, 0).

Let students know that ordered pairs (x, y) represent coordinates on the coordinate system; the first number represents the x-coordinate and the second number represents the y-coordinate.

Now, talk about the quadrants. Explain to students that the first quadrant is the top right one and we count counter clockwise for 2, 3, and 4. Also, discuss with students the types of values you find in each of these quadrants. Show students that Q1 is (+x, +y), Q2 is (-x, +y), Q3 is (-x, -y), and Q4 is (+x, -y).

Ask students why the sign changes from positive to negative (& vice versa) moving from quadrant to quadrant. Be sure the students label quadrants and the values that exist in each quadrant.

An example of a fully labeled coordinate system is below.



Extend

Give students a copy of Appendix A (a number line) and give them the following problem to explore and ponder:

> You are at school and want to go to the playground with your friends after school. The playground is 4 blocks East and 5 blocks North of the school. How can you represent this by modifying the number line provided?

If students struggle, ask them how they think they can represent East and North. You could even have students point with their hand left, right, up, or down to represent East, West, North, South - this may help them to realize they need a vertical line to represent North (see Appendix D for a vertical line that students can to use). Tell students to be creative with their thinking and give them some time to discuss with a partner and determine the coordinates of the playground. Bring students back to the whole group to share out ideas and discuss their thoughts. Once the group has agreed on the coordinates of the playground, let partner groups write a set of directions to the park using coordinate points.

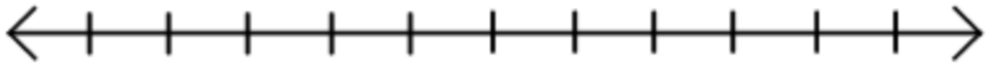
For example, Begin at (0,0), go east to (2,3), etc. To encourage creativity, challenge students to make lines that look different on the coordinate plane. Can you make a straight line? Can you make a curved line? Which way would get you to the playground the quickest? Why? See if students notice any patterns in the coordinates for the different lines they create.

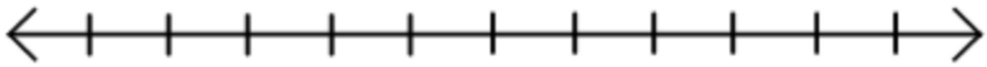
Evaluate

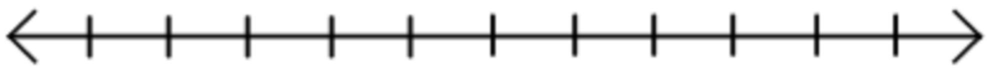
During the lesson, observe and listen to student conversation and answers to questions during the Denver Zoo activity. Notice if students are understanding the concept of how the coordinate plane numbers show a point on the graph. Students may be confused by which axis is the first number and which is the second number in the ordered pair. If there are several students who are struggling with this, go back to the Denver Zoo activity and have students write the numbers on the graph to correspond to the x and y axis. Then have them practice finding different places in the zoo using the coordinates.

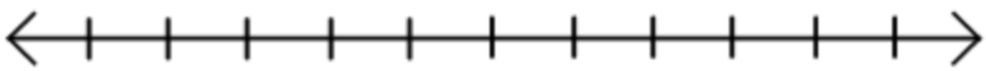
For the summative evaluation, collect the graphs and directions the students created to get from the school to the playground. Evaluate the use of coordinates and the path they created. Does it actually get you to the playground? Evaluate the students' creativity by observing the different lines they were able to create and their explanation for patterns in the coordinates to create those lines.

Appendix A. Number Line (Vertical)

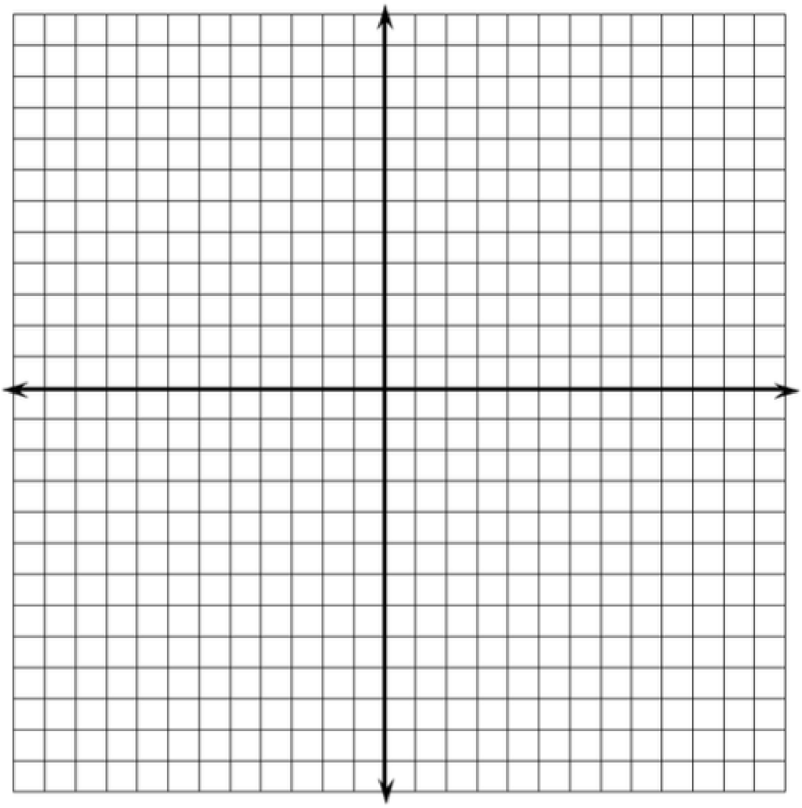






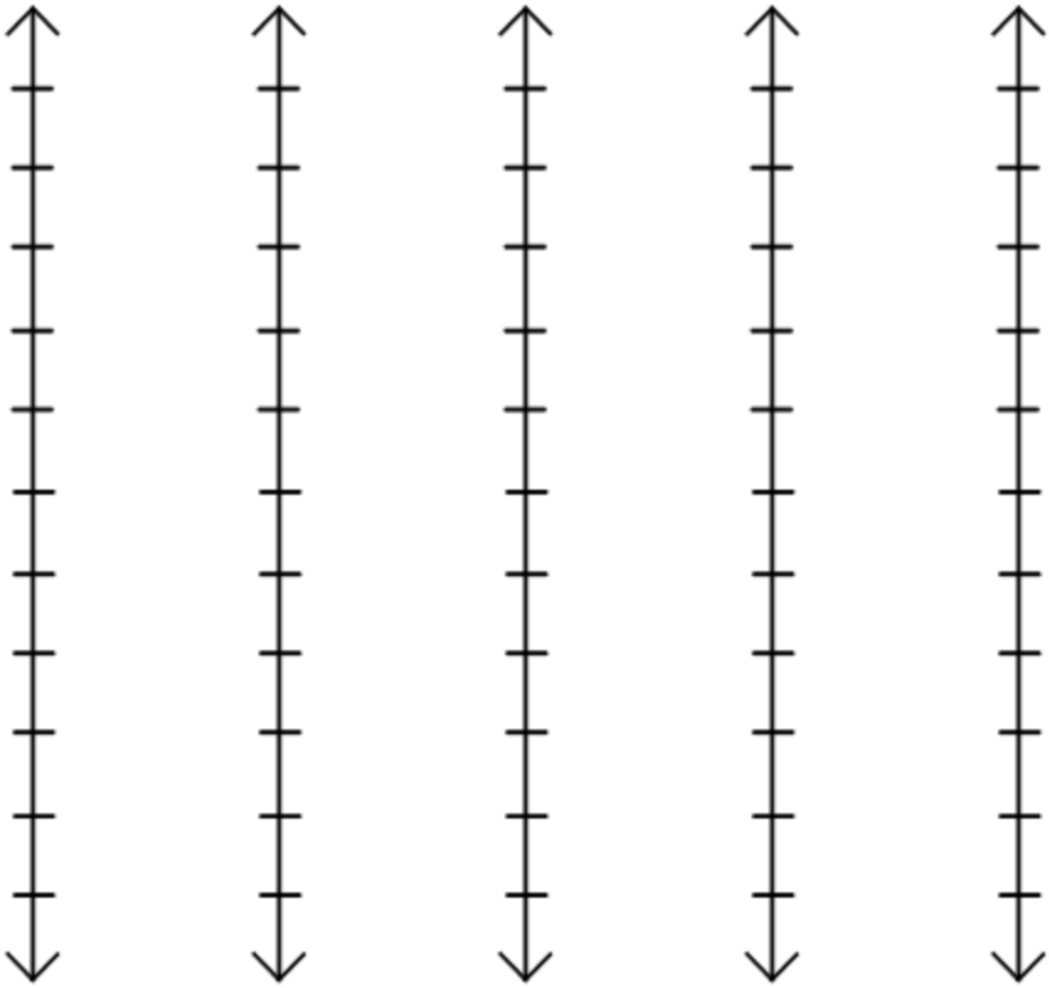


Appendix B. The Coordinate System/Plane



Appendix C. Denver Zoo Map

Appendix D. Number Line (Vertical)



Citation

Osborne, M., Tudor, M., & Gundlach, M. (2023). Plotting Directions. In A. Bicer (Ed.), *Creativity-Directed Mathematical Tasks for 5th Grade Common Core Classrooms* (pp. 575-584). ISTES Organization.

Task 42 - Traveling through the Park

Aylin S. Carey, Fay Quiroz, Traci Jackson

Mathematical Content Standards

CCSS.Math.Content.5.G.A.1:

Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate).

CCSS.Math.Content.5.G.A.2:

Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.

Supporting Standards

CCSS.Math.Content.6.NS.C.6:

Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.

Mathematical Practice Standards

CCSS.MATH.PRACTICE.MP4

Model with mathematics.

CCSS.MATH.PRACTICE.MP5

Use appropriate tools strategically.

CCSS.MATH.PRACTICE.MP6

Attend to precision.

CCSS.MATH.PRACTICE.MP7

Look for and make use of structure.

Vocabulary

x-coordinate, *y*-coordinate, x-axis, y-axis, perpendicular, parallel, first quadrant, second quadrant, third quadrant, fourth quadrant, ordered pair, origin

Materials

For the *Engagement* piece: Copy of Appendix A for each student (pieces must be cut and placed in bags *prior to the activity* or paper clipped together), plastic bags or paper clips, dividers to go between students; For the *Explore* piece: Copy of Appendix B for each student; For the *Extend* piece: Copy of Appendix C for each student (Note: Depends on the activity whether it's an individual or a group, teachers may decide on the number of copies.), markers, and dry erase lapboards.

Lesson Objective

In this lesson, students will be able to plot points on a coordinate plane. Students will be able to identify the *x*- and *y*-coordinate of a point that is plotted. Students will be provided with tasks to practice plotting points in the first quadrant of the coordinate plane and naming coordinates of points. These tasks will help adapt to plot points with negative coordinates.

The most important objective of this lesson is that students creatively use coordinate planes to show the distance between various locations and explore real world situations by graphing and interpreting the relationship between the coordinate values of points and the position of a location. This lesson will be completed in two-class time.

Engagement

(30 minutes) Bring the class together and ask students, “Have you ever been to a park? Do you have a favorite park?” Then ask, “What are some things you might find at the park?” As students answer, teachers should record these ideas on the board. Students’ answers may vary such as: playground, skate park, basketball court, community garden, lake, graffiti wall, exercise stations and flower beds. As students are describing these features, ask them to explain what each item/place is so that all students may understand. For instance, a student who is describing the skate park might say,

> “A skate park would be fenced off from the grassy area of the park and has different structures made of all concrete and metal. Kids meet here to hang out to do tricks on their bikes, skateboards or skates. All types of skill levels can practice tricks on the stairs, half-pipes, banked ramps, and ledges.”

After students brainstorm and explain the features at a park, teachers can hand out the plastic bags or paperclipped appendix A (see materials section above because these should be done ahead of time). In each bag are two empty cards on which the class may describe two additional features to be added to the park, this is optional for the teacher. If so, the class can decide which two features to add, and students can be given the opportunity to create a drawing and write the name of the feature on the cards before moving on. Next, ask students to think about these newly added features, just discussed, and where they would be located at the park. Have a short discussion about students’ ideas. Take a notice whether students realize that usually not all features are all in one area, but spread out around the entirety of the park.

Introduce the next part of the game to the students. Each student needs their park cards (from Appendix A). Place students in groups of two with their desks facing each other, and a divider in between them so they cannot see one another’s desks. Tell students they will take turns creating a version of their park, while their partner tries to guess the location of the specific places at the park.

For example, student A will take all of their park cards and place them on their desk like they want at their park. Then, student B will make guesses. As student B makes guesses they should use their features to try and recreate their partner’s park. The goal of the game is to see how many features each pair can place correctly. Remind students they will only have one turn each, and if they think any features were placed in the exact location, they must notify the teacher. Teachers should make a table on the board to keep track of each team's correctly placed features that occur during each round. Please note teachers should not give any help to students as they give, or keep track of their clues. Let the students use creative ways to communicate with their partner during this game to generate a need for a system to accurately locate the park features.

An example of how the game could be played is student A might say,

> "The playground is at the top of the park. The community garden is in the middle next to the lake. The skatepark is in the bottom part close to the edge. The exercise station has different machines/movements all over the park. The basketball courts are on the right side in the corner."

After all of the features have been placed by student B, they can lift the divider, and even move their desks side to side if needed, to see how close student B's park is to student A's park. Students should notify the teacher if they think any features are in the exact location. Before students switch, teachers should prompt partners to discuss what they noticed about the location of the features based on the directions that were given.

Additionally, students should discuss how they might plan to improve how they give their directions before switching roles. Students will clear their parks and switch roles to play one more time.

After students have had an opportunity to be both student A and student B, bring the class together for a whole class discussion. To begin, ask students to share their experiences during the game, and what they noticed about giving directions. Observe if any students created unique ways to give the directions to their partners.

For example, students might answer that they connected map directions like north, south, east or west or latitude and longitude. Perhaps other students used markers to draw lines on their lapboards like paths around the park. Compare student responses to the table created by the teacher during the game (see Figure 1). Students may deduce accuracy of located features relates to the various successful strategies employed.

Teams	Correct Features Round 1	Correct Features Round 2	Total Correct Features	Reasons for Results
Team 1	0	3	3	Arguing with the partner.
Team 2	1	7	8	Drew grid lines like a map.
Team 3	1	4	6	Plan for round 2- use directions (N, E, S, W)
Team 4	0	1	2	Partners didn't understand one another. Directions made the places upside down.

Figure 1. A Possible Table created by the Teacher to use during and after the Game

Explore-Explain Part I

(30 minutes) After students share their experiences of the game and compare their map to others, ask students to provide teachers directions to get to the white board from the classroom door (Note: Teachers may decide any location in the classroom such as from windows to the teacher's table, shelf, student A's desk, etc.). The goal is to have students use the words *right*, *left*, *up*, and *down*. For example, *Walk 5 steps up and 7 steps left*. Provide students with opportunities to create different ways to describe directions (i.e. Is there another way to get to my desk from the classroom door?) If students need to move around, count their steps to create a new path, have students explore. Also, encourage students to visualize paths and estimate their steps to strategize and get creative with their paths such as quickest, adventurous, slowest, or safest. After students share their directions, ask if they would modify their map from the previous game. Look for modifications such as using clear and accurate directions.

Now that students have explored determining directions of places through a map game and in the classroom, introduce a coordinate plane on the board and explain how coordinate planes can be useful in seeing the distance between various locations. Explain to students that coordinate points can help them realize which direction to go in and how far one place is from the other. Explain that *x* (*x*-axis) represents going left or right, and *y* (*y*-axis) represents going up and down. Show on the coordinate plane how *x*- and *y*-axis (perpendicular lines) are intersected at zero, and that this point is called the origin. Use math vocabulary such as *x*-axis being a vertical line and *y*-axis being a horizontal line, and their intersection forms a two-dimensional plane called a coordinate plane. Choose one or two examples from students' paths or ask for a new example of a destination, such as 4 left and 6 right. With students' participation, show them the location on the graph (see Figure 2).

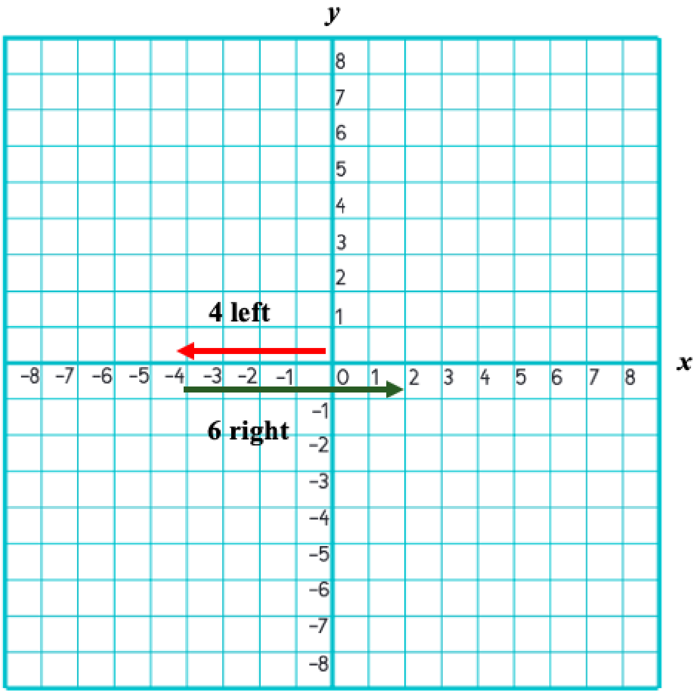


Figure 2. Example of a Destination, 4 Left and 6 Right

Explain how to write coordinates using this format (i.e., What if coordinates included right/left for the *x*-axis and up/down for the *y*-axis: (*x*, *y*)?). Show students the negative numbers in the coordinate plane and mention how left and down have negative numbers. Have students explore the idea of negative numbers and quadrants in the *Extend* part, based on their readiness. Draw a few letters on the graph (see Figure 3) and invite students to come and write the coordinate points in the (*x*, *y*) format for that letter. Also, have students explain to the class how they got that point. For example, for the letter A with the coordinate points (2, 2), listen for an explanation such as 2 units right and 2 units up. Similarly, for the letter B: 1 unit right, 5 units up, C: 5 units right, 1 unit up, and finally for the letter D: 4 units right, 6 units left. To challenge, teachers may ask students to describe the location of the letter C with respect to letter A (i.e., letter C is 3 units right and 1 unit down of the letter A).

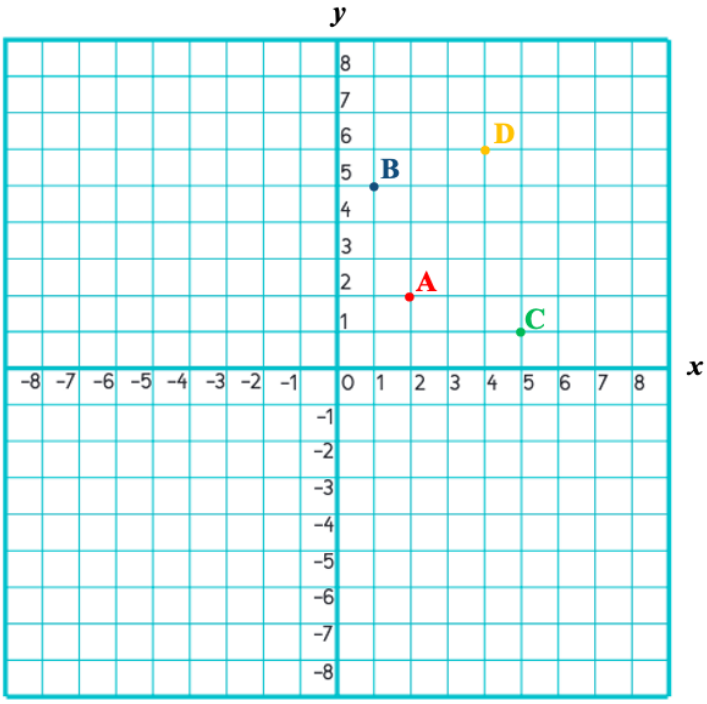


Figure 3. Example of Letter Locations on the Graph

Demonstrate that students can also use coordinate planes to draw shapes. For example, invite the students to place the points (1, 2), (1, 4), (4, 4), and (4, 2) on the graph, using one color marker (see Figure 4). After that, ask students to connect the points and what shape they notice. They should be able to answer a *rectangle*. Then have students create a triangle to check understanding whether they could generate coordinate points that connect to form a triangle (see Figure 4). Drawing shapes on the coordinate planes would help with the second *Explore* piece and interpreting coordinate values of points in the context of the situation.

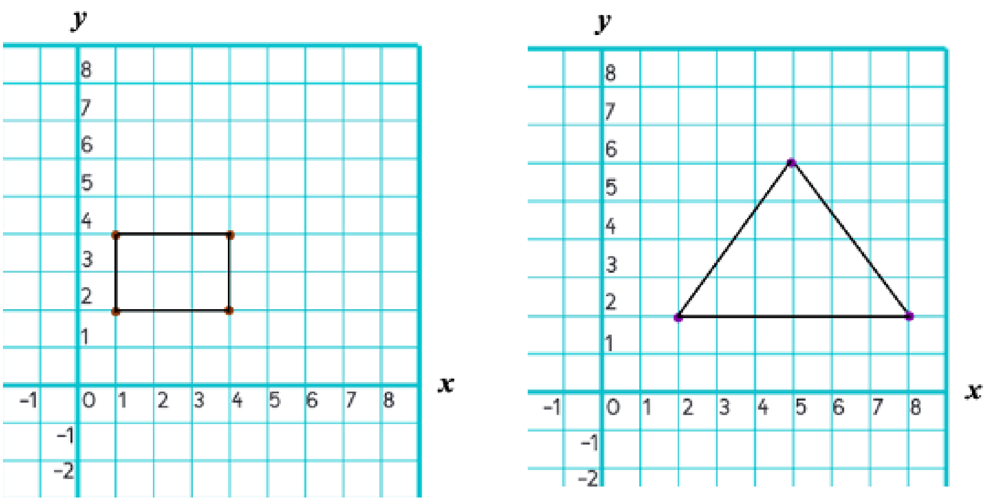


Figure 4. Example of a Rectangle and Possible Student Work of a Triangle

After drawing shapes on the coordinate planes, review perpendicular and parallel lines with students on the graph. Remind students that perpendicular lines cross (or intersect) each other at right angles (90 degree). Letter L would be a good example to share, especially with those who are new to the concept. After that example, mention that parallel lines run in the same direction without crossing each other.

Explore-Explain Part II

(20 minutes) After determining directions of places through a map game and in the classroom, instruct students to write directions they would need to get to the park if they start from the classroom door. Have them start at (0, 0), *origin*, and put a letter consecutively for each movement.

For example, if they walk 5 steps ahead, they should put A on (0, 5) and so on. Once students arrive at the park, direct them to write the coordinate for each letter on the back and have them connect the letters to see their route on a graph.

Now students have a map to the park from their classroom. After finishing their map, ask students to locate the playground (there could be multiple ones for a different age group), skate park, basketball courts, community garden, lake, graffiti wall, flower bed on the Write the Coordinates worksheet (see Appendix B).

The game that students have played during the *Engagement* part now becomes a real world task. Remind them to start at (0,0), the center of the park. The park may not have all the listed items from the game; however, encourage students to observe various places in the park, write the name or include a drawing of the place with the corresponding coordinates on the graph (see Figure 5).

Have students share their map and the worksheet with the class the next day (Note: Teachers may decide how to manage both *Explore* activities to provide students with time to work on the park activity.) Compare students' maps and worksheets and have them explain the location of places (i.e., "I had to go right 7 and up 6 to get to the playground."). Encourage students to ask their classmate questions (i.e., "Instead of going right 7 and up 6, could you go differently to get to the playground?) and have them check for accuracy.

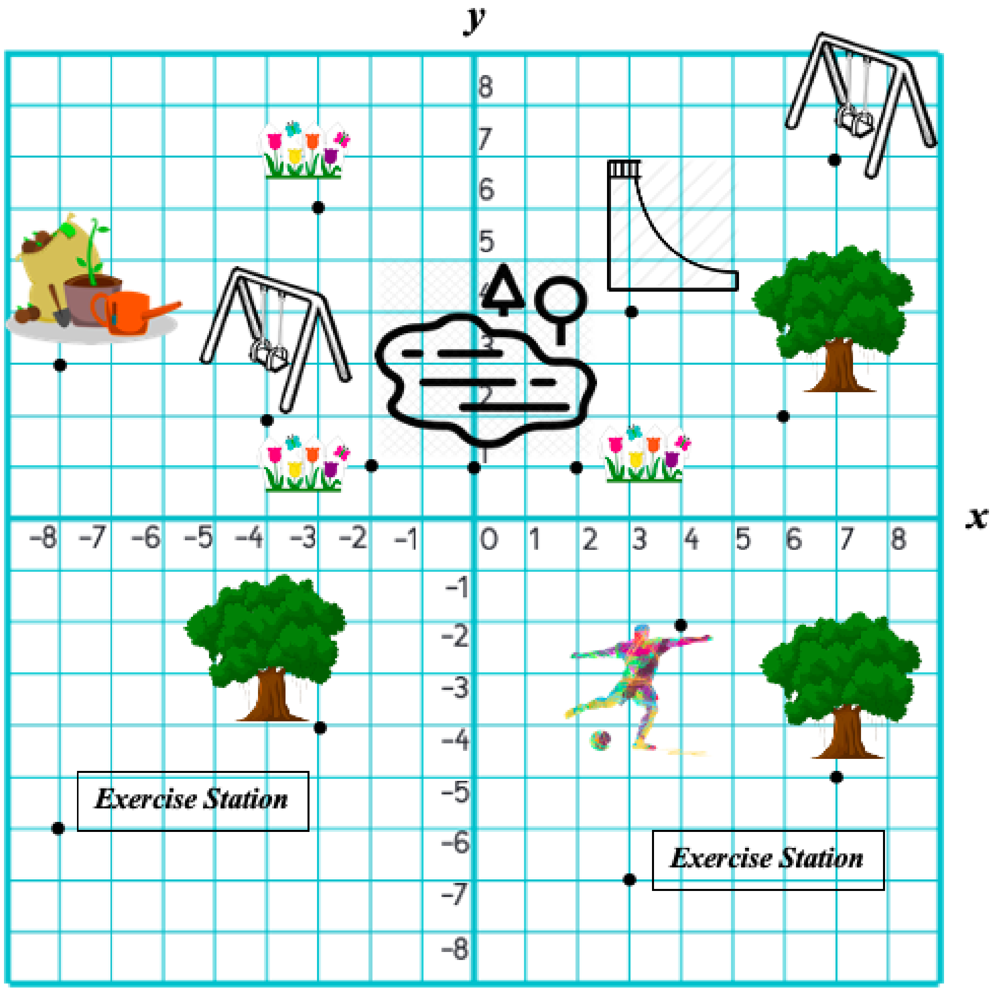


Figure 5. Possible Student Example of the Write the Coordinates Worksheet

Check whether students question the coordinate points for the second, third, and fourth quadrants since these coordinates include negative numbers. While there was a mention of how left and down has negative numbers during the first *Explore* part, students may not recall or have not had the chance to develop understanding.

Remember that negative numbers in the coordinate plane or the quadrants beside the first one have not been introduced to students. However, the park activity and visuals should help students realize that there are other parts to the coordinate plane, and these parts also have coordinate points in the (x, y) format. Ask students how the playground in the second quadrant can be written with coordinate points (Note: Teachers may point to each *quadrant* since they may not be familiar with the vocabulary.) Hopefully, students can write the location with the points (-4, 2) since they have had plenty of exercise with working on the first quadrant. Provide more exercise with the first quadrant if necessary.

Extend

After students have practiced navigating using ordered pairs in the first quadrant, ask how they might represent an object that is to the left of (0, 0). Take all responses, they might include having the left numbers be different colors, have different marking, or that they might represent them as negative numbers. Mention that negative numbers represent the left side of a number line. Ask students how they could represent a place below (0, 0). Students should connect that the *y*-axis is also a number line and below (0, 0) can be represented by a negative *y* coordinate. Have students practice with a partner saying different ordered pairs including negative numbers and placing their finger where the coordinate is (Note: Provide a copy of a coordinate plane or have students draw on their lapboard.) After their practice, let students know that the different sections have different numbers usually represented with Roman Numerals. Remind students that the *quadrant* they have worked during the *Explore* activity is called quadrant I; the one to the left is called quadrant II; below quadrant II is quadrant III; and to the left of quadrant III is quadrant IV (see Figure 6a). Ask a few questions such as "Where are both the *x* and *y* coordinates negative?", "Where is the *x* coordinate negative and the *y* coordinate positive?" (Note: Students can hold up the finger number for the quadrant number for a quick understanding check) (see Figure 6b).

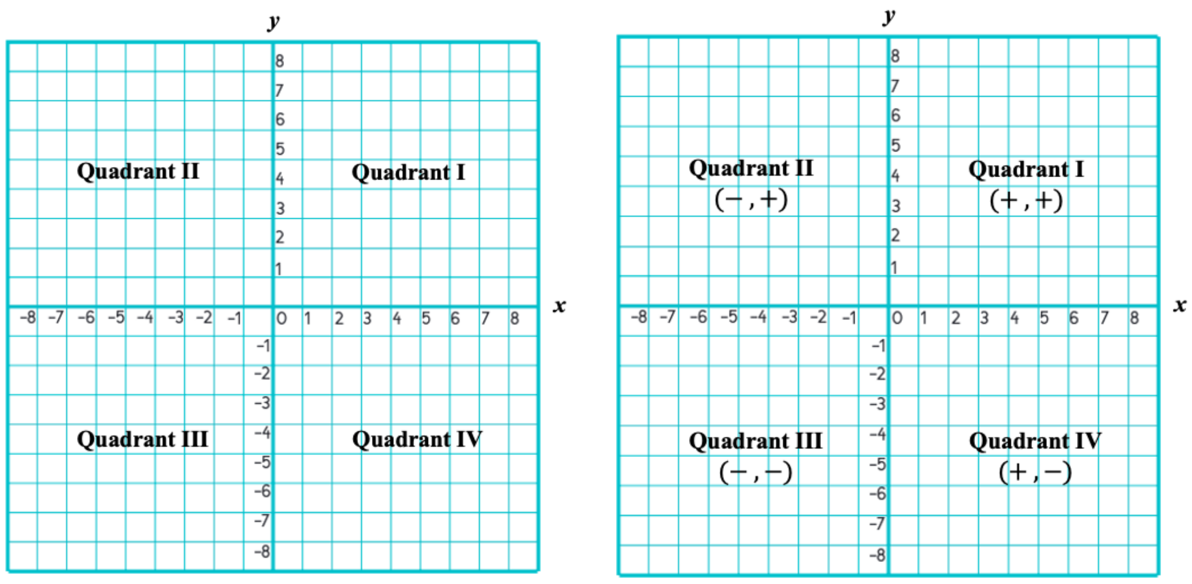


Figure 6a and 6b. Coordinate Plane with Quadrants and x and y Values

Next, Students will use this understanding to place some of the park features on a full coordinate plane with all 4 quadrants. Each student will have copies of the park features (Appendix A), and a large coordinate plane (Appendix C). Place the divider once again between the two students. Student A will choose a feature and place it on their coordinate plane. Student B will then choose another feature, place it on the coordinate plane and describe directions using the coordinate plane (the swing is left 5 and up 7.) Student A will follow the directions, find the location as an ordered pair and ask student B if the ordered pair is correct. Student A will then choose another feature, place it on the plane and describe the location to student B. This can continue for as long as time allows.

Evaluate

Teachers can look for students' understanding of the *x*-axis is being the horizontal line and the *y*-axis is being the vertical line. Students should generalize its similarity to a number line. The origin is zero (0, 0) for the *x*-axis, and moving along the *x*-axis to the right the numbers increase by one in the positive direction. The same is true for the *y*-axis. When beginning at the origin or zero on the *y*-axis, moving upwards increases by one in the positive direction. Teachers should share the vocabulary word *quadrant* with students, and students need to understand how to find places in the first quadrant where both the *x* and *y* axes are positive. Some students with mathematical flexibility may generalize that the numbers continue in the opposite direction along the *x*-axis and *y*-axis in a negative direction.

Once students' realize how the first quadrant is organized, teachers should introduce the term ordered pair, meaning the first number represents the distance from the origin on the *x*-axis, and the second number is separated by a comma, and represents the distance from the origin on the *y*-axis. As students are practicing finding and labeling the ordered pair, teachers should help redirect students who may have misunderstandings about finding the *y*-axis first, or writing the *y*-axis number before the *x*-axis number in the ordered pair. Similarly, students should be given an ordered pair, and be able to locate points on the coordinate grid by starting with the number on the *x*-axis, and then finding the number on the *y*-axis.

Look for students understanding that it is not the space in-between, but the intersection of the two perpendicular lines from the *x* and *y* coordinates that marks the coordinate location.

As students gain familiarity with finding points on the coordinate grid, teachers can ask students to strategize the shortest route or find distance between two points. Students should follow the grid lines from one point to the other by counting how many moves it takes to get from one point to the other.

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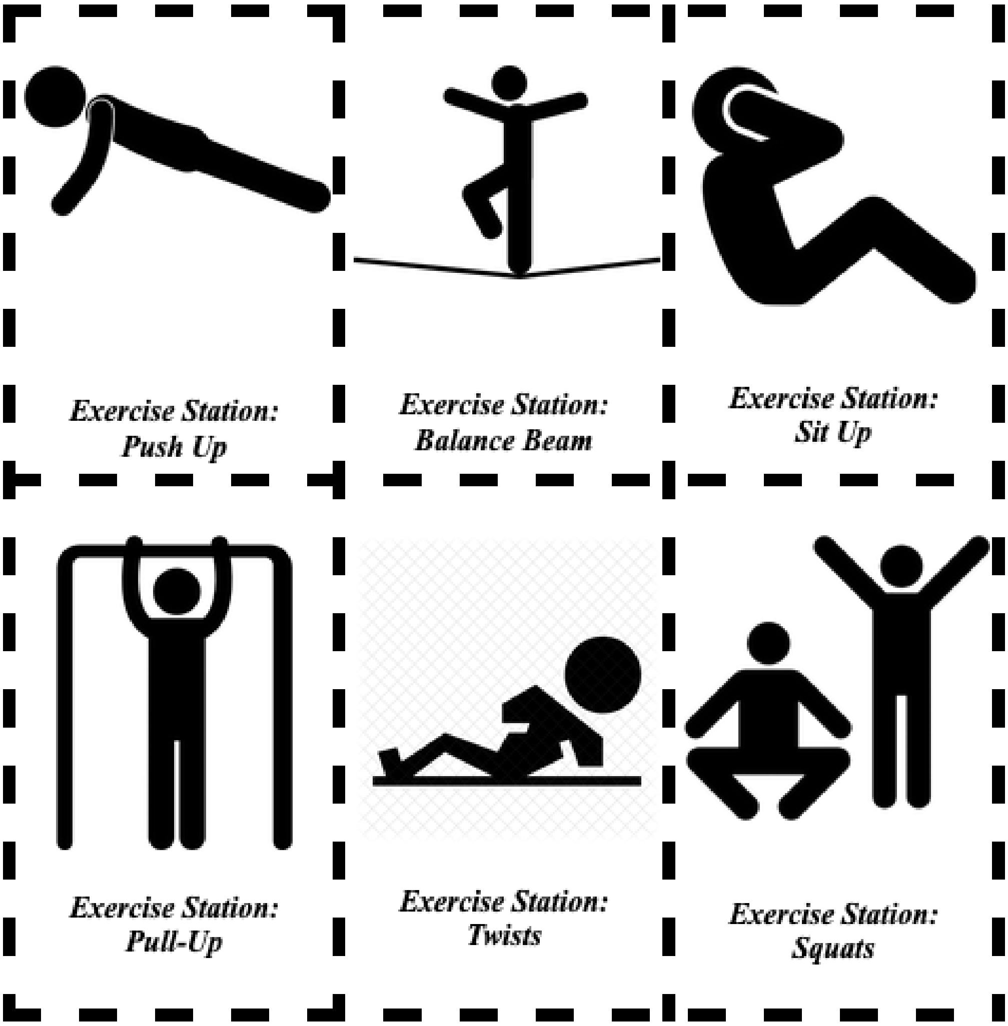
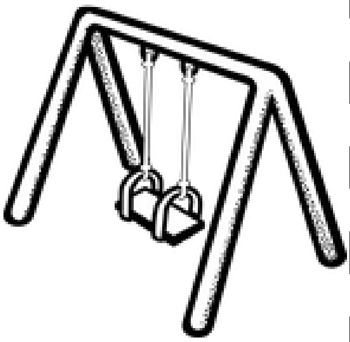
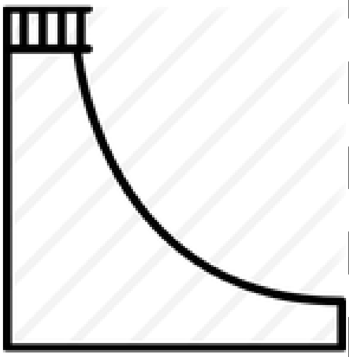
Appendix A. Park Cards

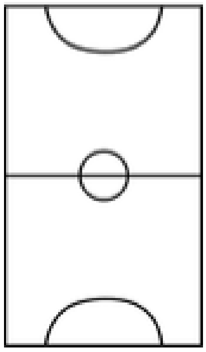




Playground

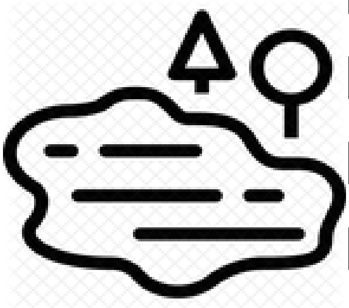


Skate Park



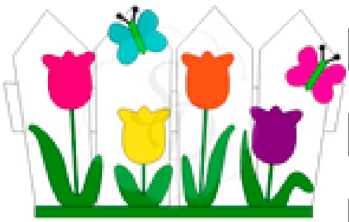
Basketball Courts

Community Garden



Lake

Graffiti Wall



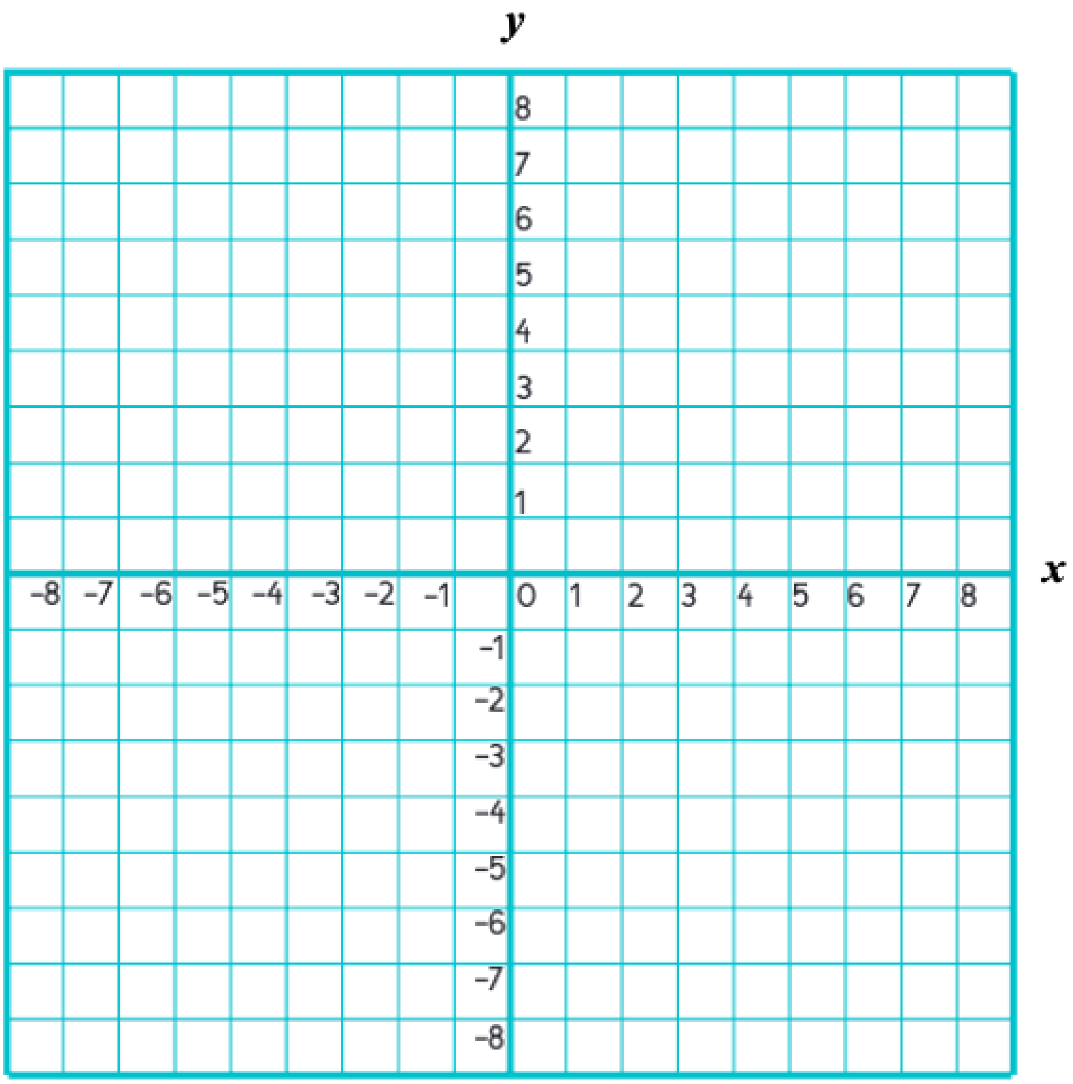
Flower Bed

*Our Idea:*____________

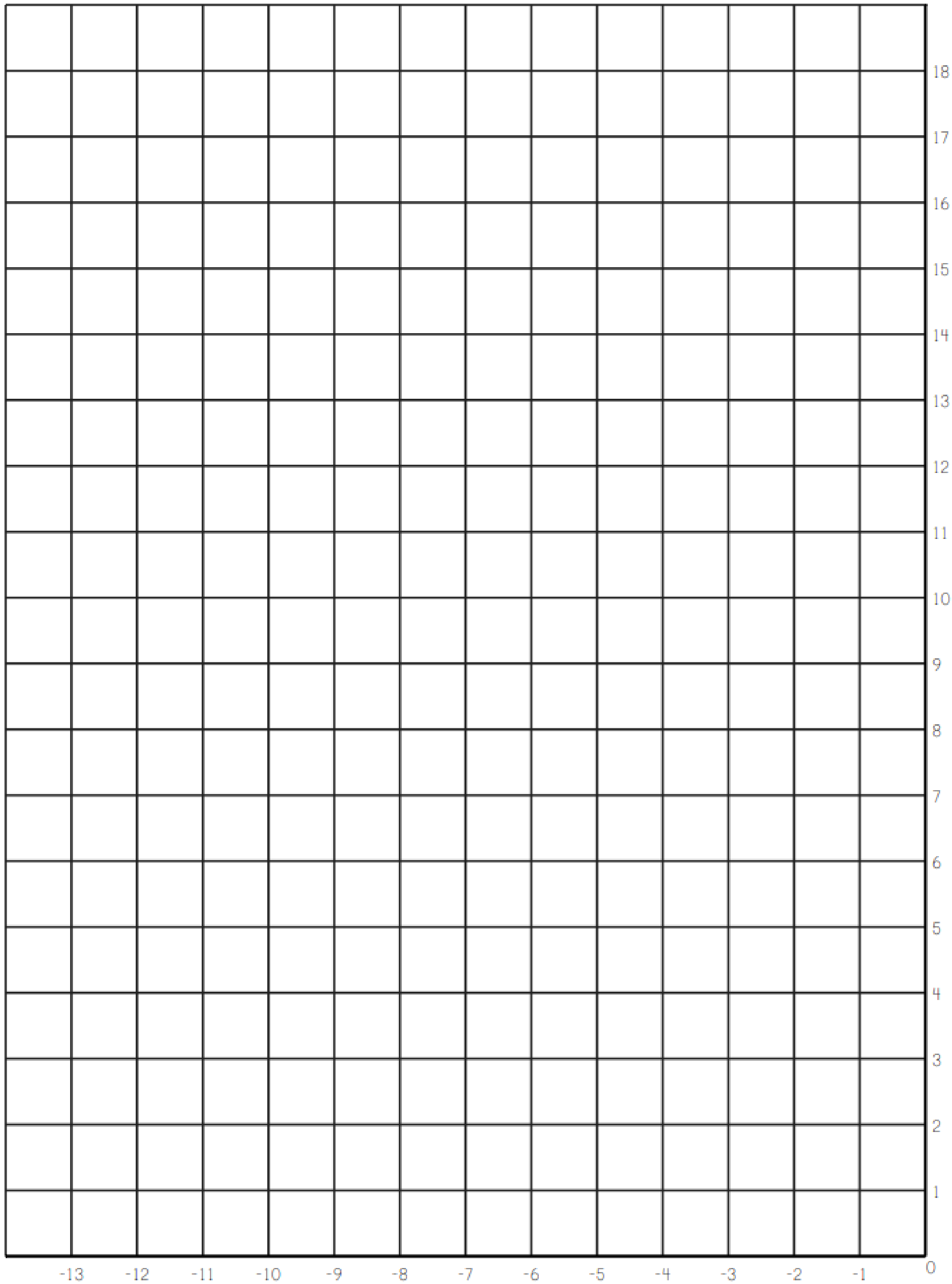
*Our Idea:*____________

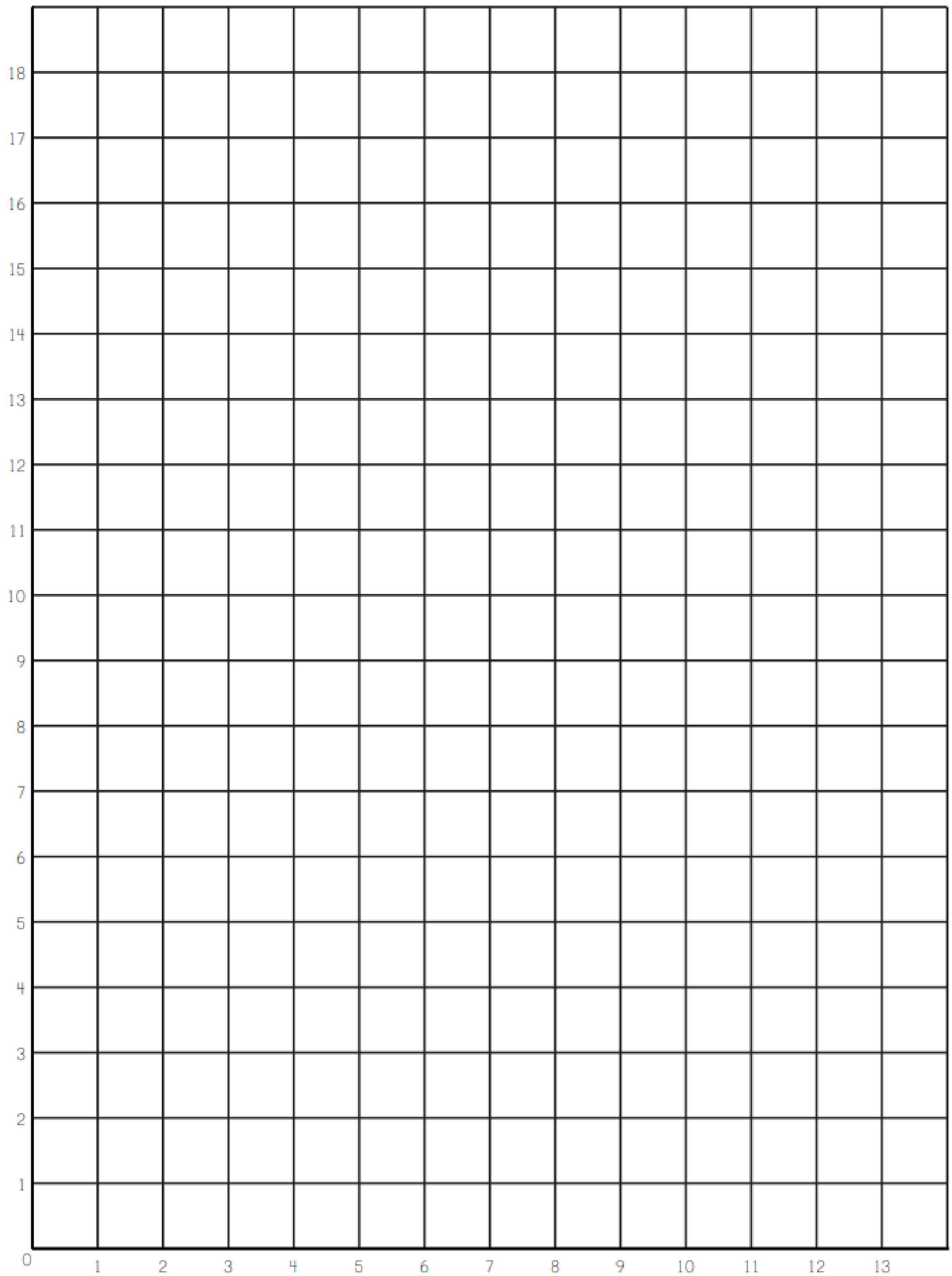
Appendix B

Write the Coordinates



Appendix C. Coordinate System

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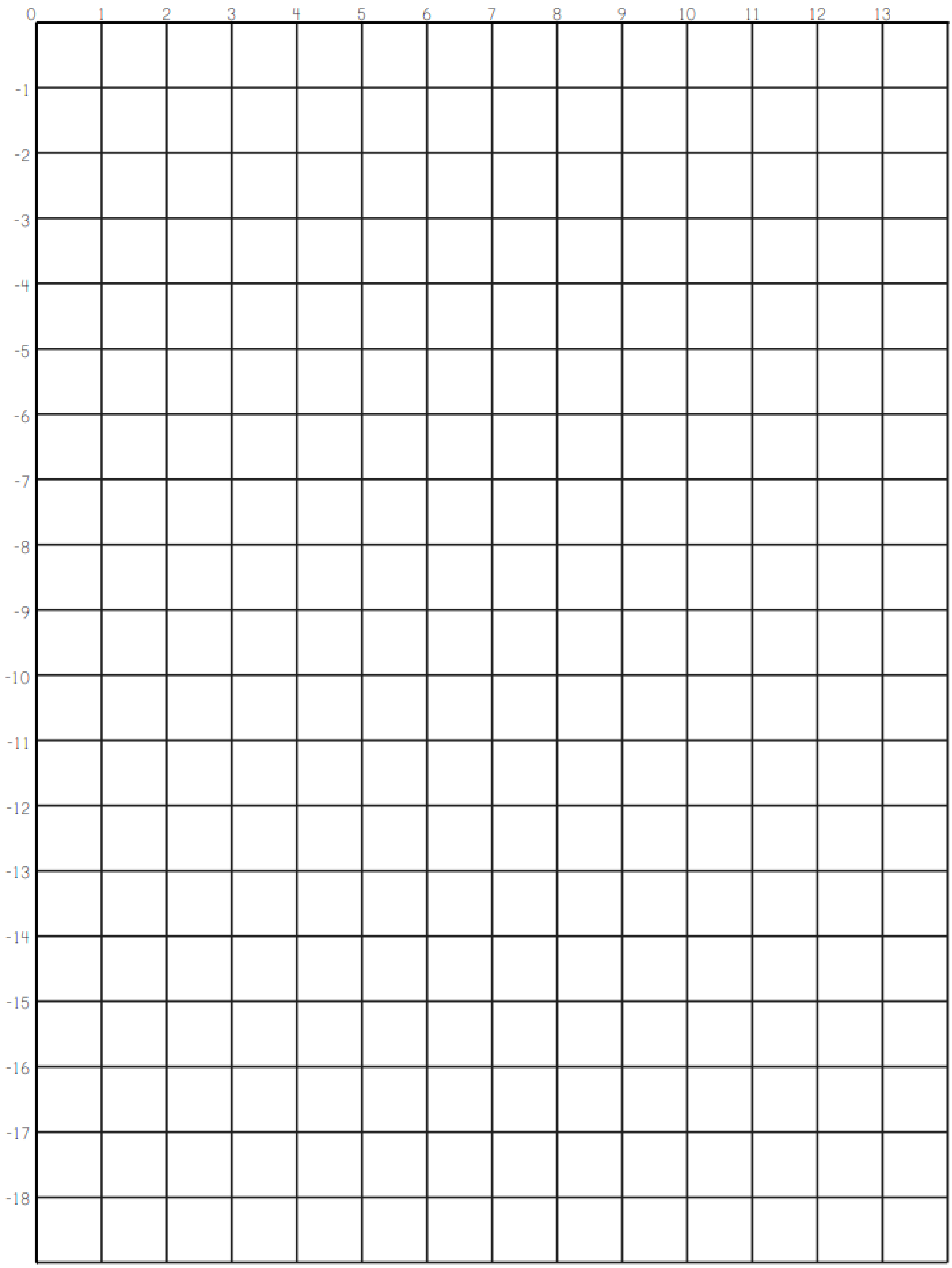
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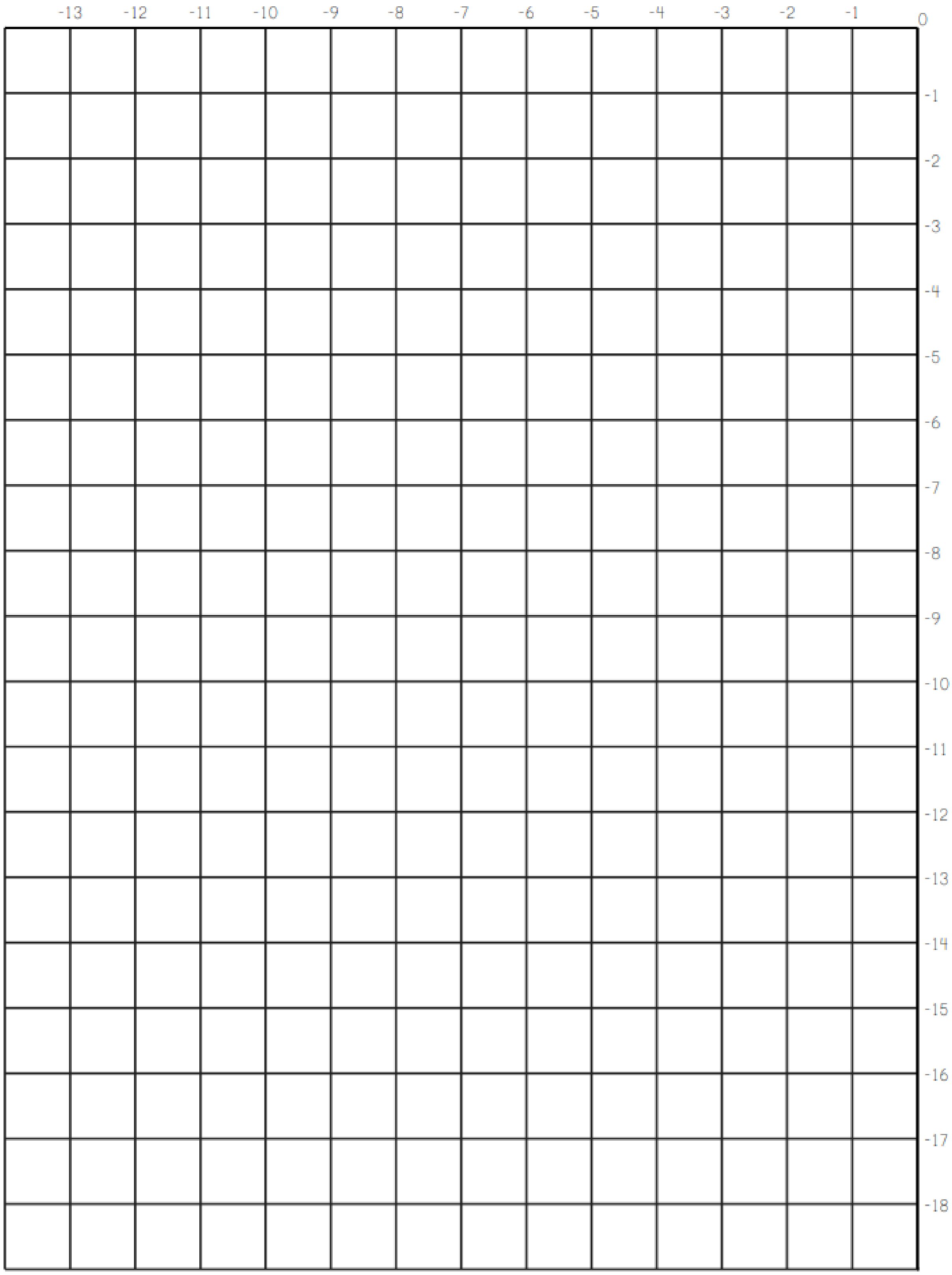
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Citation

Carey, A. S., Quiroz, F., & Jackson, T. (2023). Traveling through the Park. In A. Bicer (Ed.), *Creativity-Directed Mathematical Tasks for 5th Grade Common Core Classrooms* (pp. 585-606). ISTES Organization.

Task 43 - National Mall

Jennifer Kellner, Amy Kassel, Chuck Butler

Mathematical Content Standards

CCSS.MATH.CONTENT.5.G.A.1

Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate).

CCSS.MATH.CONTENT.5.G.A.2

Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane and interpret coordinate values of points in the context of the situation.

Mathematical Practice Standards

CCSS.MATH.PRACTICE.MP4

Model with mathematics.

CCSS.MATH.PRACTICE.MP5

Use appropriate tools strategically.

CCSS.MATH.PRACTICE.MP7

Look for and make use of structure.

Vocabulary

Monuments, memorials, origin, ordered pair, coordinate grid, x-coordinate, y-coordinate, x-axis, y-axis

Lesson Objective

Students will apply their knowledge of the coordinate system to model the monuments and memorials included in the National Mall in Washington, D.C. They will plot points on a graph to represent places on a map. This lesson will promote creative thinking by having students represent real life locations on a map with the coordinate plane. They will also create a tour consisting of several monuments and memorials.

Engagement

(20 minutes) To engage students, the teacher will begin a discussion about important landmarks, monuments, parks, buildings, etc. in their town and/or state. For example, in Wyoming, a few important places and landmarks would be Yellowstone National Park, Grand Teton National Park, and Devil's Tower. Next, the class will discuss Washington, D.C. The teacher will ask the students what they think about when they hear Washington, D.C. Students might say something about government, the Capitol, Lincoln Memorial, George Washington, White House, etc. The teacher will share the following video with students National Mall in Washington (Lonely Planet, 2012). The video describes what monuments and memorials are along the National Mall.

Explore

(30 minutes) Part 1: To provide context for the exploration part of the lesson, teachers may show students the interactive maps located at http://mallhistory.org/map (Roy Rosenzweig Center for History and New Media, n.d.) to show students the progression of the development of the National Mall.

Teachers will share that between 1860 and 1889 five monuments and memorials were added to the National Mall in Washington D.C. (Roy Rosenzweig Center for History and New Media, n.d.). Students will explore the 1860-1889 map of the National Mall indicating the five monuments and memorials that were built during that time period (Appendix A). Students will identify the whole-number coordinates for the three indicated monuments on the map. Students will then identify the location of where they would build two additional monuments or memorials on the mall by plotting and labeling the points.

Part 2: Teachers will share that between 1920 and 1949 nine additional monuments and memorials were added to the National Mall in Washington D.C. (Roy Rosenzweig Center for History and New Media, n.d.). Conduct the whole-class discussion part before asking students to explore the 1920-1949 map.

Ask students to work with a partner to identify and label the coordinates for the marked memorials and monuments. After students have labeled the coordinates, students will explore distances between markers (calculated only through horizontal and vertical travel).

Part 3: Teachers will put students in groups of 2-3 to complete the third activity. Students will work to complete the National Mall Walking Tour Activity (see Appendix C).

Explain

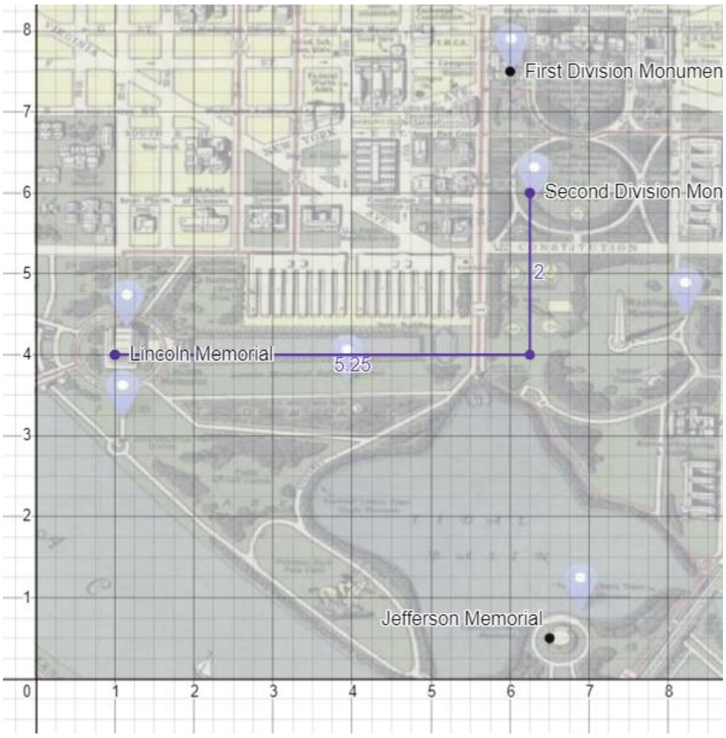
(30 minutes) Part 1: As students are working, teachers may need to support students with vocabulary such as origin, ordered pair, coordinate grid, x-coordinate, y-coordinate, x-axis, and y-axis. The x- and y-axes are just number lines the students have used in the past. The x-axis runs horizontally and the y-axis is vertical. The point where the two axes meet is called the origin, which has the ordered pair (0, 0).

Students may struggle with remembering how to write or plot the ordered pairs. Teachers may provide support by giving students an ordered pair such as $(5,2)$ and asking students to indicate where that ordered pair is graphed. If the students graph $(2,5)$ instead, the teacher may remind students that these are different points, showing students where both points are located on the coordinate grid.

Teachers may then ask students if these are the same points. Students will say they are different points. Teachers may then ask students what is important in determining how to plot an ordered pair. If students are still struggling, teachers may ask which coordinate is the x-coordinate and which axis is the x-axis, and similarly for the y-coordinate and y-axis. Students should recognize that the x-coordinate measures distance horizontally along the x-axis from the origin and that the y-coordinate measures distance vertically along the y-axis from the origin. Students may also struggle to count or plot the plots in the spaces rather than on the intersections. Teachers may show struggling students a ruler and point out that the measurements on a ruler are shown where the lines are marked. After students have written the coordinates and plotted and labeled the position of the monuments they would add, teachers may ask students to share their work on the classroom display of the map and coordinate grid. Students will plot and label the ordered pair for one of their monuments and share its name.

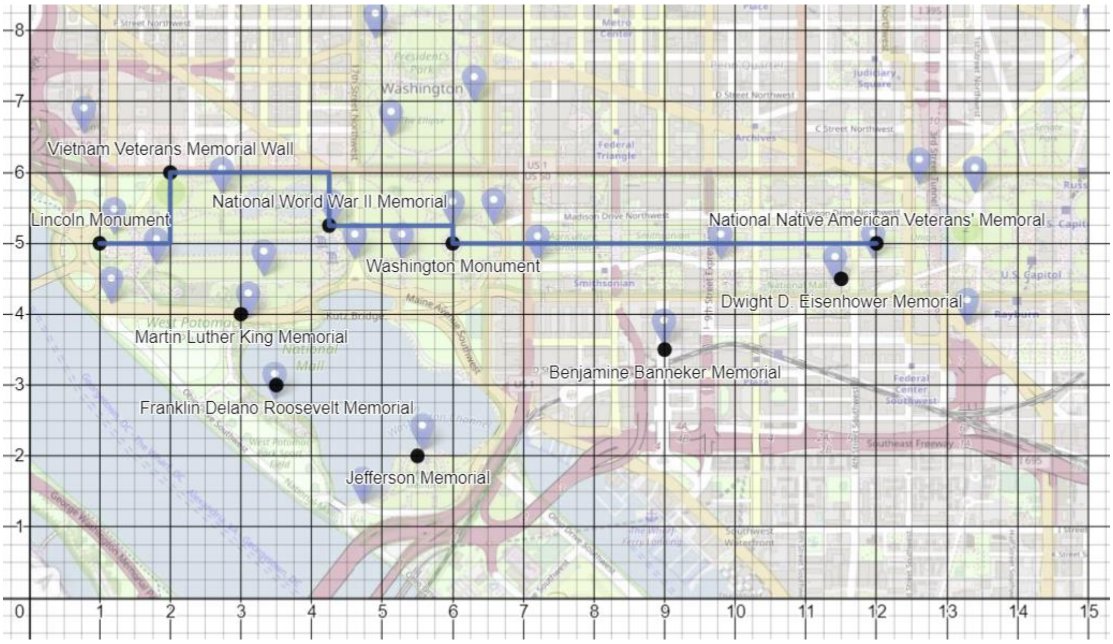
Part 2: Teachers will share the historic 1920-1949 National Mall map with the coordinate grid on the classroom display and give each student a copy of the 1920-1949 Map Handout (Appendix B). Using a whole-class discussion format, students and teachers will label the origin, x-axis, and y-axis. Teachers will ask students what they may notice is different between the first map and the second map. The student may share that there are more monuments or the coloring is different. If students do not observe that there minor gridlines are on the coordinate grid or that the plotted points are not on the intersection of the major gridlines, teachers may ask students specifically what they observe about how the axes are marked or where the points are plotted. Teachers will ask students what they observe about how the axes are labeled. If students are struggling, teachers may remind them how rulers are labeled. Students should recognize that the axes have minor gridlines marked every $\frac{1}{4}$ unit. Teachers will ask students how to label the axes. Together they will label the axes for every $\frac{1}{4}$ unit, i.e $0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 1\frac{1}{4}, 1\frac{1}{2}, 1\frac{3}{4}, 2, \ldots$ for both the x- and y-axes.

As students are labeling the ordered pairs for the five marked monuments and memorials, teachers should monitor student progress and support struggling students. Teachers may need to support students in remembering that ordered pairs are written (x, y). Teachers may also need to support students in recognizing the x-coordinate represents the horizontal measurement from the origin and the y-coordinate represents the vertical measurement from the origin. Teachers will explain how to find the distance between two points by modeling the horizontal and vertical distances between two points. Students may ask if they can travel diagonally between points. Teachers may explain, that yes, calculating diagonal distances is possible as the students learn more mathematics, but for this lesson, we will travel along the grid moving only horizontally and vertically. For example, teachers may choose to model this by selecting the Lincoln Memorial and the Second Division Monument. The teacher may highlight the horizontal distance from $(1,4)$ to $(6\frac{1}{4},4)$ and the vertical distance from $(6\frac{1}{4},4)$ to $(6\frac{1}{4},6)$ and write the distances for each length $(5\frac{1}{4}+2)$ for a total of $7\frac{1}{4}$ units.



Sample student response for distance between Lincoln Memorial and Ulysses S. Grant Memorial. Total Distance $16+2.5=18.5$ units.

Part 3: Teachers will monitor student progress. If students are struggling, they may use similar questioning as in Parts 1 and 2 to support students. Teachers may need to remind students to write in the fractional units along the axes, to label the ordered pairs, or to show the horizontal and vertical distances. Sample student response to visit the Lincoln Monument, Vietnam Veterans Memorial Wall, National World War II Memorial, Washington Monument, and the National Native American Veterans'' Memorial for a total of 13 units:



Teachers may have students share their tour stories and have them compare who walked the longest distance, who walked the shortest distance, who toured the greatest number of monuments/memorials, which monument/memorial was visited by the most/least number of students.

Extend

The teacher may choose to assign one extension to the whole class or assign different extensions to different groups based on interest, ability, and readiness.

Where could they be – At the end of part 1, the teacher tells the students that their friend is 3 coordinates away from a monument. Where could they be?

As the Crow Flies – At the end of part 1, the teacher tells the students a bird flew from the Peace Monument to the Washington Monument. Estimate how far the bird flew.

How Much – At the end of part 1 or part 2, the students estimate how much space on the coordinate plane they have available to build the monuments.

By Boat or By Car – At the end of part 2, the teacher can tell students that two people are approaching the National Mall, by boat and by car. Pick a location for each and describe that location using ordered pairs and words. Students can also give directions to the boat and car from specific monuments.

Smart or Lazy – At the end of part 3, you are located at (3,5) and want to see three monuments with the least number of steps. Describe your path and defend your claim that it is the fastest route.

What If – What if something was built at (-1, -4). Find the location and then describe what it could be.

Evaluate

- In part 1, formative assessment for content and creativity occurs as students are plotting the locations of the monuments. If a student struggles to name the location of the monument, the teacher can put a point at the location of the student's ordered pair and then ask, "What do you think could be at this location?"
- In part 2, formative assessment for content occurs as students are finding the distance between any two monuments.
- In part 3, formative assessment for content and creativity occurs as students are selecting their monuments, labeling their coordinates, calculating the total distance, and telling the story of their trip.
- In Where Could They Be, formative assessment for content and creativity occurs as students determine all the locations that are 3 coordinates from a monument. Look for students that connect the four coordinates around the monument to make a circle.
- In As the Crow Flies, formative assessment for content and creativity occurs as students estimate the distance between two monuments. If a student struggles to give an estimate, the teacher can ask students to give a range. For example, I know the distance is more than ___ but less than ___. Students should explain their reasoning.

- In How Much, formative assessment for content and creativity occurs as students find the area of an irregular shape. If a student struggles to find the area of an irregular shape, ask students if they can find the area of an easier shape.
- In By Boat or By Car, formative assessment for content and creativity occurs as students locate two coordinates based on the context of the problem. If a student struggles to contextualize the point (having a boat in a street, for example), then the teacher can have the student explain what their coordinates mean in context.
- In Smart or Lazy, formative assessment for content occurs as students are determining which path is the best option to both see three monuments and minimize the number of steps.
- In What If, formative assessment for content occurs as students are graphing points with negative number coordinates.

References

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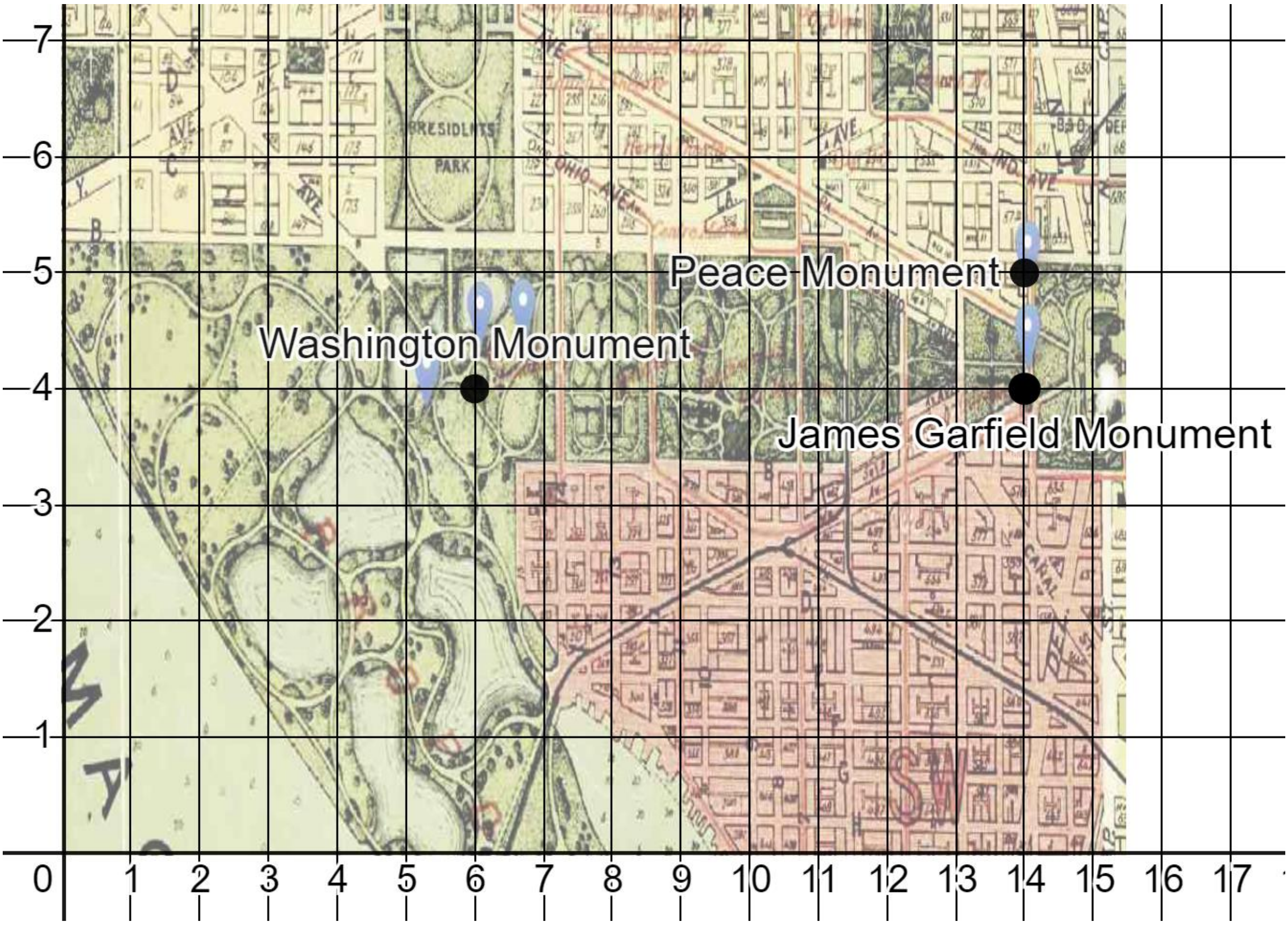
Appendix A. 1860-1889 National Mall Historic Map

Use the graph below to answer the questions.



2. Write the coordinates of the three indicated monuments.

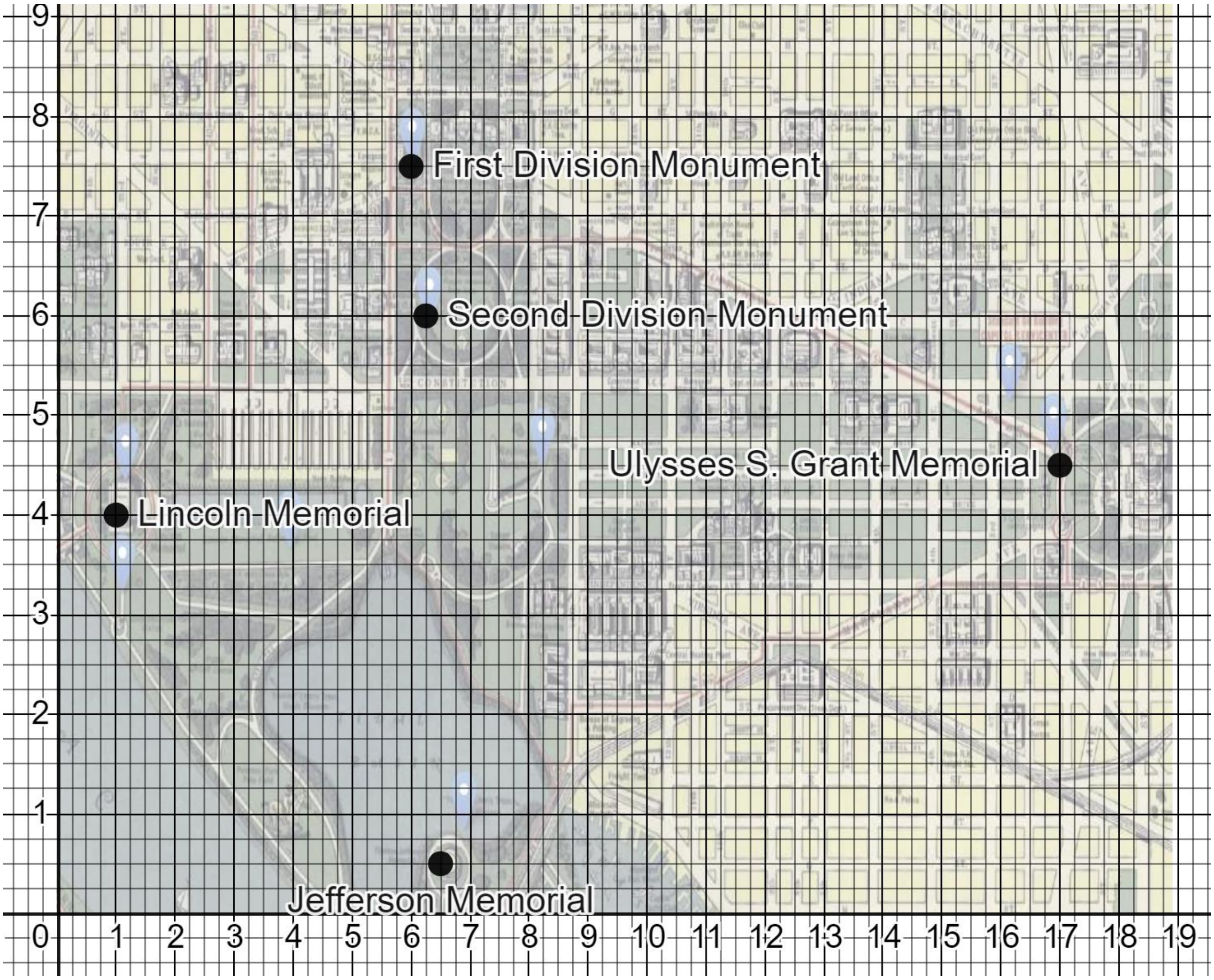
   Washington Monument ________________

   James Garfield Monument ______________

   Peace Monument _____________________

3. Plot and label the points where you think the next two monuments may be built. Write the name of your monuments.

Appendix B. 1920-1949 National Mall Historic Map



1. Write the coordinates of the three indicated monuments.

   Lincoln Memorial ________________ First Division Monument _______________

   Jefferson Memorial _______________ Second Division Monument _____________

   Ulysses S. Grant Memorial _____________________

2. Find the distance between three sets of monuments/memorials. Draw and the label the horizontal and vertical distances on the grid above.

   Distance between ___________________ and _______________________ is ___________.

Distance between ____________________ and ________________________ is ___________.

Distance between ____________________ and ________________________ is ___________.

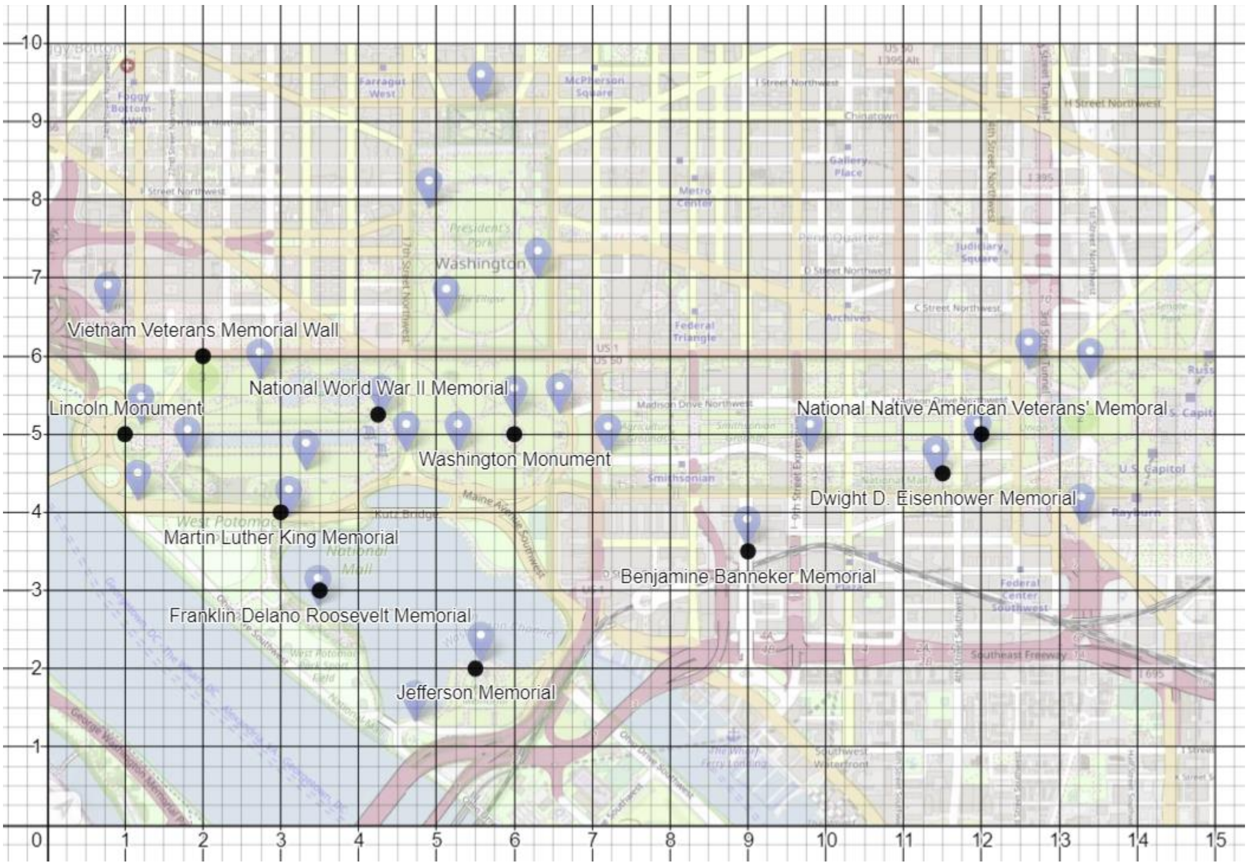
Appendix C. National Mall Walking Tour Activity

Directions: Below is a map of the present-day National Mall in Washington D.C. There are 10 monuments/memorials marked on the map along with blue markers for the other monuments/memorials located on the National Mall. You want to see at least 5 of the monuments/memorials.

Your task is to plan a route that will allow you visit at least 5 monuments/memorials and calculate the distance it will take to walk to see them all. Remember, you are only allowed to walk horizontally and vertically, and you cannot walk through the water.

1. Label the order pairs for your monuments/memorials.
2. Draw the path you will take.
3. Calculate the total distance traveled.
4. Write a story about your tour.

If you wish to visit one of the monuments/memorials not marked on the graph, visit http://mallhistory.org/map to click on the marker to see the name of the monument/memorial.



Citation

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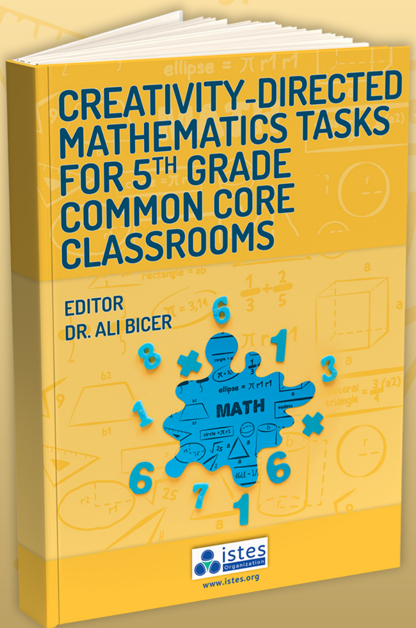
ENGAGE YOUR STUDENTS IN RICH, VISUAL, CREATIVE, HIGH-COGNITIVE DEMAND, LOW FLOOR & HIGH CEILING, OPEN-ENDED MATHEMATICAL TASKS

Creativity-directed tasks aim to develop students' creative thinking skills while they construct new mathematical ideas, concepts, and knowledge. Although research suggests that teachers should implement creativity-directed tasks in their classrooms to foster their students' mathematical creative thinking skills, it is unclear how teachers can implement these tasks while trying to address Common Core Standards. This book includes 42 research-based, Common Core content and practice standards-aligned, visual, creative, low-floor & high ceiling, high-cognitive-demand, and rich open-ended tasks that will enable your students to think deeply and reason mathematically, problem-solve, problem-pose, discuss and convince, explore multiple solutions methods, connect multiple representations, justify their thought processes, and generalize their mathematical observations. The lesson plans encourage students to understand that mathematics is a creative field in which they can be creative! There are 11 chapters addressing 10 Common Core 5th-grade content practices along with Common Core practice standards. This book provides teachers with the lesson plans about the following content standards: 1) write and interpret numerical expressions, 2) analyze patterns and relationships, 3) understand the place value system, 4) perform operations with multi-digit whole numbers with decimals to hundredths, 5) use equivalent fractions as a strategy to add and subtract fractions, 6) apply and extend previous understanding of fraction multiplication, 7) apply and extend previous understanding of fraction division, 8) convert like measurement units within a given measurement system, 9) represent and interpret data, 10) geometric measurement: understand concepts of volume, and 11) graph points on the coordinate plane to solve real-world and mathematical problems.



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